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QUANTIFIERS IN COMPARATIVES: A SEMANTICS OF DEGREE BASED ON INTERVALS*

The sentence *Irving was closer to me than he was to most of the others* contains a quantifier, *most of the others*, in the scope a comparative. The first part of this paper explains the challenges presented by such cases to existing approaches to the semantics of the comparative. The second part presents a new analysis of comparatives based on intervals rather than points on a scale. This innovation is analogized to the move from moments to intervals in tense semantics. The remainder of the paper is concerned with an interval-based semantics of degree in relation to issues other than the comparative proper. The paper begins with a discussion of the role negative polarity has played in studies on the semantics of comparatives.

1. Introduction

Scalar predicates are used to order individuals in their domain. To say that Fido is more noisy than Fida is to order him above her with respect to noisiness. Tall presupposes a height ordering, cold a temperature ordering, and so on. Generalizing on this ordering of individuals we arrive at the notion of a scale with a set of points representing the possible positions in the ordering that an individual might occupy. In many cases we invent names for these points: decibels, inches, degrees centigrade, dollar amounts, and so on. These points are the basis for comparison according to degree analyses of the comparative (Cresswell 1976; Hellan 1981; von Stechow 1984; Rullmann 1995). A simple version of this view says that if the dress is more expensive than the shirt, then there is a point on the expense scale corresponding to the dress, and it is above the point corresponding to the shirt. Similarly, if the point on the height scale corresponding to Frank is above the one for Meryl, then Frank is taller than Meryl. While this seems to be an intuitively satisfying story, the central claim of this paper is that:

(1) Scalar predicates have a semantics based on intervals, not points.

* We gratefully acknowledge the opportunity to present portions of this material at the Second Conference on (Preferably) Non-lexical Semantics, University of Paris 7, and at UC Irvine, MIT, Rutgers, SUNY Stony Brook, the University of Tübingen, the University of Pennsylvania, and before the Rutgers Undergraduate Cognitive Science Club. Special thanks to the participants of the Topics in Semantics course at Rutgers, Fall 1999. The paper has benefited from conversations with Veneeta Dayal, Irene Heim, Hans Kamp, Chris Kennedy, Cécile Meier, Arnim von Stechow, Maribel Romero, and Ede Zimmermann.

Natural Language Semantics **10**: 1–41, 2002.

The chief evidence for the claim in (1) will come from comparatives that contain quantifiers. The challenge these constructions present for degree analyses is easy to see. It may very well be that Frank is taller than everyone else is without there being a point on the scale corresponding to everyone else. Similarly, if the shirts range in price from \$20 to \$100 and the dress is \$150 then the dress is more expensive than the shirts are, yet there is no point that corresponds to the shirts. Clearly, in these cases there is an interval of the height scale corresponding to everyone but Frank which lies below Frank's height, and there is an interval of the price scale corresponding to the shirts which lies below the price of the dress.

A shift similar to the one proposed here for the semantics of scalar predicates occurred in our understanding of temporal expressions. Expressions like *before* and *after* as well as the tenses are used to order events. Generalizing on the ordering, we arrive at a time line with points on it and we have invented names for these points as well. Initially, tense logics were based on these points, called moments of time. It was later discovered that intervals of time and not moments would have to serve as the basis for a tense semantics for natural language (Bennett 1977; Bennett and Partee 1972; see also Cresswell 1985 for extensive discussion).

Comparison with temporal interval semantics will crop up at various points in our discussion. One parallel will be of particular importance. In executing a moment-based tense logic it is usually assumed that specific events are related to unique moments of time. When one moves to an interval semantics, uniqueness does not come automatically. If an event occurred during an interval it occurred during all superintervals. We will discover that the same applies in the move from points to intervals in the scalar domain.

Sections 2–4 below expose the inadequacies of degree-based analyses of the comparative. (Here and elsewhere we adopt the standard practice of using "degree" to mean 'point on a scale', contrary to our use of that term in the title. Hopefully no confusion will ensue.) While negative polarity items are a side issue here, they help to place the central issues in a broader setting, and so in section 2 we use their presence in comparatives as a starting point. Section 3 shows that the problem lies with the degrees and not with how the degree analyses are executed. Since the difficulties appear to arise only when quantifiers are present, section 4 dispenses with the possibility of solving these problems using existing mechanisms for establishing the scope of quantifiers. In sections 5–7 an interval semantics for the comparative is developed. Section 5 shows why we can't simply replace "point" with "interval" in existing frameworks. Section 6 fixes some general properties of scalar predicates as relations between individuals and intervals. Finally, the analysis of comparatives is worked out in detail in section 7. In sections 8–12 we explore various consequences of adopting a degree semantics based on intervals.

2. Negative Polarity and the Comparative

In "Negative Polarity and the Comparative," Hoeksema (1983) argues for a particular analysis of clausal comparatives on the basis of the ability of that analysis to explain the presence of negative polarity items (NPIs) in *than*-clauses like those in (2) below:

(2) a. This text is more difficult than <u>any</u> of the others were.
b. It is hotter in New Brunswick today than it <u>ever</u> was in L.A.
c. We ate more today than we've eaten <u>in weeks</u>.

Given the theory of Ladusaw (1979, 1980) and Fauconnier (1975, 1978), if the comparative were *downward entailing* with respect to the clausal argument of *than*, an NPI would be expected possible in these clauses. What Hoeksema shows is that downward entailingness indeed follows from the semantics of the comparative that he endorses. Similar arguments have since been made about other analyses of the comparative.[1]

To say that that the comparative is downward entailing with respect to the clausal argument of *than* is to say that the clause under *than* is a context in which implications are reversed. Evidence for implication reversal under the comparative often comes in the form of examples like those below from von Stechow (1984):

(3) a. Otto is fat.
b. Otto or Max is fat.

(4) a. Ede is fatter than Otto is.
b. Ede is fatter than Otto or Max is.

