More on the Difference between *more than two* and *at least three*

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Overview

I. The Generalized Quantifiers view: *three* = *more than two* = *at least three*
II. The adjectival analysis of number words
III. Krifka (1999): *three* ≠ *more than two*, *at least three*: Lack of scalar implicatures
IV. Geurts & Nouwen (2005): *more than two* ≠ *at least three*, *at least* and *at most* as modal operators
V. Problems of Geurts & Nouwen (2005)

VI. A new analysis of *at least* and *at most* as Speech Act Denegations

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I. By way of introduction...

What is the meaning of *three, at least three, more than two* in
- a. *John had three martinis.*
- b. *John had at least three martinis.*
- c. *John had more than two martinis.*

Answer of Generalized Quantifier (GQ) Theory (Barwise & Cooper 1982, etc.):
- -- *three, at least three, more than two* are all quantificational determiners;
- -- they have the same meaning, hence a, b, c mean the same.

Analysis in GQ theory, where
- ⟦*martinis*⟧ = λx[x is a martini] = m
- ⟦*John had*⟧ = λx[John had x] = jh

a. ⟦*three*⟧(⟦*martinis*⟧)(⟦*John had*⟧) = $|m \cap jh| \geq 3$
not =3 (*exactly three*), as continuation *if not four* is possible;
meaning component $\not> 3$ due to scalar implicature, not part of the literal meaning.

b. ⟦*at least three*⟧(⟦*martinis*⟧)(⟦*John had*⟧) = $|m \cap jh| \geq 3$

c. ⟦*more than two*⟧(⟦*martinis*⟧)(⟦*John had*⟧) = $|m \cap jh| > 2$, i.e. ≥ 3

One result of this talk will be:
- -- None of *three, at least three, more than two* are quantificational determiners;
- -- they all have different meanings.

II. A first correction of the GQ analysis: The adjectival analysis of number words

Analysis of *three* as a numeral (Verkuyl 1981, Link 1983):
- $[_{NP}$*three* $[_{N}$*martinis*]]: $\lambda x[m(x) \wedge |x|=3]$
- $[_{DP}$ $ø_{\exists}$ $[_{NP}$*three* $[_{N}$*martinis*]]]: $\lambda P \exists x[m(x) \wedge |x|=3 \wedge P(x)]$
- $[_{S}$ *John* $[_{VP}$ *had* $[_{DP}$ $ø_{\exists}$ $[_{NP}$ *three martinis*]]]]: $\exists x[m(x) \wedge |x|=3 \wedge h(j, x)]$

Advantages:
- Allows for compositional analysis of $[_{DP}$ *the* $[_{NP}$ *three martinis*]]: $\sigma(\lambda x[m(x) \wedge |x|=3])$,
  (σP: the maximal individual that falls under P)
  presupposition: there are exactly three martinis,
  reference to the single object in the extension of this predicate.
- Allows for existential analysis of bare nouns: $[_{DP}$ $ø_{\exists}$ $[_{NP}$ *martinis*]]
- Can be extended to a measure term analysis (cf. Krifka 1995)
  in which *martinis* denotes a measure function from entities to numbers:
  m(x)=n iff x are n martinis;
  allows modelling cases like
  $[_{S}$ *John* $[_{VP}$ *had* $[_{DP}$ $ø_{\exists}$ $[_{NP}$ *two and a half martinis*]]]]: $\exists x[m(x)=2.5 \wedge h(j, x)]$

III. Krifka (1999): ⟦*three*⟧ ≠ ⟦*more than two*⟧ , ⟦*at least three*⟧
Lack of scalar implicatures

In many contexts,

John had three martinis is understood as *John had exactly three martinis.*

Additional meaning component is a scalar implicature, as in GQ theory:

John had three martinis, if not four

Standard account of scalar implicature

(Grice 1967, Horn 1972, Gazdar 1979, Levinson 1984):

John had three martinis.

a. Meaning: $\exists x[m(x)=\underline{3} \wedge h(j,x)]$

b. Scalar implicature: $\neg\exists n>3\exists x[m(x)=\underline{n} \wedge h(j,x)]$

Generation of scalar implicature:

Alternatives to *three*: *... two, three, four, ...*

Selection of strongest proposition by Maxim of Quantity:

Say as much as you can,

say the <u>whole</u> truth.

Exclusion of stronger alternatives by Maxim of Quality.

