$\Delta\varphi(\vec r)=-\frac{\rho}{\varepsilon_0}$

$\varphi(\vec r)=\frac{1}{4\pi\varepsilon_0}\int_V\frac{\rho(\vec r\,')}{|\vec r-\vec r\,'|}\,d\vec r\,'$

$\vec E(\vec r)=\frac{1}{4\pi\varepsilon_0}\int\rho(\vec r\,')\frac{\vec r-\vec r\,'}{|\vec r-\vec r\,'|^3}d\vec r\,'$

Punktl.

$\Delta\varphi(\vec r)=-\frac{\delta(\vec r-\vec r_0)}{\varepsilon_0}q$

$\varphi(\vec r)=\frac{q}{4\pi\varepsilon_0}\frac{1}{|\vec r-\vec r_0|}$

Vollkugel

$\vec E(r)=\frac{Q}{4\pi\varepsilon_0}\cdot\vec e_r\begin{cases}\frac{r}{R^3}, & r\le R\\ \frac{1}{r^2}, & r>R\end{cases}$

$\phi(r)=\frac{Q}{4\pi\varepsilon_0}\begin{cases}\frac{1}{2r}(3-(\frac{r}{R})^2), & r\le R\\ \frac{1}{r}, & r>R\end{cases}$

$\int_V\rho(\vec r)\,d\vec r\,'=\oint_{\partial V}\vec E(\vec r)\cdot d\vec a$

$\vec p=4\pi\varepsilon_0R^3\vec E_0$

$\Rightarrow\varphi(\vec r:r>R)=-\vec r\cdot\vec E_0(1-\frac{R^3}{r^3})$

Gerade

$\phi(r)=\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r}{R}\right)$

$\vec E(r)=\frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}\cdot\vec e_r$

Dipol

$\vec p=q\cdot\vec d\quad(\vec d=\overrightarrow{q_-q_+})$

$\varphi(\vec r)=\frac{1}{4\pi\varepsilon_0}\frac{\vec p\cdot(\vec r-\vec r_0)}{|\vec r-\vec r_0|^3}$

$\vec E(\vec r)=\frac{1}{4\pi\varepsilon_0}\frac{3\vec r(\vec p\vec r)-\vec p r^2}{r^5}$ $\quad\vec r_0=\vec 0$

$W=-\vec p\cdot\vec E\qquad M=\vec p\times\vec E$

$F=p\nabla\vec E$

$W_{p_2}=-\vec p_2\cdot\vec E$

$=\frac{1}{4\pi\varepsilon_0}\left(\frac{\vec p_1\cdot\vec p_2}{r^3}-3\frac{(\vec p_1\cdot\vec r)(\vec p_2\cdot\vec r)}{r^5}\right)$

Dielektr. Halbräume

$q'=q\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\qquad q''=q\frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2}$

Kondensator

$E=\frac{\sigma}{\varepsilon_0}$

Homogen geladene Ebene

$E(z)=\frac{\sigma}{2\varepsilon_0}\,\mathrm{sgn}(z)\,\vec e_z$

Homogen pol. Kugel

$\vec E^P_{aussen}=$ Dipol $\vec p=\frac{4\pi R^3}{3}\vec P$

$\vec E^P_{innen}(\vec r)=-\frac{1}{3\varepsilon_0}\vec P$

Kugel in homogenem Feld

$\vec E^P_{innen}=\frac{1}{1+\chi/3}\vec E=\frac{3}{\varepsilon+2}\vec E$

$\vec P=\varepsilon_0\frac{3(\varepsilon-1)}{\varepsilon+2}\vec E\Rightarrow\vec E^P_{außen}$ Dip.

bzw $\varepsilon_1,\varepsilon_2$

$\Rightarrow\vec E_{innen}=\frac{3\varepsilon_1}{2\varepsilon_1+\varepsilon_2}\vec E$

Kugelförmiger Hohlraum

$\vec E_{innen}=\frac{3\varepsilon}{2\varepsilon+1}\vec E$

Dipol

$\vec A_{dip}(\vec r)=\frac{\mu_0}{4\pi}\frac{\vec m\times\vec r}{|\vec r|^3}\qquad\vec r_0=\vec 0$

$\vec B_{dip}(\vec r)=\frac{\mu_0}{4\pi}\frac{3(\vec m\cdot\vec r)\vec r-\vec m|\vec r|^2}{|\vec r|^5}$

Linienstrom

$\vec B(r)=\frac{\mu_0I}{2\pi r}\vec e_\varphi$

Linienstrom:

$\vec B(\vec r)=-\frac{\mu_0I}{4\pi}\int_C\frac{\vec r-\vec s}{|\vec r-\vec s|^3}\times d\vec s$

Mit Parametrisierung:

$\vec B(\vec r)=-\frac{\mu_0I}{4\pi}\int_a^b\frac{\vec r-\vec s(u)}{|\vec r-\vec s(u)|^3}\times\frac{d\vec s(u)}{du}du$

Zylinder

$\vec E(r)=\vec e_r\begin{cases}\frac{\rho r}{2\varepsilon_0}\\ \frac{\rho R^2}{2\varepsilon_0 r}\end{cases}$

$\varphi(r)=-\frac{\rho}{2\varepsilon_0}\begin{cases}\frac{1}{2}r^2\\ R^2\ln(\sqrt{e}\frac{r}{R})\end{cases}$

Zylindermantel

$\vec E(r)=\frac{\sigma R}{\varepsilon_0}\cdot\vec e_r\begin{cases}0, & r<R\\ \frac{1}{r}, & r\ge R\end{cases}$

$\varphi(r)=-\frac{\sigma R}{\varepsilon_0}\ln(\frac{r}{R})$

Rotierende Kugel

$\vec m=\frac{q\omega R^2}{3}\qquad dA_\theta=2\pi R^2\sin\theta\,d\theta$

Stromdurchflossener Streifen

$\vec B(\vec r)=\frac{\mu_0I}{2\pi a}\begin{pmatrix}\arctan(\frac{x-a}{y})-\arctan(\frac{x}{y})\\ \frac{1}{2}\ln(\frac{x^2+y^2}{(x-a)^2+y^2})\\ 0\end{pmatrix}$

Transformation

$\vec r\,'=\gamma(\vec r-\vec Vt)+(\gamma-1)\frac{(\vec r\times\vec V)\times\vec V}{V^2}$

