

The accomplished mathematician he claims to be: A semantic account of Modal Compatibility Relatives

Alexander Grosu, University of Tel Aviv
Manfred Krifka, Humboldt Universität Berlin

1. The Phenomenon

1.1 Examples of MCRs

- (1) *The accomplished mathematician Bill claims to be should be able to solve this simple problem without difficulties.*
- (2) a. *Der grosse Mathematiker, der du zu sein vorgibst, sollte dieses simple Problem ohne Schwierigkeit lösen können*
b. *Le grand mathématicien que tu prétends être ne devrait avoir aucune difficulté à résoudre ce simple problème.*

1.2 Properties of MCRs

1.2.1 Gap in postcopular (generally predicational) position

- (3) a. *[The accomplished mathematician Bill claims to be _] should be able to solve this simple problem.*
b. *The accomplished mathematician that many people view [Bill as _] should be able to solve this simple problem.*

O.k. but not of interest here:

- (4) *[The accomplished mathematician John claimed to have met _] should be able to solve this problem.*

1.2.2 Definiteness restriction

The relative clause must be headed by a definite article.

- (5) *#[An accomplished mathematician that Bill claims to be _] should be able to solve this problem.*

1.2.3 Internal modality requirement

The relative clause contains some modal component

- (6) *#[The accomplished mathematician that Bill is _] should be able to solve this problem.*

1.2.4 External modality requirement

The clause that the relative clause is embedded in contains some modal element.

- (7) *#[The accomplished mathematician Bill claims to be _] solved the problem.*

1.2.5 Compatibility between the modalities

External and internal modality have to correspond to each other.

- (8) *#[The accomplished mathematician Bill claims to be _] seems to be working on a very hard problem.*

Intuition: Bill should be able to solve this problem **because** he supposedly is an accomplished mathematician.

1.3 The explanation

In a nutshell

The worlds of the modal operator of the relative clause are the ones with respect to which the main clause are evaluated. For (1):

- (9) The worlds that are compatible with what Bill claims to be (in which Bill is an acclaimed mathematician), are worlds in which Bill is able to solve this simple problem.

This precludes that the main clause is interpreted w.r.t. the actual world (external modality requirement).

And it requires that the worlds at which the main clause is evaluated are included in those at which the relative clause is evaluated (compatibility between the modalities).

Some technical aspects in advance

- (10) *[The accomplished mathematician Bill supposedly is _] should have solved the problem.*

The subject NP [*the accomplished mathematician Bill supposedly is _*] refers to an **individual concept** (a function that maps worlds to individuals).

Specifically, it refers to the function that maps every world *i* in which Bill is identical to an accomplished mathematician (i.e., in which Bill is an accomplished mathematician) to Bill.

Reference to possible worlds is of course triggered by the modal operator, *supposedly*.

The VP *should have solved the problem* makes an assertion about this individual concept; in order to do so, it has to access the worlds for which the individual concept is defined,

This access is accomplished by the modal operator, *should*.

The sentence says that in all worlds in which the individual concept is defined, its value has solved the problem

Task: Arrive at this in a compositional way!

2. The interpretational framework

2.1 Outline

Types

- (11) a. Basic types: *e*, *t*, *s* for entities, truth values, world indices (variable: *i*);
b. if *τ*, *σ* are types,
then (*τ*) is the type of (possibly **partial**) functions from *τ*-entities to *σ*-entities;
write *f* if *τ* is a simple type.

Explicit quantification over indices

(12) MISS AMERICA(i): the individual that is Miss America in i, MISS AMERICA is of type se.

Predicates apply to individual concepts

(13) e.g., LAW STUDENT(i)(MISS AMERICA):
 ‘Miss America [the concept] is a law student in i.’,
 i.e. LAW STUDENT is of type s(se)t.

Cf. Montague (1973) for this analysis of verb meanings (*the temperature is rising*); Gupta (1980) for noun meanings (*passenger*).