Don't say anything for which you lack evidence,

say <u>nothing but</u> the truth.

Question:

Why do *more than two* and *at least three* not trigger this implicature?

Why no scalar implicatures with *at least* / *more than*

Following Krifka (1999), 'At least some determiners aren't determiners':

- Focus introduces background-focus partitioning, with alternatives (von Stechow 1981, 1990; Jacobs 1984): $\langle B, F, A\rangle$
  John had thrée$_F$ martinis: $\langle\lambda n\exists x\,[m(x)=n \wedge h(j,x)], 3, \{1, 2, 3, 4, ...\}\rangle$
- Assertion: Background applied to Focus: $B(F)$,
  Implicatures are denegations of assertions with respect to alternatives,
  e.g. if $X \in A$ and $X \neq F$, then $B(X)$ is not asserted
  (because it is too weak, or too strong, etc.)
  Example: ASSERT [*John had thrée$_F$ martinis*]:
  a. ASSERT $\exists x[m(x)=n \wedge h(j,x)]$
  b. Implicature: Every stronger alternative is not asserted:
  $\forall n>3\ \sim$ASSERT $\exists x[m(x)=n \wedge h(j,x)]$,
- Determiners *at least / more than / at most / less than* associate with focus:
  e.g. ⟦*more than*⟧ $(\langle\lambda n\lambda X[...n...], F, A\rangle) = \lambda X\ \exists n\in A, n>F\ [...n...]$
- Focus is "used up", no focus remains to be exploited at illocutionary level.
  ASSERT [*John had* [*more than*]$_1$ *twó$_{F1}$ martinis*]:
  ASSERT $\exists n>2\exists x[m(x)=n \wedge h(j,x)]$

Under this analysis:

⟦*thrée$_F$ martinis*⟧ ≠ ⟦*at least thrée$_F$ martinis*⟧ , ⟦*more than twó$_F$ martinis*⟧ ,

but ⟦*at least thrée$_F$ martinis*⟧ = ⟦*more than twó$_F$ martinis*⟧ ,

(for integer number determiners, e.g. not *two and a half*)

IV. Geurts & Nouwen: ⟦*at least three*⟧ ≠ ⟦*more than two*⟧

Geurts & Nouwen (2005), '*At least* et al.: The semantics of scalar modifiers'

more than two, less than three: comparative modifiers

at least three, at most three: superlative modifiers

Meanings of comparative and superlative modifiers differ in subtle ways:

- Intuitive inference patterns (⊨)
  (a) *John had three martinis.*
  ⊨ *John had more than two martinis.*
  ⊭ *John had at least three martinis.* (lack of scalar implicature of (a))
  ⊨ *John had fewer than five martinis.*
  ⊭ *John had at most four martinis.* (contradicts scalar implicature of (a))
- Superlative modifiers have fewer distributional restrictions (cf. also Kay 1992):
  *John had three martinis at most / *fewer than.*
  *At least/*More than, John had three martinis.*
  *Mary danced with at most / *more than John.*
- Missing readings:
  You may have fewer than three martinis.
  a. 'You are allowed to have fewer than three martinis (but you may have more)'
  (makes sense if you don't like martinis)
  b. 'You may have up to two martinis (but not more)'
  You may have at most two martinis.
  only b. 'You may have up to two martinis (but not more)'

Geurts and Nouwen's Proposal:

Comparative modifiers: *more than*

Analysis as in Krifka (1999), as focus sensitive, narrow-scope NP modifiers,

e.g. *John had* $[_{DP}$ ø$_\exists$ $[_{NP}$ *more than* $[_{NP}$ *twó martinis*$]]]$

$\exists n>2\exists x[m(x)=n \wedge h(j,x)]$

Superlative modifiers: *at least*

Analysis as modal constructions, here adapted and simplified for our example

- *John had at least* $[_{DP}$ ø$_\exists$ $[_{NP}$ *thrée$_F$ martinis*$]]$
  $\Box\exists x[m(x)=3 \wedge h(j,x)]$
  $\wedge\ \exists n>3\Diamond\exists x[m(x)=n \wedge h(j,x)]$
  'It is (epistimically) necessary that John had three martinis,
  and it is possible that he had more than three martinis.'