$t'=\gamma\left(t-\frac{\vec r\cdot\vec V}{c^2}\right)$

Ampère

$\int_A\vec j\cdot d\vec a=\oint_{\partial A}\vec H\cdot d\vec s$

Zwei leitende Kugeln C_1,C_2

$C_{11}\approx C_1\left[1+\frac{C_1C_2}{(4\pi\varepsilon_0r)^2}\right]\qquad C_{12}\approx-\frac{C_1C_2}{4\pi\varepsilon_0r}$

$C_{22}\approx C_2\left[1+\frac{C_1C_2}{(4\pi\varepsilon_0r)^2}\right]$

Randbedingungen

Dirichlet $\varphi(\vec r\in\partial G_i)=f_i(\vec r)$

V. Neumann $\vec n\cdot\mathrm{grad}\,\varphi(\vec r\in\partial G_i)=g_i(\vec r)$

$\nabla\varphi|_{x=0}=\frac{\partial\varphi}{\partial x}\Big|_{x=0}=-\frac{\sigma}{\varepsilon_0}$ Leiter-Vak.

$\Rightarrow\frac{\partial\varphi}{\partial y}\Big|_{x=0}=\frac{\partial\varphi}{\partial z}\Big|_{x=0}=0$ Spiegell.

$q'=q\frac{\varepsilon_r-1}{\varepsilon_r+1}$ Dielektr.-Vak.

$p'=p\frac{\varepsilon_r-1}{\varepsilon_r+1}\qquad\alpha'=-\alpha$

Anschlussbedingungen

$\frac{D_A^t}{\varepsilon_A}=\frac{D_B^t}{\varepsilon_B}\qquad\varepsilon_0\varepsilon_AE_A^n=\varepsilon_0\varepsilon_BE_B^n$ Leiter-Vak.

$E_A^t=E_B^t\qquad D_A^n=D_B^n\qquad E_A^n-E_B^n=\frac{\sigma}{\varepsilon_0}$

$E^t=0\qquad D^n=\sigma$ Leiter-Dielektrikum

$\nabla\cdot\vec D=\rho_{frei}\qquad\nabla\cdot\vec E=\frac{1}{\varepsilon_0}[\rho_{frei}(\vec r)+\rho_{pol}(\vec r)]$

Clausius-Mossotti $\alpha=\frac{3\varepsilon_0}{\rho}\frac{\varepsilon-1}{\varepsilon+2}\Rightarrow\vec P=\rho\alpha\vec E,\quad\varepsilon=\frac{3+2\rho\alpha/\varepsilon_0}{3-\rho\alpha/\varepsilon_0}$

Kraft auf Leiter

$\vec F=\oint\frac{\varepsilon_0}{2}\vec E(\vec r)d\vec a$

Dielektr. Halbräume

$\varphi(r)=\frac{q}{2\pi\varepsilon_0(\varepsilon_1+\varepsilon_2)r}$ Punktl. auf Grenze

Spannungstensor

$T_{\alpha\beta}=E_\alpha D_\beta-\frac{1}{2}\delta_{\alpha\beta}\sum_{\gamma=1}^3E_\gamma D_\gamma$

$\frac{d\vec P}{dt}=\int_V\nabla\hat T\,d\vec r=\oint_{\partial V}\hat T\,d\vec a$

Feldstärketensor

$$F^{\mu\nu}=\begin{pmatrix}0 & -E_x/c & -E_y/c & -E_z/c\\ E_x/c & 0 & -B_z & B_y\\ E_y/c & B_z & 0 & -B_x\\ E_z/c & -B_y & B_x & 0\end{pmatrix}$$

Maxwell

$\nabla\cdot\vec E=\frac{\rho}{\varepsilon_0}$ Gauß

$\nabla\cdot\vec B=0$

$\nabla\times\vec E=-\frac{\partial\vec B}{\partial t}$ Faraday

Durchflutung $\nabla\times\vec B=\mu_0\vec j+\varepsilon_0\mu_0\frac{\partial\vec E}{\partial t}$

$\vec D=\varepsilon\varepsilon_0\vec E=\varepsilon_0\vec E+\vec P\qquad\vec P=(\varepsilon-1)\varepsilon_0\vec E$

$\vec H=\frac{1}{\mu\mu_0}\vec B=\frac{1}{\mu_0}\vec B-\vec M\qquad\vec p=\int_V\vec P\,d\vec r$

Materialgleichungen

Kontinuität

$\nabla\cdot\vec j+\frac{\partial\rho}{\partial t}=0$

Ohm

$\vec j=\sigma\vec E\qquad U=\frac{I}{R}$ Leiter

Kirchhoff

$\sum_iI_i=0$ Knoten

$\sum_kU_k=0$ Masche

Leistung

$P=\int_V\vec j(\vec r)\vec E(\vec r)\,d\vec r$ allgemein

$P=\int_V\sigma(\vec r)E^2(\vec r)d\vec r=\int_V\frac{j^2(\vec r)}{\sigma(\vec r)}d\vec r$ Ohmscher Leiter $\quad P=R\cdot I^2=\frac{U^2}{R}=U\cdot I$ Draht

Strom und Stromdichte

$\vec j=nq\vec v\qquad I=\int_A\vec j\,d\vec a\qquad\nabla\cdot\vec j=0$ bei $\frac{\partial\rho}{\partial t}=0.$

$=\rho\vec v$ $\quad\rho$ bewegliche Lad. $\Rightarrow\int_{\partial V}\vec j\,d\vec a=I=0$

$\vec j=\sigma(\vec E+\vec E_{ext})=\sigma(\vec E+\frac{\vec F_{ext}}{q})\quad I=0\Leftrightarrow\vec E=-\vec E_{ext}$

EMK $\quad\mathcal E=-E_{ext}\ell=E\cdot\ell\qquad R_i=\frac{\ell}{\sigma A}$

$U=\mathcal E+IR_i\qquad I=\frac{1}{R_i}(U-\mathcal E)$

Anschlussbedingungen 2 Leiter σ_1,σ_2

$E_1^t=E_2^t\qquad j_1^n=j_2^n$

Widerstände

$R=\frac{\ell}{\sigma\cdot A}$ $\quad$ Reihe $\quad R_{ges}=\sum_iR_i$

Parallel $\quad\frac{1}{R_{ges}}=\sum_i\frac{1}{R_i}$

Joulesche Wärme $\quad W=U\cdot I\cdot\Delta t$

Lineare Schaltkreise

① Knotenregel: $\vec U=U_i\vec e_i$, $U_{ij}=U_i-U_j\quad I_{ij}=\frac{U_{ij}}{R_{ij}}$ passiv $\quad I_{ij}=\frac{U_{ij}-\mathcal E_{ij}}{R_{ij}}$ aktiv

