More on the Approximative Interpretation of Number Words

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From the land of bankers and watchmakers



Street sign in Kloten, Switzerland.

The general goals of this talk:

-- Explain why this is peculiar.

-- Draw general conclusions about language and language use.

Overview of the talk:

- 1. Precise/Approximate Interpretations of number words: The basic phenomenon
- Selection of optimal expressions and interpretations with preference for short expressions, approximate interpretations, and bidirectional OT: Krifka 2002
- Selection of optimal expressions and interpretations with preference for short expressions and strategic communication: Krifka 2005
- 4. Selection of optimal scales
- 5. Principles for the construction of optimal scales
- 6. Adaptation of scales to language use: shortening of common expressions
- 7. Adaption of language use to scales: under-use of complex expressions
- 8. A new theoretical perspective on language and language use

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Precise vs. Approximate Interpretations of Number Words: The basic phenomenon

There were forty people at the workshop.

Speaker may use precise or approximate interpretation (or rather: interpretations of various precision levels).

The context may favor particular precision levels (informal talk vs. report to funding agency) but in many cases leaves choice of precision level open within a given range.

Precision level can be indicated explicitly: *There were <u>exactly</u> forty people at the conference. There were <u>about</u> forty people at the conference.*

Precision level can be indicated implicitly with choice of number words: *There were forty people at the workshop.* => approximate interpretation *There were thirty-eight people at the workshop.* => precise interpretation

How does speaker infer precision level by choice of number words?

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Explanation by general pragmatic principles

(Krifka 2002: 'Be brief and vague! And how bidirectional OT allows for verbosity and precision')

1. A preference for simple expressions

(cf. G. K. Zipf (1929), Relative Frequency: Principle of least effort A. Martinet Elements de linguistique générale (1960): Speaker economy Horn 1984 ff: R-Principle Levinson Presumptive Meanings 2000: I-Principle) forty > thirty-eight (where a > b: 'a preferred over b')

- A preference for approximate interpretations (cf. P. Duhem 1904, balance between precision an certainty; Ochs Keenan 1976, vagueness helps to save face; reduction of cognitive effort) aproximate > precise
- 3. Interaction of the two principles following Bidirectional Optimality Theory (Blutner, 'Some aspects of optimality in natural language interpretation', *JS* 2000 Jäger, 'Some notes on the formal properties of bidirectional OT', *JoLLI* 2002)

Optimal expression-interpretation pairs

 Interaction of the two principles following Weak Bidirectional OT (Blutner, Jäger): An expression-interpretation pair ⟨F, M⟩ is **optimal** iff there are no other **optimal** pairs ⟨F', M⟩ or ⟨F, M'⟩ such that ⟨F', M⟩ > ⟨F, M⟩ or ⟨F, M'⟩ > ⟨F, M⟩.



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Models of Precise and Approximate Interpretations

- Approximate interpretations of various granularity levels; various ways of implementation:
- 1. Precise / approximate by intervals: [[thirty-eight]]_{prec} = 38, [[thirty-eight]]_{appr} = [34..42]
- 2. Precise and approximate by intervals: [[thirty-eight]]_{prec} = [38..38], [[thirty-eight]]_{appr} = [34..42]
- 3. Various levels of precision: $[[n]]_{1/m} = [n \pm n \cdot 1/m]$, e.g. $[[thirty-eight]]_{1/10} = [38 \pm 3.8] = [34.2 .. 41.8]$ precise interpretation as borderline case: $[[thirty-eight]]_{1/\infty} = [38 \pm 0] = [38..38]$
- Various levels of precision, normal distribution indicating level of fit, indicating standard deviation as acceptable levels of fit. [[thirty-eight]]_{1/10} = (38 ±3.8) = (34,2...41,8)



Explanation by Principles of Strategic Communication

cf. Krifka (2005/to appear), 'Approximate interpretation of number words: A case for strategic communication'