Superlative modifiers: *at most*

- *John had at most* $[_{DP}$ ø$_\exists$ $[_{NP}$ *thrée$_F$ martinis*$]]$
  $\Diamond\exists x[m(x)=3 \wedge h(j,x)]$
  $\wedge\ \neg\exists n>3\Diamond\exists x[m(x)=n \wedge h(j,x)]$
  'It is possible that John had three martinis,
  but it is not possible that he had more than three martinis.'

Explanation of differences between comparative and superlative modifiers

- Intuitive inference patterns:

  John had three martinis.
  meaning: $\exists x[m(x)=3 \wedge h(j,x)]$
  implicature: $\neg\exists n>3 \exists x[m(x)=n \wedge h(j,x)]$

  $\not\models$ *John had at least three martinis.*
  $\Box[......] \wedge \exists n>3 \Diamond \exists x[m(x)=n \wedge h(j,x)]$
  inconsistent with implicature!

  $\not\models$ *John had at most four martinis.*
  $\Diamond \exists x[m(x)=4 \wedge h(j,x)] \wedge \neg\exists n>4 \Diamond[......]$
  inconsistent with implicature!

- Distributional restrictions:

  At most, John had thrée$_F$ martinis.
  John had at most thrée$_F$ martinis.
  John had thrée$_F$ martinis at most.

  as a modal operator, *at most* has scope over the sentence.

  *Mary danced with at least / *more than John*

  modal operator does not require number scale; Kay 1993 for NP, AP, AdvP, VP uses

- Missing readings:
  John may have at most two martinis.
  only possible reading: $\Diamond \boxdot \exists x[m(x)=3 \wedge h(j,x)] \wedge \neg\exists n>3 \Diamond \boxdot \exists x[m(x)=n \wedge h(j,x)]$,
  where $\Diamond$: epistemic possibility, $\boxdot$: deontic necessity,
  deontic operator cannot scope over epistemic operator

V. Problems of the analysis of Geurts & Nouwen

- The comparative morphology for comparative modifiers is captured by the analysis:
  e.g. *John had more than three martinis*
  $\exists x[m(x) \underline{> 3} \wedge h(j,x)]$
  But the superlative morphology for superlative modifiers is not captured by the analysis:
  e.g. *John had at least three martinis.*
  $\Box \exists x[m(x) = 3 \wedge h(j,x)] \wedge \exists n>3 \Diamond \exists x[m(x) = n \wedge h(j,x)]$
- There are restrictions on embeddings of superlative quantifiers that are difficult to explain under the modal analysis:
  Whenever you have less than 50 $ in your pocket, go to the bank to get more.
  ? *Whenever you have at most 50 $ in your pocket, go to the bank to get more.*
- The analysis does not predict that *at most three* licenses NPIs, as no general downward-entailing context is created:
  At most three people have ever been in this cave.
  * It is possible that 3 people have ever been in this cave $\wedge$
  o.k.: $\exists n>3$ ¬It is possible that n people have ever been in this cave

VI. A new analysis: *at most* / *at least* as Denegations of Speech Acts

Basic idea for superlative modifiers:

- Superlative speech act modifiers presuppose a particular context question:
  How many martinis did John have?
  Set of question alternatives:
  {'John hat no martini', 'John had one martini', 'John had two martinis',...}
  Congruent answers eliminate alternatives.
  John had thrée$_F$ / at most thrée$_F$ / at least thrée$_F$ martinis.
- Upper-bound superlatives:
  John had at most thrée$_F$ martinis.
  'The <u>highest</u> n such that $\exists x[m(x)=n \wedge h(j,x)]$ can be <u>asserted</u> is n = 3',
  that is, *John had n martinis* cannot be asserted for n > 3,
  as this assertion would definitely be false.
- Lower-bound superlatives:
  John had at least thrée$_F$ martinis.
  'The <u>lowest</u> n such that $\exists x[m(x)=n \wedge h(j,x)]$ can be <u>asserted</u> is n = 3'
  that is, *John had n martinis* cannot be asserted for n < 3;
  as this assertion would be too weak,
  generating the wrong implicature that John drank only fewer than three martinis.

Example

Example situation:

Assume there is **very good evidence** that your guest John had **two** martinis
(you have seen it with your own eyes),
there is **good evidence** that John had **three** martinis
(someone told you),
and there is **weak evidence** that John had **four** martinis
(a quantity of four martinis is missing from your martini bottle).

The question is: How many martinis did John have?