(Knoten 1 als Ursprung wählen mit $U_1=0$)

② Maschenregel (minimal): $\sum_iU_i=\sum_k\mathcal E_k\Rightarrow$ LGS

Elektroden $\sigma\to\infty$

$\Rightarrow\varphi_{el}\approx$ const. $\quad I=\oint_{\partial V}\vec j\,d\vec a=-\oint_{\partial V}\sigma\frac{\partial\varphi}{\partial n}da\qquad\varphi_i=\sum_kR_{ik}I_k$ Widerstands-koeffizienten

Lorentz-Kraft $\quad\vec F=q\vec E+q\vec v\times\vec B$

Multipolentwicklung

$\varphi(\vec r)=\frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}\int_Vd\vec r\,'\rho(\vec r\,')+\frac{1}{r^3}\int_Vd\vec r\,'(\vec r\vec r\,')\rho(\vec r\,')+\frac{1}{2r^5}\int_Vd\vec r\,'[3(\vec r\vec r\,')^2-\vec r^2\vec r\,'^2]\rho(\vec r\,')+\dots\right]$

$\vec p=\int_Vd\vec r\,'\vec r\,'\rho(\vec r\,')\qquad Q_{ij}=\int_Vd\vec r\,'(3x_ix_j-\vec r\,'^2\delta_{ij})\rho(\vec r\,')\Rightarrow\varphi(\vec r)=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}+\frac{\vec p\cdot\vec r}{r^3}+\frac{1}{2r^5}\sum_{i,j=1}^3Q_{ij}x_ix_j+\dots\right]$

Feldenergie

$w(\vec r)=\frac{1}{2}(\vec E\cdot\vec D)=\frac{\varepsilon_0}{2}\vec E^2(\vec r)\Rightarrow W=\int\frac{\vec E(\vec r)\cdot\vec D(\vec r)}{2}d\vec r=\frac{\varepsilon_0}{2}\int\vec E^2(\vec r)d\vec r$ Vakuum

$W=\frac{1}{2}\int\rho(\vec r)\varphi(\vec r)d\vec r$

$\mathcal E=\int_V\frac{\vec E\cdot\vec P}{2}d\vec r=\chi\varepsilon_0\int_V\frac{E^2}{2}d\vec r$ lin. Med.

$U=q\cdot\varphi_{ex}(0)-\vec p\cdot\vec E_{ex}(0)+\frac{1}{6}Q_{ij}\frac{\partial E_i}{\partial x_j}+\dots$ Äußeres Potential

Magnetische Medien

$\vec A_{mat}(\vec r)=\frac{\mu_0}{4\pi}\int_V\frac{1}{|\vec r-\vec r\,'|}\nabla\times\vec M(\vec r\,')d\vec r\,'\qquad\nabla\times[\vec B(\vec r)-\mu_0\vec M(\vec r)]=\mu_0\vec j(\vec r)$

$\vec H(\vec r)=\frac{1}{\mu_0}\vec B(\vec r)-\vec M(\vec r)\qquad\nabla\times\vec H(\vec r)=\vec j_{frei}(\vec r)$

$M=\chi H\Rightarrow\vec B=(1+\chi)\mu_0\vec H=\mu\mu_0\vec H$

Stetigkeitsbedingungen

$H_A^t=H_B^t\qquad B_A^n=B_B^n$

$\mu\sim10^5$: Ferro
$\mu>1$: Para
$\mu<1$: Dia

Potential- und Kapazitätskoeff.

$q_i=\sum_{j=1}^nc_{ij}\varphi_i\qquad\hat s=\hat C^{-1}\qquad\varphi_i=\sum_js_{ij}q_j$

$\varphi(\vec r)=\sum_j\varphi_i\phi_j(\vec r)\qquad c_{ij}=-\oint_{\partial V_i}\varepsilon(\vec r)\varepsilon_0\nabla\phi_j\cdot d\vec a$

$C_{Kugel}=4\pi\varepsilon_0R$

Dielektr. Halbräume

$C_{Kugel}=2\pi\varepsilon_0(\varepsilon_1+\varepsilon_2)R$

$\sigma_i=\frac{q\varepsilon_i}{2\pi(\varepsilon_1+\varepsilon_2)R^2}$ Kugel

Magnetmoment $\quad m=\frac{1}{2}\int_Vd\vec r\,'(\vec r\,'\times\vec j(\vec r\,'))=\frac{I}{2}\oint_C\vec r\times d\vec s\qquad\vec m=I\cdot\vec a$ planare Stromschleifen

Gyromagnetischer Quotient $\quad\vec m=\frac{q}{2m}\vec L$

Wechselwirkungsenergie $\quad U=\frac{\vec m_1\cdot\vec m_2}{r^3}-\frac{3(\vec m_1\cdot\vec r)(\vec m_2\cdot\vec r)}{r^5}$

Kondensatoren

$Q=CU\Leftrightarrow U=\frac{Q}{C}\Leftrightarrow C=\frac{Q}{U}\qquad W=\frac{1}{2}U^2C=\frac{1}{2}\frac{Q^2}{C}$

$C=\frac{C_{11}+C_{22}+2C_{12}}{C_{11}C_{22}-C_{12}^2}\qquad\boxed{C=\varepsilon_0\varepsilon\frac{A}{d}}$ Platten $\quad=\frac{1}{2}Q\cdot U$

$E=\frac{\sigma}{\varepsilon_0}=\frac{U}{d}=\frac{Q}{\varepsilon A}=\frac{Q}{C\cdot d}\qquad W=\frac{1}{2}\varepsilon_0\varepsilon^2Ad$

Parallelschaltung $\quad$ Flächen addieren $\quad C_{ges}=\sum_iC_i$

Reihenschaltung $\quad$ Abstände addieren $\quad\frac{1}{C_{ges}}=\sum_i\frac{1}{C_i}$

Kraft zw. Stromfäden $\quad\vec F_{12}=\frac{\mu_0}{4\pi}I_1I_2\oint_{C_1}\oint_{C_2}\frac{d\vec l_1\times(d\vec l_2\times\vec r_{12})}{(r_{12})^3}=\frac{\mu_0I_1I_2}{4\pi}\oint_{C_1}\oint_{C_2}(d\vec l_1\cdot d\vec l_2)\frac{\vec r_{12}}{r_{12}}=-\vec F_{21}$