Basic assumptions:

- · Preference for short expressions, as before.
- No general preference for approximate interpretations
- Strategic communication, cf. Parikh 1991, "Communication and strategic inference"; Parikh 2000, *Communication, meaning and interpretation* General principle:
- -- For hearer: Assume the speaker intends the most likely interpretation, given the choice of expressions and a-priori likelihood of message
- -- For speaker: Assume the hearer selects the most likely interpretation, given the choice of expressions and a-priori likelihood of message

Question:

Given the a-priori-likelihood of the communicated information and general interpretation strategies,

what is the most likely interpretation of an ambiguous or vague form?

Strategic Communication: Explanation of Precise/Approximate Interpretations

Assumptions (repeated):

- General tendency for short expressions
 provided that they are interpretatively equivalent
- No preference for precise or approximate interpretations:
 - e.g. two interpretations, prec and appr, with p(prec) = p(appr) = 0.5
- Within a given range,
 a-priori probability of values is comparable or equal,
 e.g. p(38) = p(39) = p(40) = p(41) = p(42) = 0.02
- Values that differ minimally (under discriminatory threshold) are interpretatively equivalent:
 - e.g. $[[thirty-eight]]_{1/10} \approx [[forty]]_{1/10}$,
 - as [[*thirty-eight*]]_{1/10} = (38 ± 3.8) = (34.2 .. 41.8) [[*forty*]]_{1/10} = (40 ± 4) = (36 .. 44) 40∈[34.2 .. 41.8] 38∈[36 .. 44]

Strategic Interpretation: Explanation of Precise/Approximate Interpretations



Strategic Interpretation: Explanation of Precise/Approximate Interpretations



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Length of expression does not always count

Preference for short **expressions** cannot explain all interpretation preferences:

	I did the job in twenty-four hours. I did the job in twenty-three hours. I did the job in twenty-five hours.	approximate precise precise
	The house was built in twelve months. The house was built in eleven months. The house was built in thirteen months.	approximate precise precise
	Two dozen bandits attacked him. Twenty-four bandits attacked him.	approximate precise
. 6	and sometimes even makes the wrong pre	dictions:
	Mary waited for forty-five minutes. Mary waited for forty minutes.	approximate precise
	I turned one hundred and eighty degrees. I turned two hundred degrees.	approximate precise
	Her child is eighteen months. Her child is twenty months.	approximate precise
	John owns one hundred sheep. John owns ninety sheep.	approximate precise

...

Alternative theory: A preference for simple, coarse-grained representations?

Coarse vs. fine grained levels of representation

Cf. P. Curtin (1995), Prolegomena to a theory of granularity, U Texas MA Thesis



The train will arrive in eighteen minutes. The train will arrive in twenty minutes. (possibly two scales) The train will arrive in fifteen minutes. The train will arrive in thirty minutes. (?)

Reinterpretation of approximate interpretation: Selection of coarsest level in which number word occurs

The train will arrive in fifteen minutes.

a.	3-4-5-6-7-8-9-10-11-12-1	3-14-15-16-17-	18-19-20-21-22-	-23-24-25-26-27-	-28-29-30-31	
b.	510	15	20	25	30	
с.	10		20		30	
d.		15			30 🚄	۶D
0					30	-

The train will arrive in twenty minutes.

a.	3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-21-22-23-24-25-26-27-28-29-30-31					
b.		15	20	25	30	
с.	10		20		30	F
d.		15			30	-
e.					30	

The train will arrive in eighteen minutes.

a.	3-4-5-6-7-8-9-10-11-12-13-14-15-16-17 -18 -19-20-21-22-23-24-25-26-27-28-29-30-31	-
b.		
с.	2030	
d.	30	
e.	30	