In this situation, one is entitled to say:

John had at least two martinis.
John had three martinis.
John had at most four martinis.

These choices indicate the different levels of evidence one has.

at most / at least as Indicators of Illocutionary Strength?

In previous versions of this talk:
Superlative operators as indicators of illocutionary strength,
at least: strong assertion, *at most*: weak assertion

(Cf. Searle,Vanderveken 1986 for the concept of illocutionary strength)

John had at least two martinis: Strong assertion that John had two martinis.
John had three martinis: Neutral assertion that John had three martinis.
J*ohn had at most four martinis*: Weak assertion that John had four martinis.

But: Illocutionary strength of assertion can be indicated by *certainly, presumably*,
and we seem to have all possible combinations:

John presumably had at most four martinis.
John certainly had at most four martinis. (!)
John certainly had at least two martinis.
John presumably had at least four martinis. (!)

So a direct marking of illocutionary strength seems implausible.

A closer look at *at least two*

Context question:
{'John had no martini', 'John had one martini', 'John had two martinis',...}

Lower-bound operators:
John had at least$_1$ twó$_{F1}$ martinis.
'The lowest n such that $\exists x[m(x)=n \wedge h(j,x)]$ can be asserted is n = 2'

at least expresses denegation of <u>lower</u> values:
$\forall n<2$ ~[SC-ASSERT $\exists x[m(x)=\underline{n} \wedge h(j,x)]$]

For numbers n<2 speaker explicitly does <u>not</u>:
– <u>assert</u> that John had n martinis
– while <u>implicating</u> that John had not more than n martinis.

Speaker excludes lower alternatives by denegation:
{'John had no martini', 'John had one martini', ...}

Speaker leaves open the alternatives
{'John had two martinis', 'John had thre martinis', ...}

The context question presupposes that one question alternative holds,
the alternative 'John had no martini' is excluded by denegation of lower values,
hence context question + utterance state that John had two or more martinis.

A closer look at simple scalar implicatures: *thrée*

Context question:
{'John hat no martini', 'John had one martini', 'John had two martinis',...}

Assertion with scalar implicature:
ASSERT$_1$ *John had thrée$_{F1}$ martinis*
a. ASSERT $\exists x[m(x)=3 \wedge h(j,x)]$
b. & $\forall n>3$ ~ASSERT $\exists x[m(x) =n \wedge h(j,x)]$

in short, notion of Scalar Assertion:
SC-ASSERT $\exists x[m(x)=\underline{3} \wedge h(j,x)]$
-- It is asserted that John had three martinis,
-- and it is implicated that he didn't have more martinis.

where
& is illocutionary conjunction (cf. Krifka 2001)
~ is illocutionary denegation (cf. Searle 1969, Vanderveken 1990),
speaker refrains from performing the illocutionary act,
cf. *I don't promise to come*: ~PROMISE('I come')

A closer look at *at most four*

Context question:
{'John had no martini', 'John had one martini', 'John had two martinis',...}

Upper-bound operators:
John had at most$_1$ fóur$_{F1}$ martinis.
'The highest n such that $\exists x[m(x)=n \wedge h(j,x)]$ can be asserted is n = 4'

at most expresses denegation of <u>higher</u> values:
$\forall n>4$ ~[SC-ASSERT $\exists x[m(x)=\underline{n} \wedge h(j,x)]$]

For numbers n>4 speaker explicitly does <u>not</u>:
– <u>assert</u> that John had n martinis
(while <u>implicating</u> that John had not more than n martinis)

Speaker excludes higher alternatives by denegation:
{'John had five martinis', 'John had six martinis', ...}

Speaker leaves open the alternatives
{'John had no martini', 'John had one martini', 'John had two martinis',
'John had three martinis', 'John had four martinis'}

Inclusion of 'John had no martini' captures the fact
that *John had at most four martinis* does not commit the speaker
to the claim that John had a martini.

VII. Explanation of observations

- Inference patterns:

  John had thrée$_F$ martinis. — SC-ASSERT ∃x[m(x)=3 ∧ h(j,x)], exludes values > 3 by implicature

  ⊭ *John had at least thrée$_F$ martinis.* — ∀n<3 ~SC-ASSERT ∃x[m(x)=n ∧ h(j,x)] does not exclude values > 3

  ⊭ *John had at most thrée$_F$ martinis.* — ∀n>3 ~SC-ASSERT ∃x[m(x)=n ∧ h(j,x)] does not exclude lower values < 3
- Distributional restrictions

  At most, John had thrée$_F$ martinis.
  John had at most thrée$_F$ martinis.
  John had thrée$_F$ martinis at most.

  superlative modifiers affect illocutionary level hence have sentential scope
- Missing readings

  You may drink at most two martinis.