Biot-Savart $\quad\vec B(\vec r)=\frac{\mu_0}{4\pi}\int_Vd\vec r\,'\vec j(\vec r\,')\times\frac{\vec r-\vec r\,'}{|\vec r-\vec r\,'|^3}\qquad\vec A(\vec r)=\frac{\mu_0}{4\pi}\int_Vd\vec r\,'\frac{\vec j(\vec r\,')}{|\vec r-\vec r\,'|}\qquad\vec B(\vec r)=\nabla\times\vec A(\vec r)\qquad\Delta\vec A(\vec r)=-\mu_0\vec j(\vec r)$

$\vec F=\int\vec j(\vec r)\times\vec B(\vec r)d\vec r\approx(\vec m\times\nabla)\times\vec B=\nabla(\vec m\vec B)\qquad U=-\vec m\vec B\qquad\vec M=\int[\vec r\times(\vec j\times\vec B)]d\vec r=\vec m\times\vec B(0)$

Multipolentwicklung $\quad\vec A(\vec r)=\frac{\mu_0}{4\pi}\left[\int d\vec r\,'\frac{\vec j(\vec r\,')(\vec r\cdot\vec r\,')}{|\vec r|^3}+\int d\vec r\,'\vec j(\vec r\,')\frac{3(\vec r\cdot\vec r\,')^2-\vec r^2\vec r\,'^2}{2|\vec r|^5}\right]$

linksdrehend
$\cos()\vec e_x-\sin()\vec e_y$
rechtsdrehend
$\cos()\vec e_x+\sin()\vec e_y$

Komplexe Zahlen $\quad z=x+iy,\ x,y\in\mathbb R\qquad i^2=-1$

$\varphi=\arg(z)=\arctan\left(\frac{y}{x}\right)\qquad z=|z|e^{i\varphi}\qquad z^*=x-iy=|z|e^{-i\varphi}$

$\mathrm{Im}\,z=y\qquad\mathrm{Re}\,z=x\qquad|z|=\sqrt{x^2+y^2}=\sqrt{zz^*}$

$z_1z_2=|z_1||z_2|e^{i(\varphi_1+\varphi_2)}\qquad\frac{z_1}{z_2}=\frac{|z_1|}{|z_2|}e^{i(\varphi_1-\varphi_2)}$

Schwingungsgleichung

$\ddot x+2\gamma\dot x+\omega_0^2x=0\qquad\ddot Q+\frac{L}{R}\dot Q+\frac{1}{LC}Q=0$

$\Rightarrow\omega_0=\frac{1}{\sqrt{LC}}\qquad\gamma=\frac{L}{2R}$

Stern-Dreieck $\quad$ Stern r_i, Dreieck R_{ij}

$r_A=\frac{R_{AB}R_{AC}}{R_{AB}+R_{AC}+R_{BC}}\qquad r_C=\frac{R_{AC}R_{BC}}{R_{AB}+R_{AC}+R_{BC}}\qquad R_{AC}=r_A+r_C+\frac{r_Ar_C}{r_B}$

$r_B=\frac{R_{AB}R_{BC}}{R_{AB}+R_{AC}+R_{BC}}\qquad R_{AB}=r_A+r_B+\frac{r_Ar_B}{r_C}\qquad R_{BC}=r_B+r_C+\frac{r_Br_C}{r_A}$

Mitteln

$\cos^2x\xrightarrow{T}\frac{1}{2}\qquad\cos x\sin x\xrightarrow{T}0$

$\sin^2x\xrightarrow{T}\frac{1}{2}\qquad\cos^3x\xrightarrow{T}0$

$\cos x\xrightarrow{T}0\qquad\sin^3x\xrightarrow{T}0$

$\sin x\xrightarrow{T}0$

Taylor

$\sqrt{1\pm x}=1\pm\frac{x}{2}+\mathcal O(x^2)$

$(1\pm x)^\alpha=1\pm\alpha x+\mathcal O(x^2)$

Additionstheoreme

$\cos(a\pm b)=\cos a\cdot\cos b\mp\sin a\cdot\sin b$

$\sin(a\pm b)=\sin a\cdot\cos b\pm\sin b\cos a$

$\cos(2a)=2\cos^2a-1=1-2\sin^2a=\cos^2a-\sin^2a$

$\sin(2a)=2\sin a\cos a$

$\frac{L}{T} \ll c$ $U_{Ind} = -\dot{\Phi}_B$

Faraday: $\oint_{\partial A} \vec{E} d\vec{l} = -\frac{d}{dt}\int_A \vec{B}\cdot d\vec{a} = \oint_{\partial A} \dot{\vec{A}} \cdot d\vec{l}$

Energie des Magnetfeldes

$w = \frac{1}{2}\vec{H}\cdot\vec{B}$ $W = \frac{1}{2}\int_V \vec{H}\cdot\vec{B}\, d\vec{r}$

$W = \frac{1}{2}\sum_{ik} L_{ik} I_i I_k = \frac{1}{2}\vec{I}(\mathcal{L}\vec{I})$

Wechselstromlehre

U_0, $U(t)$, T, $\nu = \frac{1}{T}$, $\omega = 2\pi\cdot\nu = \frac{2\pi}{T}$

$U(t) = U_0 \cos(\omega t)$ harmonisch

$\Rightarrow U_{eff} = \frac{U_0}{\sqrt{2}}$ $I_{eff} = \frac{I_0}{\sqrt{2}}$ $\bar{P} = \frac{1}{2}U_0 I_0$

Wechselstromleistung

$\bar{P} = \frac{1}{T}\int_0^T U(t) I(t) dt = \frac{U_0 I_0}{2}\cos\varphi = U_{eff} I_{eff} \cdot \cos\varphi$

rein aktiv: $\bar{P} = U_{eff} I_{eff}$ | rein reaktiv: $\bar{P} = 0$

$P(t) = \frac{1}{2}\mathrm{Re}(I^*(t)\, U(t))$

Ohm: $I(t) = \frac{U_0}{R}\cos(\omega t)$ $P = \frac{U_0 I_0}{2}$ $\bar{P} = \frac{U_{eff}^2}{R} = R\cdot I_{eff}^2 = U_{eff}\cdot I_{eff}$

Effektivwerte: $U_{eff} = \sqrt{\overline{U^2}}$ $I_{eff} = \sqrt{\overline{I^2}}$