Whereas (3a) entails (3b), when they are embedded under the comparative, the entailment is "reversed". There is a reading of (4b) on which it entails (4a). But how reliable is this evidence? The entailment from (4b) to (4a) surely must be related to the general phenomenon observed with *or* whereby in various contexts *or* appears to have the meaning of *and*. Given that this phenomenon occurs outside comparative *than*-clauses (Ehrenkranz

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[1] See for example Heim (1985), Klein (1991), Moltmann (1992), Rullmann (1995), von Stechow (1984).

1973)[2] one might suspect that the *or* in (3) is not truth-conditionally equivalent to the one in (4). This is essentially the view of Lakoff (1971), but it runs counter to what is assumed in accounts of the comparative, many of which we will discuss in detail below. Cresswell (1976, fn. 10), Hoeksema (1983), von Stechow (1984), and others explicitly show how their semantics for the comparative combines with a disjunctive *or* to yield the entailment from (4b) to (4a). Interestingly, the same holds for the superficially different account of comparatives found in Klein (1982). In each of these cases, the entailment goes through because (4) is essentially read as meaning that Ede is fatter than the fatter of Otto and Max. A problem with this view is that it makes similar predictions for examples containing measure phrases. Sentence (5), for example, is read on these accounts as (6):

(5) It is 14° hotter here than it is in Madrid or Mexico City.

(6) It is 14° hotter here than it is in the hotter of Madrid and Mexico City.

But if (6) were an accurate paraphrase of (5), the following reasoning would be valid:[3]

(7) #If it is 44° here, 30° in Madrid, and 10° in Mexico City, then it is 14° hotter here than it is in Madrid or Mexico City.

Compare this to (8):

(8) If Ede is 6 ft, Otto is 5 ft, and Max is 4 ft, then Ede is taller than Otto or Max is.

The difference between (7) and (8) suggests that the *or* in these examples may in fact be a negative polarity item (cf. Larson 1988) which has a conjunctive interpretation in this context, in the way that negative polarity *any* or *ever* seem to have universal interpretations in the comparative.[4] If this view is correct, then (3b) is semantically distinct from the embedded

[2] This issue has been much discussed in connection with modals, see Zimmermann (2000), Vainikka (1987).

[3] This whole discussion is confounded by the fact that *or* is ambiguous in these examples (Lakoff 1971, p. 279). The true disjunctive reading is teased out by appending “but I can’t remember which” (e.g., *Ede is fatter than Otto or Max is, but I can’t remember which*). The purpose of the antecedent in (7) is to move away from that reading.

[4] Compare *if you* ***ever*** *saw a bulldog*, which paraphrases as *if you saw a bulldog at least once in the past*, versus *hotter than it* ***ever*** *was in New Mexico*, which roughly paraphrases as *hotter than it was in New Mexico at all past times*.

clause in (4b), which means that (3)–(4) cannot bear witness to the downward entailingness of comparatives.

The foregoing discussion shows that *or* data may not provide the strongest evidence for downward entailingness. However, since the claim that comparatives are downward entailing with respect to the clausal argument of *than* is a general one, we can simply leave *or* aside and test the claim using other expressions. For example, if the comparative were in fact downward entailing, then given the entailment patterns in (9), we would expect the entailments in (10) to go through:

(9) a. Exactly 7 of my relatives are rich. → At least 4 of my relatives are rich.
b. Given that there are elephants in this room: Almost every elephant in this room is heavy. → Some elephant in this room is heavy.
c. Most of the high tech stocks were overvalued. → At least 2% of the high tech stocks were overvalued.

(10) a.#John is richer than at least 4 of my relatives were.
→ John is richer than exactly 7 of my relatives were.

b.#My car is heavier than some elephant in this room is.
→ My car is heavier than almost every elephant in this room is.

c.#Nissan is currently more overvalued than at least 2% of the high tech stocks were.
→ Nissan is currently more overvalued than most of the high tech stocks were.

In fact, as indicated, these entailments are not intuitively valid. By comparison, observe that the entailment patterns in (9) are reversed when those clauses are embedded in bona fide downward entailing contexts:

(11) a. It isn't true that at least 4 of your relatives are rich.
→ It isn't true that exactly 7 of your relatives are rich.

b. It is impossible that some elephant in this room is drunk.
→ It is impossible that almost every elephant in this room is drunk.

c. He never admitted that at least 2% of the tech stocks were overvalued.
→ He never admitted that most of the tech stocks were overvalued.

Since the entailments in (10) do not go through we are entitled to conclude that the comparative is not downward entailing. This leads to wholesale rejection of existing analyses touted for their ability to explain downward entailingness.

This conclusion, it should be noted, does not depend on adopting the Ladusaw-Fauconnier theory of negative polarity for explaining the data in (2). It merely seizes on the claim that the comparative is downward entailing in the relevant sense. It should also be noted that the argument was presented in a form that was simplified relative to the way the analysis of comparatives works in most cases. The entailment relation in (9) relates propositional type expressions, yet clauses under *than* contributes sets of degrees to the meaning of the comparative (or sets of degree modifier meanings in the case of Klein 1982). For example, the embedded clauses of (10a) contribute sets like those in (12):

(12) a. {d: at least 4 of my relatives are d-rich}
b. {d: exactly 7 of my relatives are d-rich}

But this doesn't really change things. For any degree d, if exactly 7 of my relatives are d-rich, then at least 4 of my relatives are d-rich. (12b) is therefore a subset of (12a), and so the clause under *than* in *John is richer than exactly 7 of my relatives were* entails, in a generalized sense, the clause under *than* in *John is richer than at least 4 of my relatives were*. It is in this generalized sense that Hoeksema (1983) and others intend their claim of downward entailingness. Reading the arrow in (9) in this generalized sense, the argument we've made here still goes through. The comparative is not in fact downward entailing as claimed.

Attention to the type of the argument of *than* provides us with the opportunity to address another kind of evidence adduced for the downward entailingness of comparative *than*-clauses. Linebarger (1987, p. 378) illustrates the downward entailingness of the comparative operator with examples of the following sort:

(13) a. John eats raisin bread. → John eats bread.
b. The sun rises more often than John eats bread. → The sun rises more often than John eats raisin bread.