  Superlative modifers affect illocutionary level, deontic operator is part of proposition (but see below for commands, permissions)

Licensing of NPIs by *at most*

Example: *At most three people have ever been in this cave (in the last century)*

Explanation of NPI distribution in general, Krifka (1995):

- NPIs introduce alternatives and denote the smallest or most general alternative.
  e.g *ever ... in the last century* :
  Alternatives: more or less specific times, {t | t ⊆ last_century}
  Meaning: The least specific time, last_century
- Alternatives are exploited at the level of illocutionary operators:
  ASSERT [*no person has ever been in this cave in the last century*]
  ASSERT [¬∃x[person(x) ∧ in_cave(x, last_century)] &
  ∀t⊂last_century [~ ASSERT [¬∃x[person(x) ∧ in_cave(x, t)]]

  Reason for non-assertion:
  Alternatives are less informative than the actual assertion.
  This holds for downward-entailing contexts in general.
- In general:
  NPI indicates that the strongest claim is made among the alternatives.
- Upper-bound operators indicate strongest proposition speaker is willing to defend, NPIs indicate strongest proposition as well.
- Notice that in our analysis, the speaker is not committed to the claim that any people have been in this cave.

An observations that G&N could not explain: Superlative morphology of *at least* / *at most*

Natural explanation:

John had at most three martinis
= The most speaker is willing to assert
out of the alternatives *John had ...2/3/4... martinis*
is: *John had 3 martinis*

John had at least three martinis
= The least speaker is willing to assert
out of the alternatives *John had ...2/3/4... martinis*
is: *John had 3 martinis*

VIII. Superlative modifiers in embedded sentences

Examples:

- *If you have less than 50 $ in your pocket, go to the bank to get more.*
- ? *If you have at most 50 $ in your pocket, go to the bank to get more.* o.k. as quotation
- *If you ever have more than 1000 $ in your pocket, bring it to the bank.*
- ? *If you ever have at least 1000 $ in your pocket, bring it to the bank.*

Under an analysis of superlative operators as operators on the illocutionary level, one should expect that they cannot be embedded within the propositional level.

But embeddings of superlative operators are often not as bad as predicted -- why?

Independent evidence that scalar implicatures may be triggered within *if*-clause, i.e. that illocutionary operators can be embedded, cf. Krifka (1995), Chierchia (2001) on scalar implicatures in embedded sentences:
If you have 50 $ in your pocket, you should give me 25 and keep 25 for yourself.
(= *If you have 50 $ (and not more) in your pocket, ...*)

It is possible that scalar implicature can be triggered by other operators than illocutionary operators, but these operators would behave in the same way with respect to *at least / at most*.

Superlative modifiers in embedded sentences

There are other cases of embedded sentences
in which we observe scalar implicature and superlative operators:

Mary knows that John drank three / at least two / at most four martinis.

Every man that drank three / at least two / at most four martinis had a headache the day after.

IX. Superlative modifiers in intensional contexts

Consider the following example:

(a) *John needs at least two martinis (to function properly).*
(b) *John can have at most four martinis (if he still wants to be able to drive).*

There is a reading in which *at least* / *at most* do not express strength of assertability,
but refer properties of the intensional operators *need, can*.

Example: *need*

John needs x (in order to...)
expresses the **minimal** requirements, higher values would be better.

We can assume the same basic semantics of *at least* / *at most*,
now scoping over the intensional expression, not the illocutionary operator:

Example (a):

$\forall n{<}2\ \neg$ minimal-need(john, $\exists x \exists n[m(x)=n \wedge h(j,x)]$)
i.e. the minimal needs of John are not satisfied by fewer than 2 martinis,
more might be better.

Example (b):

$\forall n{>}4\ \neg\boxdot\ \exists x \exists n[m(x)=n \wedge h(j,x)]$
i.e. the maximal number of martinis such that John is still able to drive
after he had it is 4, fewer might be better.

X. Other illocutionary operators: Commands and Permissions

Commands, prototypical situation:
Speaker wants the hearer to do something against hearer's interest,
restricts option space of hearer.