Komplexe Spannung / Strom

$U(t) = U_0 e^{i\omega t}$ $I(t) = I_0 e^{i(\omega t - \varphi)}$

Impedanzen

$Z = Z(\omega) = \frac{U(t)}{I(t)} = \frac{U_0}{I_0} e^{i\varphi}$ $\tan\varphi = \frac{\mathrm{Im} Z}{\mathrm{Re} Z}$

Ohmscher Widerstand: $Z = R$

Spule / Induktivität: $Z = i\omega L$

Kondens./Kapazität: $Z = \frac{1}{i\omega C}$

Gegeninduktivität und Induktivitätskoeffizienten

$\Phi_j = \sum_{m\neq j} L_{jm} I_m$ $L_{jm} = L_{mj} = \frac{\mu\mu_0}{4\pi}\oint_{C_j}\oint_{C_m} \frac{1}{|\vec{r}(l_j) - \vec{r}'(l_m)|} d\vec{l}_m\, d\vec{l}_j$

$U_j(t) = -\sum_{m\neq j} L_{jm}\dot{I}_m$ $-\sqrt{L_1 L_2} \le L_{12} \le +\sqrt{L_1 L_2}$

Solenoidale Spule

$H = n\cdot I$ $U = -\mu\mu_0\pi r^2 \frac{N^2}{l}\dot{I}$ $L = \mu\mu_0 \pi r^2 \frac{N^2}{l}$ $W = \frac{1}{2}L\cdot I^2$

Maschenregel

$\sum U_i = -\dot{\Phi}_m$

Schwingkreis $IR + \frac{Q}{C} = U_e - L\dot{I}$

Transformator $U_e - L_{11}\dot{I}_1 - L_{12}\dot{I}_2 = 0$

Einschaltvorgang:

$U_0 = R\cdot I + L\cdot\dot{I}$ $\Rightarrow I(t) = \frac{U_0}{R}(1 - e^{-\frac{t}{\tau}})$ $U_L(t) = U_0\cdot e^{-\frac{t}{\tau}}$ $\tau = \frac{L}{R}$

Transformator:

Ideal: $k=1$ $V_1 = V_2$ N_1, N_2

$k = \frac{L_{12}}{\sqrt{L_1 L_2}}$ $\frac{U_2}{U_1} = \frac{k\sqrt{\frac{L_2}{L_1}}}{1 + i\omega(1-k^2)\frac{L_2}{Z}}$

$-1 \le k \le +1$

Kopplungsstärke $\frac{I_2}{I_1} = \frac{-k\sqrt{\frac{L_1}{L_2}}}{1 + \frac{Z}{i\omega L_2}}$

$L_{12} = \mu_r\mu_0 V\cdot n_{12}^2$ $\sqrt{\frac{L_1}{L_2}} = \frac{N_2}{N_1}$ $\frac{U_2}{U_1} = \frac{N_2}{N_1}$

Maxwellsche Ergänzung

$\mathrm{rot}\,\vec{H} = \vec{j}_{frei} + \underline{\frac{\partial\vec{D}}{\partial t}}$ Verschiebungsstrom

Energiebilanz

$\frac{\partial}{\partial t}\left[\frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B})\right] + \nabla\cdot(\vec{E}\times\vec{H}) = -\vec{j}\cdot\vec{E}$

w: Energiedichte; $\vec{S}$: Energiestromdichte; $\vec{j}\cdot\vec{E}$: Energieproduktionsdichte

$\frac{d}{dt}(W_{Feld} + W_{sonst}) = -\oint_{\partial V} d\vec{a}\cdot\vec{S}$

Impulsbilanz

$\int_V d\vec{r}\left[\rho\vec{E} + \vec{j}\times\vec{B} + \frac{\partial}{\partial t}(\vec{D}\times\vec{B})\right] = \frac{d}{dt}\vec{P}_{mech} + \frac{d}{dt}\vec{P}_{Feld}$

$\vec{P}_{Feld} = \frac{1}{c^2}\vec{S} = \varepsilon\varepsilon_0\mu\mu_0\vec{S}$

$T_{\alpha\beta} = E_\alpha D_\beta + H_\alpha B_\beta - \delta_{\alpha\beta}\frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B})$

$\Rightarrow \frac{d}{dt}P_{ges,\alpha} = \int_V \nabla_\beta T_{\alpha\beta} = \oint_{\partial V} T_\alpha d\vec{a}$ $\Rightarrow \frac{d}{dt}\vec{P}_{ges} = \oint_{\partial V}\hat{T} d\vec{a}$

Potentiale

$\vec{B}(\vec{r},t) = \nabla\times\vec{A}(\vec{r},t)$

$\nabla\times[\vec{E}(\vec{r},t) + \frac{\partial}{\partial t}\vec{A}(\vec{r},t)] = 0$

$\vec{E}(\vec{r},t) = -\nabla\varphi(\vec{r},t) - \frac{\partial\vec{A}}{\partial t}$

$\Delta\varphi + \nabla\cdot\frac{\partial\vec{A}}{\partial t} = -\frac{\rho}{\varepsilon_0}$

$\Box\vec{A} - \nabla(\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial}{\partial t}\varphi) = -\mu_0\vec{j}$

Helmholtz-Theorem

$\forall\vec{V}: \vec{V} = \vec{V}_\rho + \vec{V}_s = -\nabla\varphi + \nabla\times\vec{A}$ $\nabla\cdot\vec{V}_s = 0$ $\nabla\times\vec{V}_\rho = 0$

Stromstärke $\vec{j} = \vec{j}_t + \vec{j}_l$ $\Box\vec{A} = -\mu_0\vec{j}_t$ Coulomb-Eichung

$\vec{j}_t = 0 \Rightarrow \Box\vec{A} = 0$

Div + Rot + Randbed. $\Rightarrow \vec{V}$ eindeutig

Eichungen und Eichinvarianz

$\vec{B} = \nabla\times\vec{A}$ $\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t}$ $\chi(\vec{r},t)$ beliebig

Eichtransformation: $\varphi' = \varphi - \frac{\partial\chi}{\partial t}$ $\vec{A}' = \vec{A} + \mathrm{grad}\,\chi$

Lorenz-Eichung (Wellen): $\mathrm{div}\,\vec{A}' + \frac{1}{c^2}\frac{\partial}{\partial t}\varphi' = 0$