The sun rises more often than John eats bread compares how often the sun rises to how often John eats bread. The arguments of the two *than*'s in (13b) are therefore along the lines of the sets in (14) below:

(14) a. {d: John eats bread d-often}
b. {d: John eats raisin bread d-often}

This means that the entailment in (13a) is irrelevant. What is relevant is the relation between the sets in (14). Those sets presumably consist of frequencies, as the anaphoric *that* in the following suggests:

(15) John eats raisin bread twice a month. Would you wanna eat raisin bread that often?

In order to show a downward entailing pattern, then, we need to show that any frequency that is a member of the set in (14b) is a member of the set in (14a) – in other words, that (14b) is a subset of (14a). If that were the case, then one could say that in a generalized sense, (14b) entails (14a) and it is that entailment which is reversed in (13b). The crucial question then is whether (14b) really is a subset of (14a). If it were, it would be reasonable to draw the following inference:

(16) If John eats raisin bread twice a month, then John eats bread twice a month.

It is unclear to us whether this inference is valid or not. If this inference does not follow directly from the meanings of the subclauses, then we may argue that (14b) is not a subset of (14a). In that case, (13) represents another species of argument for downward entailingness that falls by the wayside.

Returning to the examples in (10), we can make an even stronger point. Reversing the direction of the arrows in (10) leads to entailments that are intuitively correct. This means that the clause embedded under comparative *than* is in fact in an upward entailing context! No existing theory of the comparative explains these entailments.

A possible way to avoid this conclusion and to save existing analyses of the comparative is to suppose that the quantifier phrases embedded under *than* are not actually interpreted in the scope of the comparative. This would mean that the argument in (9)–(10) does not go though, because the quantified clauses in (9) are not the true arguments of the comparatives in (10). In the literature, there are at least two proposals for how this might happen. Von Stechow (1984) suggested that quantifiers could be scoped outside by the usual mechanisms of quantifying in. Below we examine and reject this possibility.[5] Larson (1988), attempting to solve a similar challenge to

[5] Chris Kennedy suggested to us that the analysis of Lerner and Pinkal (1992) may constitute a third option. They proposed a rule of "Nested Quantification" whereby the relevant quantifiers take scope over the degree quantifier, but crucially not higher than *than*. This protects them from some but not all of the arguments adduced below.

another theory of comparatives (Klein 1980), proposed a compositional semantics whereby the clause under *than* actually lies outside the semantic scope of the comparative. Here again, the negative polarity facts in (2) are relevant. As Ladusaw (1979) showed, negative polarity items must lie in the semantic scope of their licensers; it is not enough for them to be c-commanded by a licenser at some level. Perhaps the most striking evidence for this comes from connectedness effects in pseudoclefts (Higgins 1979; Heycock and Kroch 1999). This is illustrated with *any* in (17) below, taken from Hoeksema (2000), where the question of semantic versus syntactic scope for NPI licencing is addressed in detail.

(17) What was missing was any real interest in the murk and challenge of the real world.

Since NPIs are licenced to occur in comparative *than*-clauses, and since their occurrence requires a scopally higher licencer, it would seem to follow that *than*-clauses in fact do lie in the semantic scope of the comparative. If, as we will argue below, quantifiers cannot scope outside of *than*-clauses, we are back to our original conclusion. Existing theories of the comparative make incorrect predictions concerning entailment patterns like those in (10) involving quantifiers in the scope of a comparative. In the following section, we will turn to the details of these theories to see what goes wrong.

In this section we have used NPIs to raise the issue of down-/upward entailingness and to argue that *than*-clauses must be in the scope of *more*, as the surface order suggests. Our discussion leaves a strong suggestion that an explanation for the data in (2) will not come in the form of a simple extension of Ladusaw's theory. Unfortunately, we have nothing further to add on this issue. (See Zepter 2001 for an alternative account of NPIs which specifically addresses quantifier data like those in (10).)

3. Quantifiers in the Scope of Clausal Comparatives

Among analyses of the comparative, there are those that make reference to degrees in the object language and those that eschew reference (see Klein (1991) for this and other issues relating to the use of degrees). The bulk of the discussion will focus on degree analyses. As noted above, Larson (1988) has already shown that avoiding degree reference does not grant immunity from the kinds of problems we will be interested in here.

A typical degree analysis begins by taking the clause under *than* to denote a set of degrees, as in (18) below. The material inside the parentheses in (18a) undergoes elision. The trace is a variable over degrees, which is bound by a lambda operator. The embedded clause denotes the set of degrees in (18c).

(18) a. John is taller than [$_{CP}$ Mary is (t tall)].
b. [$_{CP}$ *Mary is* (*t tall*)] $\rightsquigarrow$ λd[Mary is d tall]
c. {d: d is Mary's height} = {d_M}

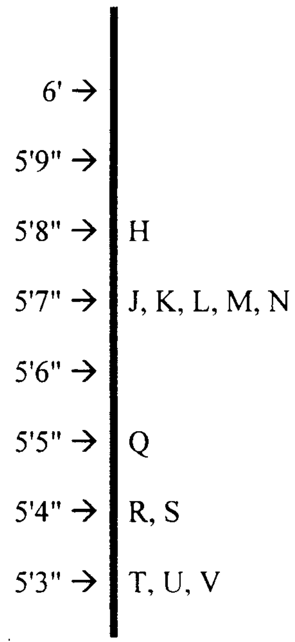
For now we are assuming that if *d* in *Mary is d-tall* is assigned height h, the result is true just in case Mary is exactly h tall. This means that the set in (18c) is a singleton. The next step is to say how this set is related to John's height. Theories differ here; in (19) the options are listed:

(19) [John is taller than CP] iff: John's height is greater than:

a. some height in CP	(Hellan 1981)
b. the height in CP	(Russell 1905)
c. the maximal height CP	(von Stechow 1984)
d. every height in CP	(Cresswell 1976; Klein 1982)

These theories collapse when applied to a case like (18) where the embedded clause denotes a singleton set. Predictions diverge when quantifiers occur in the embedded clause. In order to illustrate these predictions we will make reference to the scale in (20) below. It represents data on a number of suspects ranging in height from 5′3″ (Tom, Uwe, and Victor) to 5′8″ (Hubert).