Minimal and maximal commands:

(a) *Give me at least twó$_F$ dollars.*
'The smallest n such that I ask you to give me n dollars is n = 2'
'I ask you to give me 2 dollars,
and I do not ask you to give me less than 2 dollars (this would be too little)'
'I am minimally (sufficiently) satisfied if you give me 2 dollars,
more satisfied if you give me more.'

(b) ?*Give me at most fóur$_F$ dollars.*
'The greatest n such that I ask you to give me n dollars is n = 4'
'I ask you to give me 4 dollars,
and I do not ask you to give me more than 4 dollars (this would be too much)'
'I am maximally satisfied if you give me 4 dollars.'

(b) is less natural because maximal satisfaction is a strange concept,
o.k. in politeness contexts with distorted interests.

Permissions

Permissions, prototypical situation:
Speaker wants to allow the hearer to do something in his interest,
Speaker increases option space of hearer.

Maximal and minimal permissions:

(a) *You may take at most fóur$_F$ dollars.*
'The greatest n such that I allow you to take n dollars is n = 4'
'I allow you to take 4 dollars,
and I do not allow you to take more than 4 dollars (this would be too much)'
'I satisfy you most (within given limits) if I allow you to take 4 dollars,
I satisfy you less if I allow you to take less.'

(b) ?*You may take at least twó$_F$ dollars.*
'The smallest n such that I allow you to take n dollars is n = 2'
'I allow you to take 2 dollars,
and I do not allow you to take less than 2 dollars (this would be too little)'
'I satisfy you least (within given limits) if I allow you to take 2 dollars,
I satisfy you less if I allow you to take less.'

(b) is less natural because permissions normally are downward-entailing:
If I allow you to take n dollars, you can also take m dollars, m<n.
o.k. in a description (assertion) of a previously uttered permission

XI. Other Scales and Scale Direction

The account devoloped here can be extended to other scales:

John had at least [*two martínis*]$_F$

Alternatives: Other amounts of alcoholic drinks,
e.g. three beers, two martinis and a glass of wine, five caipirinhas, ...
Relevant order: size of sum individuals or amount of alcohol
'two martinis' < 'two martinis and a beer'
'two martinis' < 'seven tequilas'

John is at least [*an associate professor*]$_F$

Alternatives: Other levels of professorship,
e.g. assistant professor, full professor, named full professor

Scale direction

The direction of scale might not be fixed:

[Weight loss clinic, John was admitted to it with 300 pounds;
context question: When can he be released?]
(a) *Now, John weighs in at at least 250 pounds (if not less)*
(b) *Now, John weighs in at at most 275 pounds (if not more)*

Problem for current interpretation of (a):
∀n<250 ~SC-ASSERT [weigth(j, n pounds)]
'The least n such that it is assertible that John weighs in at n pounds is 250'

Proposal: The alternatives are ordered with respect to the **relevance** for the question
(cf. Ducrot 1972, Merin 1999):
$<_R$ = ... 'John weighs 255 lb' < 'John weighs 250 lb' < 'John weighs 245 lb' ...

New interpretation of (a):
'The least relevant proposition one can assert is that John weighs 250 lb'
∀p $<_R$ weigh(j, 250 lb) ~SC-ASSERT[p],
e.g., ~SC-ASSERT weigh(j, 255 lb),
as this could lead to the false conclusion that he still has to stay in the hospital..

New interpretation of (b):
'The most relevant proposition one can assert is that John weighs 275 lb'
∀p $>_R$ weigh(j,275 lb) ~SC-ASSERT[p],
e.g., ~SC-ASSERT weigh(j, 270 lb),
as this could lead to the false conclusion that he can already leave the hospital.

XI. Wrapping things up...

Generalized Quantifier Theory:
Same meaning for *three, at least three, more than two*

We have seen that this is not the case:

- *three* is a number word specifying number argument of measure function, existential quantifier is independent of that.
- *more than two, fewer than four* are comparative modifiers of this number argument
- *at least three, at most four* are superlative modifiers operating on a higher level,
  sometimes at the level of illocutionary operators
  expressing minimally and maximally relevant illocutionary acts,
  sometimes at the level of modal operators
  relating to ordering sources.

Morale:
Natural Language "Quantification" is much richer, and quite different from what Generalized Quantifier Theory has suggested.