$\Rightarrow \Box\varphi' = -\frac{\rho}{\varepsilon_0}$ $\Box\vec{A}' = -\mu_0\vec{j}$ $\Box\chi = -(\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial}{\partial t}\varphi)$

Coulomb-Eichung: $\Delta\chi = -\nabla\vec{A} \iff \mathrm{div}\,\vec{A}' = 0$

$\Delta\varphi' = -\frac{\rho}{\varepsilon_0}$ $\Box\vec{A} = -\mu_0\vec{j} - \frac{\mu_0}{4\pi}\nabla\int d\vec{r}'\frac{\nabla\cdot\vec{j}(\vec{r}',t)}{|\vec{r} - \vec{r}'|}$

$\varphi'(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int d\vec{r}'\frac{\rho(\vec{r},t)}{|\vec{r} - \vec{r}'|}$

Drehimpulsbilanz

$M_{\alpha\delta} = \sum_{\beta\gamma}\varepsilon_{\alpha\beta\gamma} x_\beta T_{\gamma\delta}$ $\vec{L}_{Feld} = \int d\vec{r}(\vec{r}\times(\vec{D}\times\vec{B}))$

$\frac{d}{dt}(\vec{L}_{mech} + \vec{L}_{Feld}) = \int_V \nabla\hat{M} = \oint_{\partial V}\hat{M}\, d\vec{a}$

M.D.+E.Q.

$A_{EQ} = -i\omega\frac{\mu\mu_0}{24\pi}\left(\frac{1}{r} - ik\right)\frac{e^{ikr}}{r}\left(\vec{Q} + \vec{n}\int d\vec{r}'(r')^2\rho(\vec{r}')\right)$ $\vec{Q} = \hat{Q}\vec{n}$

Strahlungszone

$A_{MD} \simeq -ik\frac{\mu\mu_0}{4\pi}\frac{e^{ikr}}{r}(\vec{n}\times\vec{m})$ $\frac{dP_{MD}}{d\Omega} = \frac{u^{-5}\omega^4 m^2\sin^2\theta}{32\pi^2\mu\mu_0}$

$w = \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{uk^6}{288\pi}(\vec{n}\times\vec{a})^2$ $\frac{dP_{EQ}}{d\Omega} = \frac{u^{-5}\omega^6 Q^2 f(\theta,\varphi)}{1152\pi^2\varepsilon\varepsilon_0}$

Strahlung

$A_\omega(\vec{r}) = \int d\vec{r}'\frac{\vec{j}_\omega(\vec{r}')\exp(ik|\vec{r} - \vec{r}'|)}{|\vec{r} - \vec{r}'|}$ $k = \frac{\omega}{u}$

$\vec{E}(\vec{r},t) = i\frac{u^2}{\omega}e^{-i\omega t}\left[\nabla\times(\nabla\times\vec{A}(\vec{r}))\right]$

E.D. $\vec{A}(\vec{r}) = \frac{\mu\mu_0}{4\pi}\frac{e^{ikr}}{r}\int_\Omega d\vec{r}'\vec{j}_\omega(\vec{r}') + \frac{\mu\mu_0}{4\pi}\left(\frac{1}{r} - ik\right)\frac{e^{ikr}}{r}\int_\Omega d\vec{r}'\vec{j}_\omega(\vec{r}')(\vec{e}_r\cdot\vec{r}') + \dots$

$\vec{A}_\omega(\vec{r}) = -i\omega\frac{\mu\mu_0}{4\pi}\vec{P}_\omega\frac{e^{ikr}}{r} \Rightarrow \vec{B}_\omega = uk^2\frac{\mu\mu_0}{4\pi}\left(1 - \frac{1}{ikr}\right)\frac{e^{ikr}}{r}\vec{n}\times\vec{P}_\omega$

$\vec{E}_\omega(\vec{r}) = \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{e^{ikr}}{r}\left[k^2(\vec{n}\times\vec{P})\times\vec{n} + \frac{1}{r}\left(\frac{1}{r} - ik\right)(3\vec{n}(\vec{P}\cdot\vec{n}) - \vec{P})\right]$

Strahlungszone: $\frac{dP}{d\Omega} = \frac{1}{32\pi^2\varepsilon\varepsilon_0}u^{-3}\omega^4 P^2\sin^2\theta$

$P_s = \frac{1}{12\pi\varepsilon\varepsilon_0}u^{-3}\omega^4 P^2$

Nahzone: $\vec{E} \approx \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{3\vec{n}(\vec{P}\cdot\vec{n}) - \vec{P}}{r^3}$

$\vec{B} \approx iu\frac{\mu\mu_0}{4\pi}\frac{k}{r^2}(\vec{n}\times\vec{P})$

$k = \frac{\omega}{c}$

Bewegte Ladungen

$\vec{D}_{ret} = \vec{r} - \vec{R}(t_{ret})$ $\vec{n}_{ret} = \frac{\vec{D}_{ret}}{|\vec{D}_{ret}|}$ $K_{ret} = 1 - \frac{\vec{n}_{ret}\vec{V}_{ret}}{u}$ $\beta_{ret} = \frac{\vec{V}(t_{ret})}{u}$

$\varphi(\vec{r},t) = \frac{q}{4\pi\varepsilon\varepsilon_0|\vec{D}_{ret}|K_{ret}}$

$\vec{A}(\vec{r},t) = \frac{q\mu\mu_0}{4\pi}\frac{\vec{v}(t_{ret})}{|\vec{D}_{ret}|K_{ret}}$

v=const.: $\varphi(\vec{r},t) = \frac{q}{4\pi\varepsilon\varepsilon_0|\vec{r} - \vec{R}(t)|}\frac{1}{\sqrt{1 - v^2/u^2\sin^2\alpha}}$, $\vec{A}(\vec{r},t) = \frac{1}{u^2}\vec{v}(t_{ret})\varphi(\vec{r},t)$

$\frac{dP_s}{d\Omega} = \frac{q^2\mu\mu_0}{16\pi^2 u}\frac{[\vec{n}_{ret}\times((\vec{n}_{ret} - \vec{\beta}_{ret})\times\vec{a}_{ret})]^2}{(K_{ret})^5} \underset{\beta\parallel a}{=} \frac{q^2\mu\mu_0 a_{ret}^2}{16\pi^2 u}\frac{\sin^2\theta}{(1 - \beta_{ret}\cos\theta)^5}$