Options for interpretation for Coarse/Fine-Grained Scales

```
1. Values of measure functions are intervals, not numbers, yet named by numbers
  min_{a}(d) = [18]. named 18
  \min_{b}(d) = [17.5 .. 22.5], named 20
  min_{a}(d) = [15 ... 25]. named 20
  min_{d}(d) = [7.5 .. 22.5], named 15
  \min(d) = [15 ... 45], named 30
a. 3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-[18]-19-20-21-22- 23-24-25-26-27-28-29-30-31
d. ______15______30_____30____
  P
2. Values of measure functions are numbers, but their domain is constrained;
  durations are mapped to the number with best fit.
  DOM(min_{2}) = \{1, 2, 3, 4, ...\}
  DOM(min_{b}) = \{5, 10, 15, ...\}
  DOM(min_{o}) = \{10, 20, 30, ...\}
  DOM(min_d) = \{15, 30, 45, ...\}
  DOM(min_{e}) = \{30, 60, 90, ...\}
Notice:
  In either case: Additivity does not hold, as min(x y) = m(x) + m(y) is not always satisfied;
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e.g. if x and y are durations of 14 minutes each, then x^y is a duration of 28 minutes, min_c(x) = 10, min_d(y) = 10, min_c(x^y) = 30, but min_c(x) + min_c(y) = 20 (!)

Explanation of coarseness level choice by strategic communication

Assume each coarseness level is selected with equal probability,

p(a) = p(b) = p(c) = p(d) = p(e) = 0.2

Assume a-priori equal probabilities of durations, p(15) = p(16) = ... = p(20) = 0.01



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Coarse scales and simple expressions: An evolutionary perspective on brevity?

It is hardly an accident that for many, perhaps most scales, coarse-grained scales have expressions of reduced complexity (cf. Krifka 2002)

Example: Complexity of number words on scales, average number of syllables

a. one, two, three, four, one hundred:	273/100 = 2.73
b. one, five, ten, fifteen, one hundred:	46/20 = 2.3
c. one, ten, twenty, thirty, one hundred:	21/10 = 2.1

Suspicion:

Scales develop in a way to enable complexity-based optimization, expressions of coarse-grained scales tend to be simpler.

An evolutionary perspective on brevity? The optimization of scales.

Scales and hierarchies of scales of different granularity have to satisfy certain requirements to be useful for communication:

- Requirement for scales: Equidistance of units (additive, sometimes logarithmic, cf. decibel; deci-/milli-/micro-/nano-/pico-, kilo-/mega-/giga-/tera-)
- 2. Requirements for scale hierarchies of different granularity: Scales of increasing granularity $S_n S_{n+1} S_{n+2}$ should increase granularity by the same factor,

example: powers of 10 [10, 20, 30, 40, 50, 60, ...] [100, 200, 300, 400, 500, ..] [1000, 2000, 3000, 4000, ...]

where the most natural step is decrease granuality by factor 1/2:



cf. hour scale: [1h, 2h, ...], [30min, 1h, 1h30min, ...], [15min, 30min, 45min, 60min, ...]

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An evolutionary perspective on brevity? The optimization of scales

The expressions of values of scales align with the optimization of scales Example: Expression of half points between powers of ten Roman number writing (also motivated iconically, by shape of hand)

I II III IV V VI VII VIII IX X X XX XXX XL L LX LXX LXXX XC C C CC CCC CD D DX DXX DXXX XM M

Simplification of number word 'five':

English: *fifteen* (**fiveteen*), *fifty* (**fivety*):

loss of diphthong, shortening
OE *fi:f* as word vs. *fif-* as prefix; vowel shift only affected i: (> ai)

Simplification of 'fifteen', 'fifty' in colloquial German: *fuffzehn*, *fuffzig* vs. regular *fünfzehn*, *fünfzig*:

unrounding ü > u, loss of n, shortening (3 morae to 2 morae)

Simplification of 'half' in German *anderthalb* 'one and a half', lit. 'the second half' vs. regular *eineinhalb*Simplification of 'fifty' in Danish: *halvtreds* vs. older *halvtredsind-s-tyve*,

similarly for 'seventy', 'ninety'