(20)



The statement below, made in reference to (20), is intuitively false:

(21) Q is taller than everybody else is (t tall). (false of (20))

What do our theories say? To answer this question we calculate the meanings contributed by the clauses under *than*:

(22) a. λd[everyone else is d tall]
b. {d: everyone but Q is d tall}
c. ∅

Since the suspects are not all the same height, this set is empty. According to the definite analyses in (19b,c), we should find (21) uninterpretable due to presupposition failure. According to the universal theory (19d), (21) should be true, vacuously. The existential analysis in (19a) correctly predicts the sentence to be false but for the wrong reason. It would predict any sentence of the form *X is taller than everyone else* false, including the true sentence *H is taller than everyone else is*.

As noted above, we are taking 'is d-tall' to mean 'is exactly d-tall'. If instead we take it to mean 'is at least d-tall' we get the following meaning for the clause under *than*:

(23) a. λd[everyone else is d-tall]
b. {d: H, J. K, L, M, N, R, S, T, U, and V are all at least d-tall}
c. {d: d ≤ 5′3″}

Since Q's height is greater than 5′3″, all the theories in (19) predict that (21) should be true or truth-valueless when in fact it is false.

In deciding whether Q is taller than everybody else is, we don't look for a point corresponding to everyone else, but rather we scan the scale to check everyone's height. This simple observation is missed by degree analyses. Points on the scale corresponding to individuals are "too small" to carry information about several individuals at once. This leaves us with two options. We either retain a degree analysis appealing to some mechanism for removing the offending quantifiers or we revise the analysis of comparatives so that degrees do not play a central role. We will take up these two options in turn.

Difficulties inherent in the degree analyses were illustrated using an example with a universal quantifier. However, this is by no means necessary. According to all of the analyses above the sentences below are both true:

(24) H is taller than exactly 5 of the others are.

(25) H is taller than only one of the others is.

And for each of the theories there are further mistaken predictions generated by considering quantifiers of various types.

4. Why Quantifier Raising Won't Work

Crucial to the argument in the previous section was the assumption that quantifiers occurring inside a *than*-clause have in-situ scope. There are some very good reasons for making this assumption. As observed in section 2, we must assume that quantifiers are at least generated in the scope of the comparative, since the *than*-clauses containing them are. This leaves the possibility that quantifiers are moved or 'scoped' away from this position. As Larson (1988) observed, an argument against such a move stems from the well-known parallels between constraints on quantifying in and those imposed on wh-movement. Wh-words may not be moved from inside a clause under *than* either overtly as in (26a) or covertly as in (26b):

(26) a.* [Which bird]$_i$ are you taller than t_i was?
b.* She asked who was richer than who else was.

If quantifier scope opportunities are constrained in roughly the same way that wh-movement is, then it should be impossible to grant the quantifiers in question scope outside a *than*. An additional violation would arise on a QR account with examples like (27) below:

(27) a. Alice is richer than George was and than most of his children will ever be.
b. [most of his children]$_i$ [Alice is richer than George was and than t_i will ever be.

In order to save the degree analysis here the quantifier *most of his children* would have to be removed from inside a conjoined clause.

Furthermore, in the normal case when quantifiers can have scope outside their surface structure position, the wide scope reading exists in addition to the narrow scope surface reading. But, again as Larson (1988) observed, existing analyses of comparatives would have to assume an *obligatory* rule of QR. This point has been obscured in the past because researchers have tended to look at examples with simple universal or existential quantifiers. In these cases the potential ambiguity would arguably be hard to detect because the two readings are often related by entailment. Furthermore, in theories where definiteness plays a role, presupposition failure has been used to explain away the lack of ambiguity (Rullmann 1995). Wilkinson (1998) showed, however, that if non-monotonic quantifiers are used the two readings are logically independent and both can arise in the same context. If von Stechow's theory in (19c) were correct, for example, and if a rule of QR could scope quantifiers outside the comparative, the sentence in (28a) would be ambiguous between the paraphrases in (28b) and (28c):

(28) a. Hubert is taller than exactly 5 of the others are.

b. Find the largest height h, where exactly 5 individuals other than Hubert are h tall. Hubert is taller than that. (narrow scope)

c. There are exactly 5 individuals that are shorter than Hubert. (wide scope)

Neither reading is ruled out by presupposition when uttered with respect to (20) since there is indeed a unique height, 5′7″, that exactly 5 individuals have. (28b) is clearly true and (28c) is clearly false. Yet (28a) is intuitively univocal and false. In other words, only the wide scope reading is available, showing that if QR accounts for the wide scope it must be an obligatory rule in this case.

Another property of QR is that it is prohibited from applying to quantifiers like those in (29) below. To save degree analyses, however, this prohibition would have to be relaxed.

(29) a. Lucy paid more for her suit than they <u>both</u> paid in taxes last year.

b. It is colder in Stony Brook today than it <u>usually</u> is in New Brunswick

Paralleling the discussion in the previous section, we observe that (29a) could be true even if there is no single amount that they both paid. Similarly, (29b) could be true even in the likely case that there is no single temperature (degree of coldness) that characterizes New Brunswick most of the time.

Finally, even if a QR solution could be maintained, it makes the wrong predictions in some cases where another scope-taking element lies between the quantifier and the comparative over which it will take scope. Imagine John predicts that most of his students will get between 80 and 90 points on the national exam. When the exam is over, Bill receives a score of 96 and Alex receives a score of 70. In that case, it would be fair to say that:

(30) Bill did better than John predicted most of his students would do.

But it would not be true to say that:

(31) Alex did better than John predicted most of his students would do.

After QR applies to the embedded quantifier in (30) we arrive at the paraphrase below:

(32) Most of John's students are x such that: Bill did better than John predicted x would do.

This has the welcome result that the clause under *than* can safely pick out degrees associated with individual students. The problem is that it doesn't accurately capture the meaning of (30). Since John did not make any prediction about a particular student or about a particular degree, for any value of x, the set denoted by (33) is empty:

(33) λd[John predicted x would do d well]

Sentence (30) is therefore incorrectly predicted to be false according to the existential theory (19a) and truth-valueless according to the definite theories (19b, c). (30) comes out true on the universal theory (19d), but then (31) would come out true for the same reasons.