Wellen

$\Box\vec{A} = -\mu_0\vec{j}$ $\Box\varphi = -\frac{\rho}{\varepsilon_0}$ $\Box A_i = 0$ $\Box\varphi = 0$ Vakuum $\Box\vec{E} = \mu_0\frac{\partial\vec{j}}{\partial t} + \frac{1}{\varepsilon_0}\nabla\rho$ $\Box\vec{B} = -\mu_0(\nabla\times\vec{j})$

Ebene Wellen

$\vec{E} = \vec{E}_0\exp(i(\vec{k}\cdot\vec{r} - \omega t))$

$\vec{B} = \vec{B}_0\exp(i(\vec{k}\cdot\vec{r} - \omega t))$

$\vec{E}_0 \perp \vec{k} \perp \vec{B}_0$ $\vec{k}\times\vec{B}_0 = -\frac{\omega}{c^2}\vec{E}_0$

Polarisation

Allgemein: $\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = 1$ $a = b$: zirkular, $\delta = \pm\frac{\pi}{2}$

δ Phase zw. $E_{0,x}, E_{0,y}$ $\vec{E} = E_0[\cos(kz - \omega t + \phi)\vec{e}_x \pm \sin(kz - \omega t + \phi)\vec{e}_y]$

$\frac{\partial\alpha}{\partial t} = 0$ für $\delta \in \{0, \pm\pi\}$ (linear polarisiert)

Ellipsengl. $\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} - 2\frac{E_x E_y}{ab}\cos\delta = \sin^2\delta$

Lorentz-Transformation

Boost entlang x-Achse mit $\vec{V}$

$\hat{L} = \begin{pmatrix}\gamma & -\beta\gamma & 0 & 0\\ -\beta\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$ $\gamma = \frac{1}{\sqrt{1 - V^2/c^2}}$ $\beta = \frac{V}{c}$ $\hat{L}^{-1} = \begin{pmatrix}\gamma & \beta\gamma & 0 & 0\\ \beta\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$

Pseudonorm $s^2 = x_0^2 - \vec{r}^2$ (invariant)

$\vec{u}' = \frac{-\vec{V} + \vec{u}_\parallel + \frac{1}{\gamma}\vec{u}_\perp}{1 - \frac{\vec{u}\cdot\vec{V}}{c^2}}$ $\vec{u}_\parallel = \vec{u} - \frac{(\vec{u}\cdot\vec{V})\vec{V}}{V^2}$ $\vec{u}_\perp = \vec{u} - \frac{\vec{V}\times(\vec{u}\times\vec{V})}{V^2}$

Vierervektoren $\underline{X} = (x_0, x_1, x_2, x_3) = (x_0, \vec{r}) = (ct, \vec{r})$ $s^2 = \underline{X}\cdot\underline{X} = g_{ij}x^i x^j$

Metrik $g_{ij} = g^{ji} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$ $\Lambda^i{}_j = \frac{\partial x'^i}{\partial x^j} \Rightarrow A'^i = \Lambda^i{}_j A^j$ $\Lambda^i{}_k\Lambda^k{}_j = \delta^i_j$

$X' = \hat{L}X$

Teilchen im Feld

Lagrange $A^i = (\frac{\varphi}{c}, \vec{A})$ $A_i = (\frac{\varphi}{c}, -\vec{A})$

$\mathcal{L} = -mc^2\sqrt{1 - \frac{v^2}{c^2}} = -\frac{mc^2}{\gamma}$ freies Teilchen Dreierimpuls $p_\alpha = \frac{\partial\mathcal{L}}{\partial v_\alpha} = \frac{mv_\alpha}{\sqrt{1 - v^2/c^2}} = mv_\alpha\gamma$

Viererimpuls $p^i = mv^i\gamma = mcu^i$ $\Rightarrow p^0 = mc\gamma = \frac{H}{c}$ Hamilton $H = E = \vec{p}\vec{v} - \mathcal{L} = mc^2\gamma$

$p^i = (\frac{H}{c}, \vec{P})$ $\Rightarrow H = \sqrt{m^2c^4 + p^2c^2}$ $\mathcal{L}_{nichtrel} = \frac{1}{2}mv^2 - q\varphi + q\vec{v}\cdot\vec{A}$ Bew.gl. $\frac{dp_k}{ds} = qF_{ki}u^i$

$\mathcal{L} = -\frac{mc^2}{\gamma} - qcA_0 + q\vec{A}\cdot\vec{v}$ Kanonischer Impuls $P_\alpha = \frac{\partial\mathcal{L}}{\partial v_\alpha} = mv_\alpha\gamma + qA_\alpha$ $\vec{P} = \vec{p} + q\vec{A}$

$\Rightarrow \vec{H} = \sqrt{m^2c^4 + c^2(\vec{P} - q\vec{A})^2} + q\varphi$ $\xrightarrow{\vec{P}\text{ klein}}$ $\vec{H} = mc^2 + \frac{1}{2m}(\vec{P} - q\vec{A})^2 + q\varphi$

Feldtensor $F_{ki} = A_{i,k} - A_{k,i}$

Minkowski-Raum

Eigenzeit $dt' = \frac{ds}{c} = dt\sqrt{1 - V^2/c^2}$ $\Delta t' = \int_{t_1}^{t_2}\sqrt{1 - V^2/c^2}$

Vierergeschwindigkeit $u^i = \frac{1}{\sqrt{1 - v^2/c^2}}(1, \frac{\vec{v}}{c})$ $u_i = \frac{1}{\sqrt{1 - v^2/c^2}}(1, -\frac{\vec{v}}{c})$

Vierervektoren $A'^i = \Lambda^i{}_j A^j$ Tensoren $T'^{ij} = \Lambda^i{}_i\Lambda^j{}_j T^{ij}$ $\partial^i = (\frac{1}{c}\frac{\partial}{\partial t}, -\nabla)$

Kontraktion $\mathrm{Sp}\,T = g_{ik}T^{ik} = g^{ik}T_{ik}$ Diff.operatoren $\partial_i = (\frac{1}{c}\frac{\partial}{\partial t}, \nabla)$

$\partial_i\partial^i = \Box$

Nachdifferenzieren $\vec{v} = \frac{d}{dt}\vec{r} = \frac{dt'}{dt}\frac{d\vec{r}}{dt'}$

Transformationen der Felder

Boost $V\vec{e}_x$

$\underline{A}' = \hat{\Lambda}\underline{A} = \hat{L}\underline{A} \Rightarrow \varphi' = \gamma(\frac{\varphi}{c} - \frac{V}{c}A_x)$ $A'_x = \gamma(A_x - \frac{V}{c}\frac{\varphi}{c})$ $A_y' = A_y$ $A_z' = A_z$