The optimization of scales in language change

Development of number system in Old World history:

Babylonian system: based on 60;

60 and multiples thereof as estimation number e.g. in Ancient Greek (K. Menninger, 1962, *Number words and number symbols*):

The swineherd always sent them the best one of the fattened pigs for them to feast on, and the number of the swine remaining was only three hundred and sixty. (Odyssey)

Different complexity of number words in ancient languages; break after "60"; has disappeared in modern languages

	Greek	Anglo-Saxon	Gothic	Celtic
50	penté-konta	fiftig	funf-tigjus	coí-ca
60	hexé-konta	sixtig	saihs-tigjus	ses-ca
70	hebdomé-konta	hund-seofontig	sibunt-e-hunt	secht-moga
80	ogdoé-konta	hund-ahtatig	ahtaút-e-hund	ocht-moga

e.g. Gothic: 'five-ten', 'six-ten' / 'seven-ness', 'eight-ness'

Evidence for preferred reference points: Frequency of number words

- If fine-grained / coarse-grained scales are used to report measurements, and if with coarse-grained scales, only certain number words occur, then these number words should occur more likely
- in a natural linguistic corpus containing measurement reports.
- Cf. Dehaene & Mehler (1992), Cross-linguistic regularities in the frequency of number words, *Cognition* 43,

Jansen & Pollmann (2001), On round numbers: Pragmatic aspects of numerical expressions. Journal of Quantitative Linguistics 8,

Corpora of English, French, Dutch, Japanese, Kannada:

- Between 10 and 100, the powers of ten occur most frequently
- Frequency decreases with higher powers of 10, but local maximum for 50
- Between 10 and 20, local maxima at 15, also at 12 ("dozen")
- Example: Occurrences of number words in British National Corpus, after H. Hammarström (2004), Properties of lower numerals and their explanation... (ms.)



Evidence: Frequency of number words

Frequency of round numbers on *-aine* in French French web sites of Google, April 11, 2005, search for strings "une quarantaine de"

dixaine	4.230	vingtaine	737.000
onzaine	19	trentaine	866.000
douzaine	262.000	quarantaine	272.000
treizaine	16	cinquantaine	490.000
quatorzaine	47	soixantaine	159.000
quinzaine	540.000	septantaine	614
seizaine	6	quatre-vingtaine	85
dix-septaine	1	quatre-vingtdixaine	0
dix-huitaine	11	centaine	548.000
dix-neuvaine	1		

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Still an effect of complexity of expression?

In vigesimal number systems, '50' is more complex than '40'/'60' Question:

Is '50' nevertheless used as approximate number word? Conflict cognitive preference / communicative preference

Cf. Hammarström (2004), Number bases, frequencies and lengths cross-linguistically.

Inspired by this work: Investigation of occurrences of number words on Norwegian vs. Danish web sites of Google (March 4, 2005)

Number	Norwegian	Occurrence s	Danish	Occurrence s
20	tjue	61300	tyve	121000
30	tretti	43700	trediv e	25400
40	førti	39200	fyrre	26800
50	femti	81200	halvtreds	15500
60	seksti	19400	tres	36400
70	sytti	10200	halvfjerds	581
80	åtti	13100	firs	3740
90	nitti	13500	halvfems	540

Norwegian / Danish number words



Experiment on number estimation: Decimal vs. vigesimal systems

Problem of corpus analysis:

There might be a wide variety of reasons for number frequency

Hence: Experimental evidence is necessary.

Number estimation experiment:

Subjects should estimate number of dots on a screen; all numbers between 12 and 100 were shown once

in nearly random order;

(two subsequent pictures differed by at least 15 points).

Hypothesis:

Speakers of languages with a vigesimal system use complex multiples of 10 less often, (i.e. number words for 50, 70, 90)

than speakers of languages with decimal system.

First results:

Danish (thanks to Anton Benz; 20 subjects) Norwegian (thanks to Torgrim Solstad; 20 subjects); Georgian (thanks to Rusudan Asatiani, 10 subjects); Basque, German, Dutch, English, Spanish...: Not yet completed.

Complexity of multiples of 10 in Danish, Basque, Georgian

Example of test items

Danish	Basque	Georgian
ti	hamar	ati
tyve	hogei	otsi
tredive	hogei ta hamar	otsdaati
fyrre	berrogei	ormotsi
halvtreds (halvtredsindstyve)	berrogei ta hamar	ormotsdaati
tres	hirurogei	samotsi
halvfjerds (halvfjerdssindstyve)	hirurogei ta hamar	samotsdaati
firs	larrogei	otkhmotsi
halvfems (halvfemsindstyve)	larrogei ta hamar	otkhmotsdaati
hundrede	ehun	asi









Example of test items

Number of dots: 79 - 30 - 73 - 11

Notice:

dots are spaced out regularly to allow for numerosity-based estimations (Stanislas Dehaene 1997, *The number sense*)

Results of Number Estimation Experiment, Norwegian / Danish / Georgian



- Higher numbers words are used less often, especially by Georgian and Danish speakers (this is significant)
- No significant difference between Norwegian / Danish / Georgian speakers for 50
 -- against hypothesis.
- But there is a peak for Norwegians at "70", not for Danish and Georgian speakers; significant difference for "70", < 0.01 (for Norwegian vs. Danish)

Norwegian/Danish number estimation experiment: Discussion

Lack of valley at "50" for Danish/Georgian speakers:

This scale position has high cognitive prominence (at least for the population the subjects are drawn from -- university students)

- that the complexity of number words is irrelevant.
- Explanation of peak at 70 with Norwegian speakers:
- 1. "50" is a natural estimation number, cf. Dehaene and others.
- Differences of +/- 5 are difficult to recognize for sets around size 50; differences of +/- 10 are recognizable.
- 3. As "50" is a natural attractor, "60" is a less natural attractor than expected, as it is not sufficiently distinct from "50"
- 4. "70" is a better attractor, as it is sufficiently distinct from "50"
- Explanation of lack of peak for "70" for Danish and Georgian speakers: Even though "70" is a natural attractor for cognitive reasons, the complexity of the number word for "70" mitigates against it.

Why difference between corpus study and experiment?



Possible explanation (Jason Mattausch):

Use of arabic numbers in written text; should appear more frequently in Danish

-- planned corpus study



Numerals in Norwegian vs. Danish Web Sites

• Frequent occurrence of number base "20" in Danish in spellect-out form

• Slight summit for "50" in Danish Arabic numerals, but more pronounced in Norwegian.

Problems of the experiment

- Influence of second native language, e.g. Basque: Spanish/French, Welsh: English
- Influence of formal education in school, better don't take university students as subjects!
- How to make subjects estimate higher numbers: Run experiment with numbers n, 40 < n < 100

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A new perspective on language and language use

- Common functionalist belief: "Grammars do best what speakers do most" (DuBois 1987)
- But perhaps we also have to assume: Sometimes speakers do most what grammars do best,

or perhaps: Speakers do least what grammars do worst.

Cf. Codability Hypothesis, Brown & Lenneberg (1954): (Color words and color recognition):

Meanings that are difficult to encode are expressed more rarely.

A new perspective on language

An analogy to economics: Optimization of transaction costs Example:

- Typical prices of goods at Dutch fleamarkets before January 1, 2002: 2.5 guilders (ca. 1.136 Euros) or multiples thereof (Rijksdaalder)
- Typical prices of goods at Dutch fleamarkets after January 1, 2002: 1 Euro (2.2 guilders) or multiples thereof



A new analogy:

language is money,

• meanings are goods.

The End

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