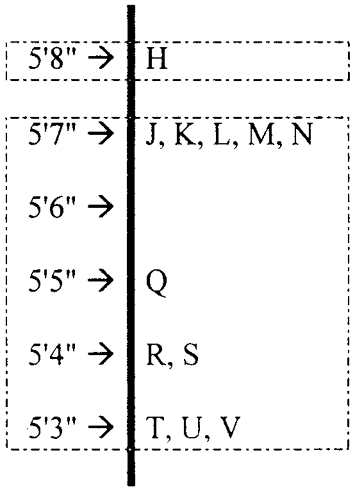
Hopefully the evidence amassed in this section will deter anyone from supposing that quantifiers under *than* are routinely scoped outside the comparative. This spells doom for the analyses surveyed in section 3. Salvation will come in the following sections as our horizons broaden to encompass larger expanses of the scales on which degrees are marked.

5. Towards a Solution: Intervals in the Semantics of Comparatives

An analysis of comparatives using intervals might begin with the following observation.[6] *Hubert is taller than everyone else is* would be true if there was an interval of a tallness scale containing the heights of everyone but Hubert, and above that there was an interval containing Hubert's height – something like in the figure below:

[6] The idea to use intervals as the basis for a semantics of comparatives is not novel. One finds this for example in Seuren (1984), Bierwisch (1987), and more recently in Kennedy (1997, 2001). Our point of departure is different from those approaches. They are in a certain sense extensions of degree approaches and are similarly challenged by quantifiers under *than*, as Bierwisch himself noted.

(34)



We intend to preserve this view of the comparative as a relation between intervals of the scale; however, the exact nature of the relationship will need sharpening. The examples below give an indication of what will be needed:

(35) H is taller than exactly 5 of the others were.

(36) H is 1 inch taller than everyone else is.

Sentence (35) is intuitively false according to the information in (34). However, there is an interval which contains the heights of exactly 5 of the others, namely those of J, K, L, M, and N, and which lies below an interval covering H's height. (36) is likewise intuitively false according to the scale in (34). But the upper interval depicted in (34) is in fact 1 inch above the lower one, the one that covers everyone but H's height. If this view is to work, we need somehow to restrict the choice of intervals that participate in the comparative relation.

In order to address this issue, we should say a bit more about relations between individuals and intervals of the scale and between one interval and another.

6. Intervals and Differentials

The underlined phrases in the examples below are what von Stechow (1984) called differentials:

(37) John is 1 inch taller than Mary was.

(38) Felix is a fair amount richer now than he was last year.

(39) Harry is a lot more fascinating than his father was.

(40) It's a little bit more tasty than it was when we first made it.

(41) She's no slower than she usually is.

(42) Bill Clinton wasn't any happier than Bill Gates was.

(43) Maxine wasn't that much faster than I thought she would be.

Differentials measure parts of the scale. According to (37), there has to be a 1-inch portion that lies between an interval or point on the scale associated with John and one associated with Mary. According to (39), there has to be a large portion on the fascination scale between Harry and his father. Given that these differentials are predicates applying to parts of the scale, it is interesting to notice that they are all symmetric quantifiers that apply in the mass domain. Their standard uses are illustrated below:

(44) He has 1 inch of rope in his pocket.

(45) Felix put a fair amount of gasoline in the tank.

(46) Harry drank a lot of milk.

(47) There was a little bit of evidence to support her alibi.

(48) She has no interest in your proposal.

(49) I don't have any rice.

(50) He didn't drink that much wine.

In general, count quantifiers don't perform well as differentials in adjectival comparatives. **John is every/many taller than his mother was* is ill-formed. And while the negative polarity *any* is potentially a symmetric mass quantifier ((49)) and therefore can be used as a differential ((42)), free choice *any* is not a mass quantifier:

(51) # Choose any rice. (≠ choose any amount of rice)

Free choice *any* is also not symmetric and cannot be used as a differential:

(52) * With enough alcohol, you can feel any richer than Bill Gates feels.

Expressions such as *inches* and *degrees* are classifiers, which combine with count quantifiers to form mass quantifiers. They therefore enable the use of count quantifiers in differentials:

(53) He bought several *(inches) of rope.

(54) He is several *(inches) taller than he used to be.

(55) It is a few *(degrees) colder now than it was an hour ago.

The key property of the mass domain is its homogeneity. If something is rope, then any relevant subpart is rope. A scale is similarly a mass-like object. Any interval of the scale has parts which are also intervals of the scale. The part-of relation (part-of: $\sqsubseteq$, proper part-of: $\sqsubset$) defined on intervals of the scale exists in addition to the ordering relation (less-than: $\prec$, less-than or equal to: $\preceq$) necessary for scalehood. Both of these relations are partial, for if two intervals partially overlap, then neither one is part of the other nor is one less than the other. These relations exclude one another in the sense that if A is less than B, then A isn't part of B, and if A is part of B, then A cannot be less than B.

Since differentials measure gaps between intervals we need to define a 'subtraction' operation. Assuming I is above K, we want [I – K] to pick out that part of the scale that is below I and above K:

(56) For intervals I, K:

If $K \prec I$, then:	$\forall J: (J \prec I \ \& \ K \prec J) \leftrightarrow J \sqsubseteq [I - K]$
Otherwise	$[I - K] = 0$

I – K names the interval that lies in between I and K, below I and above K. Differentials are understood as predicates that apply to the gaps named with the subtraction operation. One differential that deserves special mention is SOME.[7] It says that an interval is equal to or greater than some contextually specified minimum:

(57) SOME(J) = 1 iff the size of J equals or exceeds δ, where δ is determined by context.

SOME is used to interpret comparatives that do not have an overt differential. If it is true that *this board is longer than that board is* then there must be *some* difference in the lengths of the boards. The context-dependent component of (57) reflects the fact that two boards which are close in length will count as equally long in some contexts, but not in other

[7] The discussion here has greatly improved thanks to remarks and calculations by Arnim von Stechow. We have not managed to incorporate all of his suggestions. We hope to return to that on another occasion.

contexts where the level of precision is greater. The semantics of SOME is modeled on the mass quantifier *some*, which is similarly context dependent, as the following pair illustrates:

(58) There is some wood in my eye.