$E'_x = E_x$ $E'_y = \gamma(E_y - VB_z)$ $E'_z = \gamma(E_z + VB_y)$

$B'_x = B_x$ $B'_y = \gamma(B_y + V\frac{E_z}{c^2})$ $B_z' = \gamma(B_z - V\frac{E_y}{c^2})$

$\vec{B} = 0$: $B_y' = \gamma V\frac{E_z}{c^2} = \frac{V}{c^2}E_z'$ $\vec{B}' = -\frac{1}{c^2}\vec{V}\times\vec{E}'$

$B_z' = -\gamma V\frac{E_y}{c^2} = -\frac{V}{c^2}E_y'$

$\vec{E} = 0$ $\vec{E}' = \vec{V}\times\vec{B}'$

Feldinvarianten

$I_1 = B^2c^2 - E^2$ $I_2 = \vec{B}\cdot\vec{E}$ $\Rightarrow (\vec{B}\cdot\vec{E} \gtrless 0 \Rightarrow \forall\Sigma': \vec{B}\cdot\vec{E} \gtrless 0)$ $\vec{B}\vec{E} = 0$: $\Rightarrow (B > \frac{E}{c} \Rightarrow \exists\Sigma': \vec{E} = 0)$, $\Rightarrow (B < \frac{E}{c} \Rightarrow \exists\Sigma': \vec{B} = 0)$

$I_1\varepsilon_0 = \vec{B}\cdot\vec{H} - \vec{E}\cdot\vec{D}$ $\Rightarrow (\vec{B}\cdot\vec{E} = 0 \Rightarrow \forall\Sigma': \vec{B}'\vec{E}' = 0)$ $\Rightarrow (B \gtrless \frac{E}{c} \Rightarrow \forall\Sigma': B' \gtrless \frac{E'}{c})$

Vektoranalysis

$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$ $\nabla(\vec{A}\cdot\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} + (\vec{A}\cdot\nabla)\vec{B} + \vec{B}\times(\nabla\times\vec{A}) + \vec{A}\times(\nabla\times\vec{B})$ $\nabla\times(\phi\vec{A}) = \phi\nabla\times\vec{A} - \vec{A}\times(\nabla\phi)$

$\nabla\times(\vec{A}\times\vec{B}) = \vec{A}\cdot(\nabla\cdot\vec{B}) - \vec{B}(\nabla\cdot\vec{A}) + (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B}$

$\nabla\cdot(\phi\vec{A}) = \phi\nabla\cdot\vec{A} + \vec{A}\cdot\nabla\phi$ $\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$ $\nabla\cdot\nabla\phi = \Delta\phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi$

$\nabla\cdot(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) + (\vec{B}\cdot\nabla)\vec{A} + (\vec{A}\cdot\nabla)\vec{B}$

$\nabla\times\nabla\phi = 0$ $\nabla\cdot(\nabla\times\vec{A}) = 0$ $\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \Delta\vec{A}$ $\Delta\vec{A} = \Delta A_x\vec{e}_x + \Delta A_y\vec{e}_y + \Delta A_z\vec{e}_z$

$\oint_{\partial A}\vec{B}\cdot d\vec{s} = \int_A \nabla\times\vec{B}\, d\vec{a}$ $\oint_{\partial V}\vec{E}\cdot d\vec{a} = \int_V \nabla\cdot\vec{E}\, d\vec{r}$ $\oint_{\partial V} u\nabla v\, d\vec{a} = \int_V u\Delta v\, d\vec{r} + \int_V \nabla u\nabla v\, d\vec{r}$ $\int_V \nabla\psi(\vec{r})\, d\vec{r} = \oint_{\partial V}\psi(\vec{r})\, d\vec{a}$ $\int_V \nabla\times\vec{V}(\vec{r})\, d\vec{r} = \oint_{\partial V} d\vec{a}\times\vec{V}(\vec{r})$ $\Box = \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

$\delta(x - x_0) = 0\ \forall x \neq x_0$ $\int_{-\infty}^{+\infty}\delta(x)dx = 1$ $\int_{-\infty}^{+\infty} f(x)\delta(x - x_0)dx = f(x_0)$ $\int_V \delta(\vec{r} - \vec{r}_0)d\vec{r} = \begin{cases}1, & \vec{r}_0 \in V\\ 0, & \text{else}\end{cases}$ $\Delta\frac{1}{|\vec{r} - \vec{r}_0|} = -4\pi\delta(\vec{r} - \vec{r}_0)$ $\delta(f(t')) = \sum_{t_i\in\text{Pole}(f)}\frac{\delta(t' - t_i)}{\left|\frac{df}{dt'}\right|_{t' = t_i}}$

z, r, φ: $x = r\cos\varphi$ $y = r\sin\varphi$ $z = z$ Jacobi: r $\nabla\phi = \vec{e}_r\frac{\partial\phi}{\partial r} + \frac{\vec{e}_\varphi}{r}\frac{\partial\phi}{\partial\varphi} + \vec{e}_z\frac{\partial\phi}{\partial z}$ $\nabla\cdot\vec{A} = \frac{\vec{e}_r}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$ $\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}r\cos\varphi\\ r\sin\varphi\\ z\end{pmatrix}$

$\nabla\times\vec{A} = \vec{e}_r\left[\frac{1}{r}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right] + \vec{e}_\varphi\left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right] + \frac{\vec{e}_z}{r}\left[\frac{\partial(rA_\varphi)}{\partial r} - \frac{\partial A_r}{\partial\varphi}\right]$ $\Delta\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$

r, θ, φ: $x = r\sin\theta\cos\varphi$ $y = r\sin\theta\sin\varphi$ $z = r\cos\varphi$ Jacobi: $r^2\sin\varphi$

$\nabla\phi = \vec{e}_r\frac{\partial\phi}{\partial r} + \frac{\vec{e}_\theta}{r}\frac{\partial\phi}{\partial\theta} + \frac{\vec{e}_\varphi}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}$ $\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$ $\nabla\times\vec{A} = \vec{e}_r\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right] + \vec{e}_\theta\left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}\right] + \vec{e}_\varphi\left[\frac{1}{r}\frac{\partial(rA_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial\theta}\right]$

$\Delta\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$ $(x, y, z)^T = r(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)^T$