Predications on individual concepts often can be reduced to predications about individuals

(14) There is a predicate LAW_STUDENT*, type set, such that for every individual concept x,
 $i[\text{LAW_STUDENT}(i)(x) \text{ LAW_STUDENT}^*(i)(x(i))]$

In particular, the meaning of the copula can be reduced to individuals:

(15) $[[is]] = IS = i x y[y(i) = x(i)]$

Computation of meaning of complex expressions:

(16) $[[[]]] = i[[[]](i)([](i))]$ or $i[[[]](i)([](i))]$, whichever is wellformed.

2.2 Example; identificational statement

Example derivation:

- (17) a. $[[[Erika Harold [is [Miss America]]]]]$
 b. $= i[[[is [Miss America]]](i)([Erika Harold](i))]$
 c. $= i[[is](i)([Miss America](i)([Erika Harold](i)))]$
 d. with $[[is]] = IS = i x y[y(i) = x(i)]$, type s(se)(se)t,
 $[[Miss America]] = i[MISS AMERICA]$, type sse,
 $[[Erika Harold]] = i[ERIKA HAROLD]$, type sse
 e. $= i[IS(i)(MISS AMERICA)(ERIKA HAROLD)]$
 f. $= i[i x y[y(i) = x(i)](MISS AMERICA)(ERIKA HAROLD)]$
 g. $= i[ERIKA HAROLD(i) = MISS AMERICA(i)]$

Example model:

Assume 5 indices i_0, i_1, i_2, i_3, i_4 , and 5 individuals, a, b, c, d, e.

- (18) a. MISS AMERICA = $[i_0 e, i_2 a, i_3 c, i_4 e]$
 (short for $\{ i_0, e, i_2, a, i_2, c, i_4, e \}$)
 b. ERICA HAROLD = $[i_0 e, i_1 e, i_2 e, i_3 e, i_4 e]$

(19)

	a	b	c	d	e
i_0					ME
i_1					E
i_2	M				E
i_3			M		E
i_4					ME

Table 1:

MISS AMERICA: MMMM,
 ERIKA HAROLD: EEEEE

Example interpretation:

(20) $[[Erika Harold is Miss America]]$
 $= i[ERIKA HAROLD(i) = MISS AMERICA(i)]$
 $= [i_0 1, i_2 0, i_3 0, i_4 1]$ (undefined for i_1)

2.3 Example derivation of a clause involving binding

Standard use of assignment functions

- (21) a. *Miss America is a law student.*
 b. LF, Heim & Kratzer style: $[[a \text{ law student}] \ 1 [Miss America \text{ is } t_1]]$
- (22) a. $[[[a \text{ law student}] \ 1 [Miss America \text{ is } t_1]]]$
 b. $= i[[[a \text{ law student}]](i)([[1 [Miss America \text{ is } t_1]] (i)])$
 c. $= i[[[a \text{ law student}]](i)(i x_1[[[Miss America \text{ is } t_1]]^{t_1 x_1}(i)(i))]$
 d. $= i[[[a \text{ law student}]](i)(x_1[[[Miss America \text{ is } t_1]]^{t_1 x_1}(i)])$
 e. $= i[[[a \text{ law student}]](i)(x_1[[[is]]^{t_1 x_1}(i)([t_1]]^{t_1 x_1}(i))([Miss America]]^{t_1 x_1}(i)))]$
 f. $= i[[[a \text{ law student}]](i)(x_1[IS(i)(x_1)(MISS AMERICA)])]$
 g. $= i[i P x[LAW STUDENT^*(i)(x(i)) \ P(x)](i)(x_1[MISS AMERICA(i) = x_1(i)])]$
 h. $= i x[LAW STUDENT^*(i)(x(i)) \ MISS AMERICA(i) = x(i)]$

A function that maps worlds i to truth iff there is an individual concept x that is a law student at i and that is identical to Miss America at i.