(59) There is some wood in my truck.

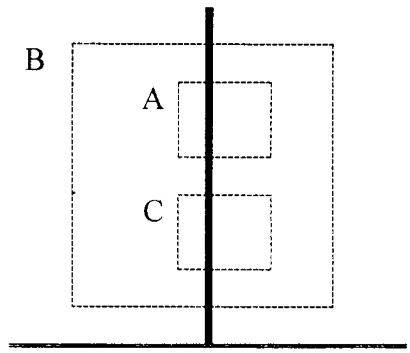
In order to illustrate the notions introduced so far, we define another differential:

(60) NO(J) = 1 iff the size of J is less than or equal to δ, where δ is determined by context.

Assuming that the distance between A and C in (61) below is greater than δ, all the following statements are true:

SOME[A – C])	NO([A – B])	NO([B – A])	$C \prec A$	$C \sqsubset B$
NO([C – A])	NO([B – C])	NO([C – B])	$C \preceq C$	$A \sqsubset B$

(61)



In this scheme, scalar predicates such as ‘tall’ denote relations between individuals and parts of the scale, or intervals. Whereas in the temporal domain, one says that a time interval contains an event, we will say that a scalar interval *covers* an individual. The temporal ‘contain’ relation is persistent, meaning that if one interval contains an event, then so do all superintervals. An event that happened in July of 1969, for example, also happened in 1969. Likewise, we will take the relation expressed by a scalar predicate to be persistent:

(62) *Persistence Principle:*
For any scalar predicate extension P, individual x, and portions of the relevant scale I,K:

$P(x,I) \rightarrow \forall I'[I \sqsubset I' \rightarrow P(x,I')]$

Among other things, this guarantees that a man who is 6 ft tall is covered by an interval corresponding to the interval on the ruler from 4 ft to 6 ft. It also means that there are intervals which cover both the 6 ft tall man and his 5 ft tall friend. The temporal analogy would be a time interval that contains two events that occurred at different times, such as a year that contains events from different months. Taking for granted that for any two intervals there is an interval that includes both of them ($\forall$I,J $\exists$K[I $\sqsubseteq$ K & J $\sqsubseteq$ K]), it follows from (62) that if a number of individuals are related by some scalar predicate P to various intervals on a scale, then there will be some interval that covers all of them:

(63) Let S be a set of individuals:
$(\forall x[x \in S \rightarrow \exists I\ P(x,I)]) \rightarrow \exists K \forall x[x \in S \rightarrow P(x,K)]$

This in turn guarantees that if everyone in the domain of discourse is in the sortal domain of a predicate like ‘tall’, there will be an interval (in fact several) that verifies quantificational statements such as ‘Everyone is I-tall’.

Another principle that we will adopt guarantees that if Jack is exactly 5 ft tall, he cannot be exactly 4 ft tall. In interval terms, this means that Jack cannot be covered by two distinct intervals, one lying above the other. The Overlap Principle is stated as follows:

(64) *Overlap Principle:*
For any scalar predicate extension P, individual x, and portions of the relevant scale I,K:

$P(x,I)\ \&\ P(x,K) \rightarrow \exists J(J \sqsubseteq K\ \&\ J \sqsubseteq I\ \&\ J \neq 0]$

The following is a corollary of (64):

(65) For any individuals x,y, scalar predicate P, and intervals I,J:
$[P(x,I)\ \&\ P(y,J)\ \&\ I \prec J] \rightarrow \sim\exists I'\exists J'[P(x,I')\ \&\ P(y,J')\ \&\ J' \prec I']$

According to (65), if there is an interval covering Mary on the height scale that is above one covering John, than there is no interval covering John that is above one covering Mary.

7. Interval Semantics for Comparatives

The sentence in (66) compares the temperature that it is in New Brunswick today to the temperature that it was in L.A. last week.

(66) New Brunswick is [2 degrees] hott**er** today **than** L.A. was last week.

So, in addition to the bracketed phrase and the comparative morphology, there are two temperature predicates. We can think of these as open sentences, sentences with variable-denoting expressions in them:

(67) New Brunswick is t-hot today.

(68) L.A. was t-hot last week.

We will call the clause embedded under *than* – in this case, *L.A. was t-hot last week* – the **subordinate clause**. The clause in which the comparative occurs, (67), will be called the **main clause**. Both of these function as predicates of intervals whose arguments are given by the traces. We will use the symbols **Mn** and **Sub** to stand for these predicates. We use the symbol **Diff** to stand for the contribution to the meaning made by the differential *2 degrees*. Comparatives with no overt differential will be taken to include an implicit existential to be written as SOME.[8]

Since it is possible that phrasal and clausal comparatives differ with respect to quantifier scope, we have been limiting, and will continue to limit, discussion to clausal comparatives (annoying as that might be in some cases).

At this point, we are prepared to state a necessary condition on the truth of the comparative:

(69) $\exists I\ \exists K\ [Mn(I)\ \&\ Sub(K)\ \&\ Diff([I - K])]$.

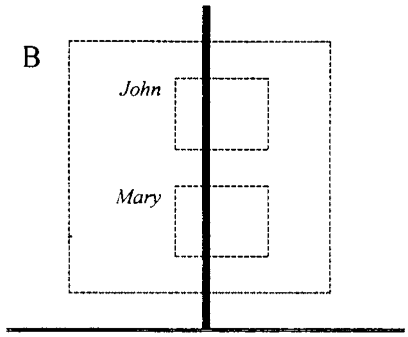
For the example in (66), this would mean that there is an interval on the temperature scale I such that New Brunswick is I-hot today, there is another interval K such that L.A. was K-hot last week, and I differs from K by 2 degrees.

Moving to a somewhat simpler case, according to (69), if *John is taller than Mary is* is true, there must be an interval of the height scale covering John that is some amount higher than one covering Mary. Below we have indicated three intervals of the height scale: one just covering John, one just covering Mary, and the interval labeled B covering both of them.