Example model (extended)

Assume the following reduced individual property LAW STUDENT*

(23) LAW STUDENT* =

i_0	[a	0, b	1, c	1, d	1, e	0]
i_1	[a	0, b	0, c	1, d	1, e	1]
i_2	[a	0, b	1, c	0, d	0, e	1]
i_3	[a	0, b	0, c	0, d	1, e	1]
i_4	[a	0, b	0, c	0, d	0, e	0]]

(24)

	a	b	c	d	e
i_0					ME
i_1					E
i_2	M				E
i_3				M	E
i_4					ME

Table 2:

Property LAW STUDENT*:
shaded area

The individual concept property LAW STUDENT is far too large to be illustrated here -- the model supports $5^5 = 3125$ total individual concepts, and $5^6 - 1 = 15624$ individual concepts if we also count the partial ones except the empty set

Example interpretation

$$(25) \llbracket \text{Miss America is a law student} \rrbracket = [i_0 \quad 0, i_2 \quad 0, i_3 \quad 1, i_4 \quad 0]$$

2.4 Example, modalized sentence

Modalized statements involve quantification over indices.

(26) *Supposedly Miss America is a law student*

$$(27) \llbracket \llbracket \text{supposedly [a law student } _1 [\text{Miss America is } t_1]] \rrbracket \rrbracket$$

$$= i \ i \ \text{SUPPOSED}(i) [\llbracket \llbracket \text{a law student } _1 [\text{Miss America is } t_1]] \rrbracket (i \)]$$

$$= i \ i \ \text{SUPPOSED}(i) \ x[\text{LAW STUDENT}^*(i)(x(i \)) \ \text{MISS AMERICA}(i) = x(i \)]$$

‘For all indices i that are compatible with what is supposed to be the case at i , Miss America at i is a law student at i .’

This gives us the attributive, de-dicto reading ‘Supposedly Miss America is a law student, whoever she happens to be’. This type of reading exists, e.g. as the preferred reading of:

(28) *Supposedly, Miss America is a celebrity.*

2.5 Referential readings

Referential readings can be generated by creating a rigid individual concept by “diagonalization”:

$$(29) \llbracket _R \rrbracket = i \ i \ u[u = \llbracket _ \rrbracket (i)(i)]$$

$$(30) \llbracket \text{Miss America}_R \rrbracket$$

$$= i \ i \ u[u = \llbracket \text{Miss America} \rrbracket (i)(i)]$$

$$= i \ i \ u[u = i \ \text{MISS AMERICA}(i)(i)]$$

$$= i \ i \ u[u = \text{MISS AMERICA}(i)]$$

$$(31) \text{ a. } \llbracket \text{Miss America}_R \rrbracket (i_0) = \llbracket \text{Miss America}_R \rrbracket (i_4) = [i_0 \ e, i_1 \ e, i_2 \ e, i_3 \ e, i_4 \ e]$$

$$\text{ b. } \llbracket \text{Miss America}_R \rrbracket (i_1): \text{ undefined.}$$

$$\text{ c. } \llbracket \text{Miss America}_R \rrbracket (i_2) = [i_0 \ a, i_1 \ a, i_2 \ a, i_3 \ a, i_4 \ a]$$

$$\text{ d. } \llbracket \text{Miss America}_R \rrbracket (i_3) = [i_0 \ d, i_1 \ d, i_2 \ d, i_3 \ d, i_4 \ d]$$

Derivation of referential reading of example

Scoping out of *Miss America* necessary in the framework adopted here; not required in framework with double indexing.

$$(32) \llbracket \llbracket \llbracket \text{Miss America}_R _2 [\text{supposedly [a law student } _1 [t_2 \text{ is } t_1]]] \rrbracket \rrbracket \rrbracket$$

$$= i [\llbracket _2 [\text{supposedly [a law student } _1 [t_2 \text{ is } t_1]]] \rrbracket (i) (\llbracket \text{Miss America}_R \rrbracket (i))]]$$

$$= i [_2 \ i \ \text{SUPPOSED}(i) \ x[\text{LAW STUDENT}^*(i)(x(i \)) \ _2(i) = x(i \)]$$

$$\quad (\ i \ u[u = \text{MISS AMERICA}(i)])]]$$

$$= i [\ i \ \text{SUPPOSED}(i) \ x[\text{LAW STUDENT}^*(i)(x(i \)) \ x(i \)]$$

$$\quad \ i \ u[u = \text{MISS AMERICA}(i)](i) = x(i \)]]$$

$$= i [\ i \ \text{SUPPOSED}(i) \ x[\text{LAW STUDENT}^*(i)(x(i \)) \ u[u = \text{MISS AMERICA}(i)] = x(i \)]$$

$$= i \ i \ \text{SUPPOSED}(i) \ x[\text{LAW STUDENT}^*(i)(x(i \)) \ \text{MISS AMERICA}(i) = x(i \)]$$

This proposition holds for i_0 in the following model:

$$(33) \quad i \text{ SUPPOSED}(i_0) \ x[\text{LAW STUDENT}^*(i)(x(i))] \quad \text{MISS AMERICA}(i_0) = x(i)$$

	a	b	c	d	e
i_0					Mm
i_1					m
i_2	M				m
i_3				M	m
i_4					Mm

Table 4:
LAW STUDENT*: shaded
[[Miss America]_R](i_0) = mmmmm
SUPPOSED(i_0) = { i_2, i_3, i_4):
double frame

2.6 Definite descriptions

A problem with standard interpretation of definite descriptions

Standard: *the law student* in i : The unique individual u such that u has the property *law student* in i .

Application in framework of individual concepts:

$$(34) \quad \llbracket \text{the [law student]} \rrbracket(i) = x [\text{LAW STUDENT}(i)(x)],$$

defined iff LAW STUDENT(i) denotes a singleton set.

Problem: Lack of uniqueness.

$$(35) \quad \text{LAW STUDENT}(i_0) \\ = \{x \mid \text{LAW STUDENT}^*(x(i_0))\} \\ = \{ [i_0 \ a], [i_0 \ a, i_1 \ a], [i_0 \ a, i_1 \ b], [i_0 \ a, i_1 \ a, i_2 \ a], \dots \}$$

Definite descriptions for individual concepts

Take the smallest individual concept of this set – i.e. the intersection of the individual concepts, if this is an individual concept.

$$(36) \quad \llbracket \text{the } \alpha \rrbracket = i[\llbracket \rrbracket(i)], \text{ if this is an individual concept, else undefined.}$$

Recall that functions are sets of pairs, e.g. $[i_0 \ a, i_1 \ b] = \{ i_0, a, i_2, b \}$.

$$(37) \quad \text{a. } \llbracket \text{the [law student]} \rrbracket(i_0) = \text{LAW STUDENT}(i_0) = [i_0 \ a] \\ \text{b. } \llbracket \text{the [law student]} \rrbracket(i_1) = \text{LAW STUDENT}(i_1) = \text{— undefined!}$$

Possible referential reading, e.g. in *the law student could be Miss America*.

$$(38) \quad \llbracket \llbracket \text{the [law student]} \rrbracket_R \rrbracket(i_0) \\ = i \ u[u = \llbracket \llbracket \text{the [law student]} \rrbracket \rrbracket(i_0)(i_0)] \\ = i \ u[u = [i_0 \ a](i_0)] = i \ u[u = a] \\ = [i_0 \ a, i_1 \ a, i_2 \ a, i_3 \ a, i_4 \ a]$$

3. The interpretation of MCRs

3.1 Intended interpretation

(39) *the accomplished mathematician Bill supposedly is*

should denote for each index i the individual concept c such that

- c is defined for the indices i that are compatible with what is supposed to be the case at i ,
- for all indices i where c is defined, Bill is an accomplished mathematician at i ,
- for all indices i where c is defined, c picks out Bill at i

(40) Exemplifying model:

	a	b	c	d	e
i_0		B			
i_1		B			
i_2		B			
i_3		B			
i_4		B			

Table 5:
Property ACC. MATHEMATICIAN*:
shaded
Individual concept BILL:
BBBBB
SUPPOSEDLY(i_0) = { i_2, i_3, i_4):
double framed area

$$(41) \quad \llbracket \text{Bill} \rrbracket(i_0) = [i_0 \ b, i_1 \ b, i_2 \ b, i_3 \ b, i_4 \ b]$$

$$(42) \quad \llbracket \llbracket \text{the accomplished mathematician Bill supposedly is} \rrbracket \rrbracket(i_0) \\ = [i_2 \ b, i_3 \ b, i_4 \ b]$$

	a	b	c	d	e
i_0					
i_1					
i_2		B			
i_3		B			
i_4		B			

Table 6:
the individual concept
 $\llbracket \llbracket \text{the accomplished mathematician} \\ \text{Bill supposedly is} \rrbracket \rrbracket(i_0)$:
BBB