[8] This is reminiscent of the implicit existential in the agent argument of a passive. Like the passive agent argument, the differential can also appear finally in a *by*-phrase:

(i) McConnell-Ginet made this observation earlier than I did **by** at least 25 years.

(70)



According to (70), (69) is true because the John interval is a possible value for I and the Mary interval is a possible value for K. To be sure, there are more intervals that could be pictured here covering John, Mary, or both. But if John is indeed taller than Mary, there are no Mary intervals that lie above John intervals (see (65) above).

Condition (69) is necessary, but not sufficient. Imagine a group of individuals of varying heights with John being the tallest. John is 6′ tall, the next person is 5′8″, and the remainder descend in height from there. In that case, (71) is true, but (72) isn't ('exactly' reading of *2 inches*). (72) entails that everyone but John has the same height:

(71) John is taller than everyone else is:

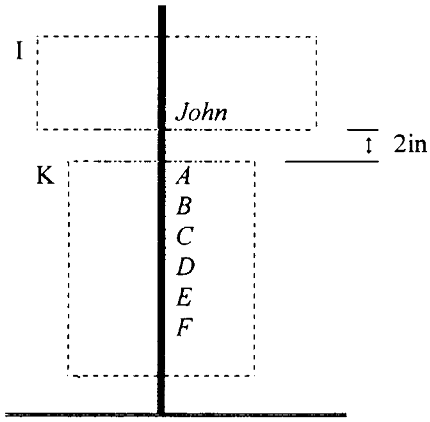
(72) John is 2 inches taller than everyone else is.

The condition in (69) is spelled out for (72) in (73):

(73) ∃I John is I tall & ∃K everyone else is K tall & 2-IN ([I – K])

As the figure in (74) below shows, the condition in (73) is met despite the fact that (72) is not true. In this figure and elsewhere italicized names indicate the position of small intervals covering the named individual:

(74)



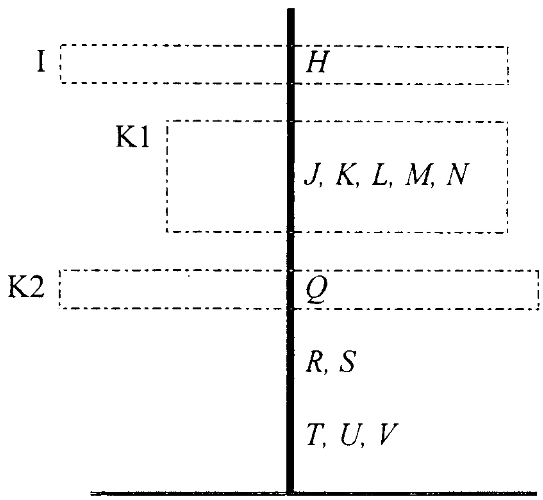
Intuitively, the differential *2 inches* needs to measure not only the distance between I and K but also the distances between I and intervals throughout K. Since in (74), K has subintervals that are more than 2 inches from I, (72) is false. The condition in (69) can therefore be strengthened requiring K to include *only* subintervals that are separated from I by the amount given by the differential:

(75) $\exists I\ \exists K\ [Mn(I)\ \&\ Sub(K)\ \&\ \forall K'[K' \sqsubseteq K \rightarrow Diff(I - K')]]$

This condition can be strengthened even further. Recall the example in (76), which was false on the ordering depicted in (77) below.

(76) H is taller than exactly 5 of the others were.

(77)



Notice that the interval labeled I satisfies the main clause, H is I-tall, and the interval K1 satisfies the subordinate clause since exactly 5 others are K1-tall. Furthermore, every part of K1 is some distance away from I:

$$\forall K'[(K' \sqsubseteq K1) \rightarrow \mathrm{SOME}([I - K']).$$

The condition in (75) holds despite the fact that (76) is false. What intuitively makes (76) false is that there are other intervals, K2 for example, all of whose parts are also some distance away from I, but K2 does not satisfy the subordinate clause. This observation leads to the following strengthened condition:

(78) $$\begin{aligned}&\exists I\ \exists K[\mathrm{Mn}(I)\\ &\quad \&\ \mathrm{Sub}(K)\\ &\quad \&\ \forall K'[K' \sqsubseteq K \rightarrow \mathrm{Diff}(I - K')]\\ &\quad \&\ \forall K''[K \sqsubset K'' \rightarrow (\exists K'[K' \sqsubset K''\ \&\ \sim \mathrm{Diff}([I - K'])])]\end{aligned}$$

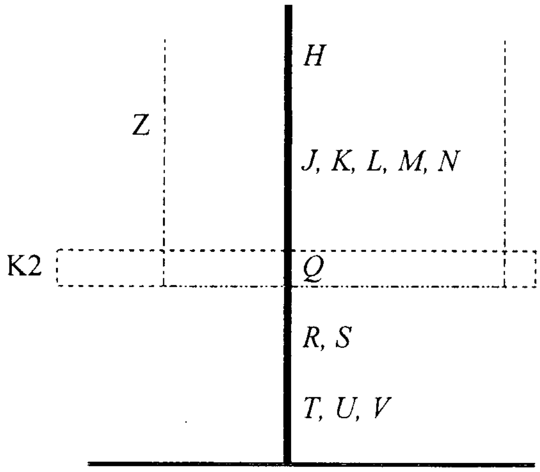
The last conjunct requires that K be the largest interval that satisfies the second-to-last conjunct. In the case of (76)/(77) above, the largest interval every part of which is some distance from I begins just below I and continues down to the bottom of the scale. That interval does not satisfy the subordinate clause, and so (76) is false.