3.2 Compositional derivation of this interpretation

3.2.1 Why the standard theory is problematic

Standard interpretation of relative clauses as properties that intersect with noun meaning:

$$(43) \quad \text{a. } \llbracket \llbracket 1[\text{that Bill met } t_1] \rrbracket \rrbracket = i \ x_1[\text{MET}(x_1)(\text{BILL})] \\ \text{b. } \llbracket \llbracket \llbracket \llbracket \text{woman } 1[\text{that Bill met } t_1] \rrbracket \rrbracket \rrbracket \\ = i \ x[\llbracket \llbracket \text{woman} \rrbracket \rrbracket(i)(x) \ \llbracket \llbracket 1[\text{that Bill met } t_1] \rrbracket \rrbracket(i)(x)] \\ = i \ x[\text{WOMAN}(i)(x) \ \text{MET}(i)(x)(\text{BILL})] \\ = i \ x[\text{WOMAN}^*(i)(x(i)) \ \text{MET}^*(i)(x(i))(\text{BILL}(i))]$$

Problem for MCRs: head noun does not get interpreted in the scope of the modal operator:

- (44) a. $\llbracket \llbracket \text{accomplished mathematician} \rrbracket \ 1[(\text{that supposedly } [Bill \text{ is } t_1])] \rrbracket$
 b. $= \lambda i \lambda x \llbracket \llbracket \text{accomplished mathematician} \rrbracket (i)(x) \rrbracket$
 $\llbracket 1[(\text{that supposedly } [Bill \text{ is } t_1])] (i)(x) \rrbracket$
 c. $= \lambda i \lambda x [\text{ACC.MATH}(i)(x(i)) \quad i \text{ SUPPOSED}(i)[BILL(i) = x(i)]]$

This property applies to individual concepts that are accomplished mathematicians at the index of interpretation i and that are supposedly identical to Bill. Not the intended result.

3.2.2 A head internal analysis of relative clauses?

Cf. Bhatt (1999); Sauerland (2002) for overview.

Phenomena to be described:

- (45) a. *the relative of his₁ that every boy₁ likes best*
 b. *the amount of headway that we made*
 c. *the longest book Bill said Tolstoy wrote*

Structure being suggested:

- (46) a. *the [relative of his]₂ that every boy₂ likes [relative of his]₁ best*
 b. *the [accompl. mathematician]₁ that supposedly [Bill is [accompl. mathematician]₁]*

Various problems, cf. e.g. Jacobson (2002), Sauerland & Hulsey (2003).

3.2.3 A head external analysis of relative clauses with individual concepts

Cf. also Sauerland & Hulsey (2003).

Proposed meaning rule for relative clauses:

- (47) $\llbracket \llbracket \text{NP} \llbracket \text{NP} \rrbracket \ n \llbracket \text{CP} \rrbracket \rrbracket$
 $= \lambda i \lambda x \llbracket \llbracket \text{DOM}(x) \llbracket \llbracket \rrbracket (i)(x) \rrbracket \llbracket n \llbracket \text{CP} \rrbracket \rrbracket (i)(x) \rrbracket$

- The nominal head restricts the individual concepts x to those concepts that are at every index for which they are defined;
- the relative clause restricts the concepts x further to those that are at the index of evaluation.

- (48) $\llbracket \llbracket \text{NP} \llbracket \text{NP} \text{ accomplished mathematician} \rrbracket \ 1[(\text{that supposedly } [Bill \text{ is } t_1])] \rrbracket \rrbracket$
 $= \lambda i \lambda x \llbracket \llbracket \text{DOM}(x) [\text{ACC.MATH}(i)(x)] \quad i \text{ SUPPOSED}(i)[BILL(i) = x(i)] \rrbracket \rrbracket$
 $= \lambda i \lambda x \llbracket \llbracket \text{DOM}(x) [\text{ACC.MATH}^*(i)(x(i))] \quad i \text{ SUPPOSED}(i)[BILL(i) = x(i)] \rrbracket \rrbracket$

- A function from indices i to functions from individual concepts x to truth values,
- to truth iff for all indices i in which x is defined, x is an accomplished mathematician, and x is identical to Bill in all worlds i that are compatible with what is generally supposed at i .

Notice: for all indices that are compatible with what is generally supposed at i , Bill must be an accomplished mathematician.

It appears as if the modal operator would scope over the head noun. This illusion comes about because the modal operator restricts the domain of the individual concept.

Example w.r.t. model of (40), Table 5, repeated here:

(49)

	a	b	c	d	e
i_0		B			
i_1		B			
i_2		B			
i_3		B			
i_4		B			

Property ACC. MATHEMATICIAN*:
 shaded
 Individual concept BILL:
 BBBBB
 SUPPOSEDLY(i_0) = $\{i_2, i_3, i_4\}$:
 double framed area

- (50) $\llbracket \llbracket \text{NP} \llbracket \text{NP} \text{ accomplished mathematician} \rrbracket \ 1[(\text{that supposedly } [Bill \text{ is } t_1])] \rrbracket (i_0) \rrbracket$
 $= \lambda x \llbracket \llbracket \text{DOM}(x) [\text{ACC.MATH}^*(i)(x(i))] \quad i \text{ SUPPOSED}(i_0)[BILL(i) = x(i)] \rrbracket \rrbracket$
 $= \llbracket \llbracket i_2 \quad b, i_3 \quad b, i_4 \quad b \rrbracket \quad 1, \rrbracket$
 $\llbracket i_2 \quad b, i_3 \quad a, i_4 \quad b \rrbracket \quad 0, \rrbracket$ (this concept b for i_3)
 $\llbracket i_2 \quad b, i_3 \quad b \rrbracket$: undefined, (not defined for all $i \text{ SUPPOSED}(i_0)$)
 $\llbracket i_0 \quad b, i_2 \quad b, i_3 \quad b, i_4 \quad b \rrbracket \quad 0, \rrbracket$ (b not an acc. math. in i_0)
 $\llbracket i_1 \quad b, i_2 \quad b, i_3 \quad b, i_4 \quad b \rrbracket \quad 1, \rrbracket$
 ...]

Predicate applies to two ind. concepts: $[i_2 \quad b, i_3 \quad b, i_4 \quad b]$, $[i_1 \quad b, i_2 \quad b, i_3 \quad b, i_4 \quad b]$.

3.2.4 Application of definite article

singles out the smallest of these individual concepts:

- (51) $\llbracket \llbracket \text{the} \llbracket \text{NP} \llbracket \text{NP} \text{ accomplished mathematician} \rrbracket \ 1[(\text{that supposedly } [Bill \text{ is } t_1])] \rrbracket (i_0) \rrbracket$
 $= \llbracket \llbracket x \mid \llbracket \text{DOM}(x) [\text{ACC.MATH}^*(i)(x(i))] \quad i \text{ SUPPOSED}(i_0)[BILL(i) = x(i)] \rrbracket \rrbracket \rrbracket$
 $= [i_2 \quad b, i_3 \quad b, i_4 \quad b]$

3.3 Predications involving MCRs

3.3.1 Modal predication on simple individual concept

- (52) a. $\llbracket \llbracket [Bill] \llbracket \text{should have solved the problem} \rrbracket \rrbracket$
 b. $= \lambda i \llbracket \llbracket \llbracket \text{should have solved the problem} \rrbracket (i) \llbracket [Bill] \rrbracket (i) \rrbracket$
 c. $= \lambda i \llbracket \llbracket x \mid \text{EXPECTED}(i)[\text{SOLVE.THE.PROBLEM}(i)(x)(BILL)] \rrbracket \rrbracket$
 d. $= \lambda i \llbracket \llbracket \text{EXPECTED}(i)[\text{SOLVE.THE.PROBLEM}(i)(BILL)] \rrbracket \rrbracket$
 e. $= \lambda i \llbracket \llbracket \text{EXPECTED}(i)[\text{SOLVE.THE.PROBLEM}^*(i)(BILL(i))] \rrbracket \rrbracket$

the proposition that maps indices i to truth iff for all indices i that are compatible with what would have been expected at i , Bill solved the problem at i .

- (53) a. $\llbracket \llbracket \llbracket \text{the accomplished mathematician that Bill supposedly is} \rrbracket \llbracket \llbracket \text{should have solved the problem} \rrbracket \rrbracket \rrbracket$
 b. $= \lambda i \llbracket \llbracket \llbracket \text{EXPECTED}(i)[\text{SOLVE.THE.PROBLEM}(i) \rrbracket$
 $\quad (\llbracket \llbracket x \mid \llbracket \text{DOM}(x) [\text{ACC.MATH}^*(i)(x(i))] \rrbracket \rrbracket \rrbracket$
 $\quad \quad \llbracket \llbracket \text{EXPECTED}(i)[BILL(i) = x(i)] \rrbracket \rrbracket \rrbracket$

This proposition maps every index i to truth iff the following holds: For every index i that is compatible with what is expected at i , the individual concept c described by *the accomplished mathematician that Bill supposedly is* solved the problem at i .

For this to be true, this individual concept c must exist at i , that is, it has to be defined for i , or technically, it must hold that $i \in \text{DOM}(c)$. As $\text{DOM}(c) = \text{SUPPOSED}(i)$, the index i must be one that is compatible with what is generally supposed to be true at i .

Visualization in model

(54)

	a	b	c	d	e
i_0					
i_1					
i_2		B			
i_3		B			
i_4		B			

Table 7:
Property SOLVE.THE.PROBLEM*:
shaded
Worlds EXPECTED(i_0):
triple frame

3.4 Explanation of observations

3.4.1 Compatibility between modalities, cf. 1.2.5

(55) # *The acc. mathematician Bill supposedly is seems to be working on a hard problem.*

It must hold that $\text{EXPECTED}(i_0) \subseteq \text{SUPPOSED}(i_0)$, as $\text{SUPPOSED}(i_0)$ is the domain of the individual concept x described by *the accomplished mathematician that supposedly Bill is*, and the main clause says that in all worlds i in $\text{EXPECTED}(i_0)$ the individual $x(i)$ solved the problem in i , that is, x must be defined for i .

3.4.2 External modality requirement, cf. 1.2.4

(56) # *The accomplished mathematician Bill supposedly is solved the problem.*

To be true, the world of evaluation i_0 must be an element of $\text{SUPPOSED}(i_0)$. Two problems:

- With modal *supposedly* it is indicated that the evaluation word i_0 might not be in the accessible world.
- If $i_0 \in \text{SUPPOSED}$, then the truth value of (56) is the same as of *Bill solved the problem*, which is shorter, blocking the longer expression.

3.4.3 Internal modality requirement, cf. 1.2.3

(57) # *The accomplished mathematician Bill is should have solved the problem.*

In an evaluation world i_0 , *the accomplished mathematician Bill is* refers to the individual concept $[i_0 \ b]$ in case Bill is an accomplished mathematician in i ; otherwise it not defined.

Problems:

- The individual concept is defined in the evaluation world; the modal *should* is not guaranteed to access the real world.
- The individual concept is defined only for the evaluation world; the modal *should* needs to access other worlds as well.

- The individual concept is defined only for one word; the modal *should* needs to access more than one world.

Possible interpretation strategy: The referential interpretation, cf. below. But then Speaker could have used the simpler expression, *Bill*.

(58) [*The accomplished mathematician Bill is*]_R *should have solved the problem.*

But notice that this expression adds the presupposition that Bill is an accomplished mathematician, which might be felicitous, cf. e.g. constructions like:

(59) (*Being*) an accomplished mathematician, *Bill should have solved the problem.*

3.4.4 The definiteness requirement, cf. 1.2.2

(60) # *An accomplished mathematician that Bill supposedly is should have solved the problem.*

Only the definite article creates the required individual concept. The indefinite NP quantifies over arbitrary individual concepts that all accomplished mathematicians and that are identical to Bill in the worlds made accessible by *supposedly*. These concepts are not restricted by *supposedly* and quite unconstrained.

4. Some further cases

4.1 Negation or confirmation of existence

(61) *The happy couple that Charles and Diana appeared to be never in fact existed / in fact existed.*

No modal element in main clause, violation of external modality requirement.

But: Negation of existence or confirmation of existence of an individual concept is informative.

(62) a. $\llbracket \text{exist} \rrbracket(i)(x) = u[u = x(i)]$
b. $\llbracket \text{doesn't exist} \rrbracket(i)(x) = \neg u[u = x(i)]$, i.e. $x(i)$ is undefined.

(63) $\neg u[u = [\{x \mid i \in \text{DOM}(x)[\text{HAPPY COUPLE}^*(i)(x(i))]$
 $i \text{ APPEAR}(i_0)[\text{CH DI}(i) = x(i)] \}](i_0)]$

4.2 Appeal to nonexisting concepts

(64) *I am addressing (this appeal to) the idealist you claim to be.*

Violation of external modality requirement.

But: Verbs like *address*, *appeal to*, *admire*, *pray to* etc. can involve concepts not realized in the world of evaluation: $\text{ADDRESS}(i)(x)$ in this sense cannot be reduced to $\text{ADDRESS}^*(i)(x(i))$.

(65) $\text{ADDRESS}(i_0)(\{x \mid i \in \text{DOM}(x)[\text{IDEALIST}^*(i)(x(i))]$
 $i \text{ CLAIM}(\text{YOU})(i) [\text{YOU}(i) = x(i)] \})$

4.3 Temporally defined concepts

(66) *I am addressing this appeal to the idealist you once were, not to the opportunist you have become.*

No modal, but temporal operator: Temporally defined individual concept.

- (67) $\llbracket \textit{idealist you once were} \rrbracket (t_0)$
 $= t[t \text{ DOM}(x)[\text{IDEALIST}(t)(x)] \quad t \text{ ONCE}(t_0)[\text{YOU}(t) = x(t)]]$,
 where $\text{ONCE}(i_0)$ selects a time interval of the past w.r.t. i_0 .

4.4 Evaluative statements

(68) *The abominable atrocity that the stoning of these women was must not go unpunished.*

(69) *#The attack that the stoning of these women was must not go unpunished.*

With the classification of an event as *abominable atrocity*, Speaker invokes deontically accessible worlds: In all those worlds, the stoning is classified as an abominable atrocity.

These appears to be characteristic of evaluative statements of this type:

- (70) a. *The stoning of these women happened to be an act watched by a witness.*
 b. *#The stoning of these women happened to be an abominable atrocity.*

Proposal for evaluative statements, cf. treatment of indefinites in (22):

- (71) *This stoning is an atrocity.*
 $i \text{ DEONT}(i_0) \quad x[\text{ATROCITY}^*(i)(x(i)) \quad \text{THIS STONING}(i) = x(i)]$

Interpretation of example, in analogy to previous one:

- (72) a. $\llbracket \textit{the atrocity that the stoning was} \rrbracket (i_0)$
 $= \{x \mid i \text{ DEONT}(i_0)[\text{ATROCITY}^*(i)(x(i)) \quad \text{THIS STONING}(i) = x(i)]]$

This singles out the smallest individual concept x that is an atrocity wherever it is defined, and it is defined for all worlds that are deontically accessible from i_0 , and is identical to the stoning in those worlds. We have to assume that the domain of i is further restricted to those worlds for which THIS STONING is defined, i.e. for those in which it exists.

4.5 A counterexample by a reviewer?

(73) *#The current US president that John claims to be created the problem.*

How is the oddness predicted? Construction of the NP:

- (74) $\llbracket \textit{current US president that John claims to be} \rrbracket (i_0)$
 $= x \quad i \text{ DEONT}(i_0)[\text{CURRENT}_{i_0} \text{ US PRESIDENT}^*(i)(x(i))]$
 $i \text{ CLAIM}(\text{JOHN}(i_0))(i_0) [\text{JOHN}(i) = x(i)] \}$

Current picks out the context world (i_0); this will either lead to an individual concept that is only defined for i_0 or to a rigid designator identical with the value of this individual concept for i_0 . On the first interpretation: individual concept is degenerated. On the second: Individual concept is defined for all worlds compatible with John's claim to be George W. Bush, and picks out John / GWB in these worlds. To say that this individual created the problem in the real world implies that John is right in his claim to be George W. Bush. But then the statement can be expressed more simply: *John / GWB created the problem.*