Consider the interval K2 again. Suspect Q is K2-tall. Next, imagine finding all the intervals that lie some distance below K2. Put them together into an interval we call J. J begins just below K2 and extends to the bottom of the scale. Exactly 5 individuals are J-tall. The condition in (78) is therefore correctly met for the true sentence *Q is taller than exactly 5 of the others are*. Keeping with the interval K2, consider the interval given by [K2 – I], that part of the scale which is below K2 and above I. There is in fact no part of the scale that fits this description; hence we get the empty interval. The same would hold if we replaced I with any interval above K2 or if we had K2 itself. Putting all these intervals K2 and above together we get an interval, call it Z. Z begins with K2 and extends upwards to the top of the scale (if there is one). The following statement is true:

(79) $$\begin{aligned}&\forall K'[K' \sqsubseteq Z \rightarrow \mathrm{NO}([K2 - K'])]]\\ &\quad \&\ \forall K''[Z \sqsubset K'' \rightarrow (\exists K'\ [K' \sqsubset K''\ \&\ \sim[\mathrm{NO}(I - K')])]\end{aligned}$$

The situation is roughly as depicted below:

(80)



Since Q is K2-tall and H is Z-tall, the condition in (78) is met for the true sentence *Q is no taller than H is*.

At this point existential quantification over K in (78) is misleading since the requirements imposed on K are satisfied by only one interval.[9] To make this more apparent, we define a maximality operator as follows:

(81) $\mu K'[\phi] = K$ iff: $\forall K'[(K' \neq 0 \ \& \ K' \sqsubseteq K) \rightarrow \phi(K')]]$
$\& \ \forall K''[K \sqsubset K'' \rightarrow (\exists K'[K' \sqsubset K'' \ \& \ {\sim}\ \phi(K')])]$

According to (81), $\mu K'[\phi]$ picks the largest interval all of whose non-empty parts are ϕ. The condition in (78) can now be rewritten as:

(82) $\exists I[Mn(I) \ \& \ Sub(\mu K'[DIFF(I - K')])$

According to (82), if a comparative statement is true we should be able to perform the following routine. First we show some interval I that satisfies the main clause. Then we find the largest interval all of whose parts are below I by the amount given by the differential. We then show that that maximal interval satisfies the subordinate clause.

It is probably wise at this point to pause to consider what's behind (81), appealing again to intuitions about temporal scales. We've used the term

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[9] Suppose you had two distinct intervals K1, K2 such that both were candidates for being $\mu K'[\phi]$ (all parts of both were ϕ parts and neither had a superinterval all of whose parts were ϕ parts). Consider M which has K1 as part and K2 as part and no part which is not also a part of K1 and K2 (M need not be convex, i.e. there could be 'gaps' in it). Neither K1 nor K2 could satisfy the second conjunct of (81) because of M ($K1 \sqsubset M \ \& \ {\sim}\exists K'[K' \sqsubset M \ \& \ {\sim}\phi(K')]$).

"maximality" and we've pointed out that the μ operator picks out a unique interval (see footnote 9). Both of these properties have at times been attributed to the definite article, suggesting that one place where we might find a similar effect is with definite descriptions of time intervals. Consider the use of the definite article in the following sentence:

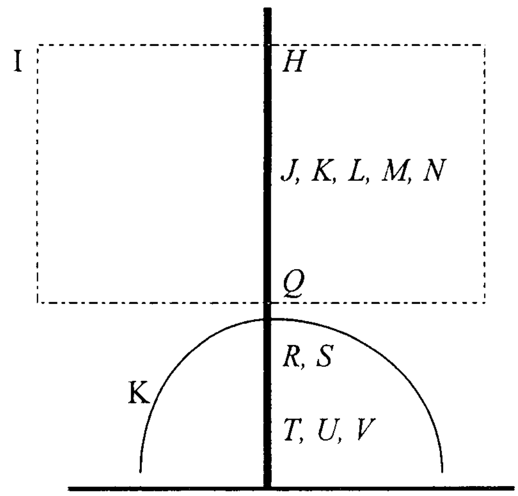
(83) A mysterious balloon floated on a wave around the time Calypso had that nasty cough.

The noun phrase beginning with *the time . . .* appears to denote a unique time interval, the one that coincided with Calypso's cough. What is that interval? Since the embedded clause is stative, there is no minimal time interval of Calypso having the cough, hence minimality (by itself) couldn't give us uniqueness. On the other hand, any time interval containing the coughing is an interval when Calypso had the cough, so there is no maximal interval either. Instead, we choose a time interval K satisfying the following two requirements: all parts of K are times of Calypso having that cough and any K″, a proper superinterval of K, includes time stretches during which Claypso doesn't have the cough. This is just μ operating in the temporal domain.[10]

In strengthening the condition in (69) we looked inside the "subordinate" interval K; the same should be done for the main clause interval. To see that, note that our earlier explanation for why (76) (*H is taller than exactly 5 of the others are*) is false in (77) would not have gone through if we had begun with a larger interval I, as in the figure below:

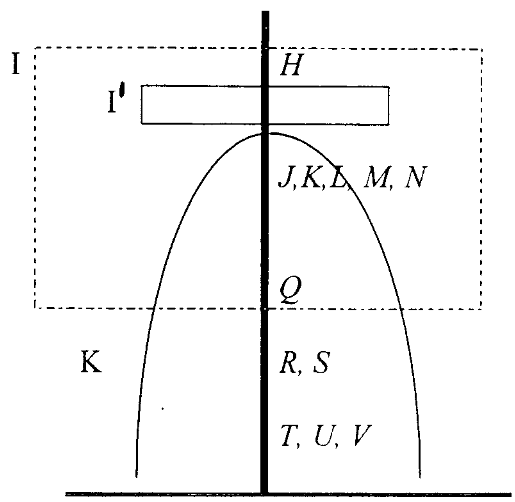
[10] Why can't we simplify μ so that μK′[ϕ] includes every K′ that satisfies ϕ? Here's one reason. Reexamine the diagram in (80). The interval that includes the entire scale pictured there is an interval K′ such that NO(K2 – K′). On the proposed simplification, μK′[ϕ] would include that interval. Since that interval covers any individual depicted, it would follow that Q is no taller than R is and Q is no taller than T is, and so on. Note that the proposed simplification is simply a sum operator of the kind used in the interpretation of the definite article in the mass and plural domains (Wheeler 1972; Sharvy 1980; Link 1983). But this is not what we want here.

(84)



K pictured here satisfies the subordinate clause, and it is the largest interval consisting just of intervals that are some distance below I. Again the problem arises because while I picks out the interval K which meets the relevant requirements, I contains subintervals, such as I′ in the figure below, which does not meet the requirements:

(85)



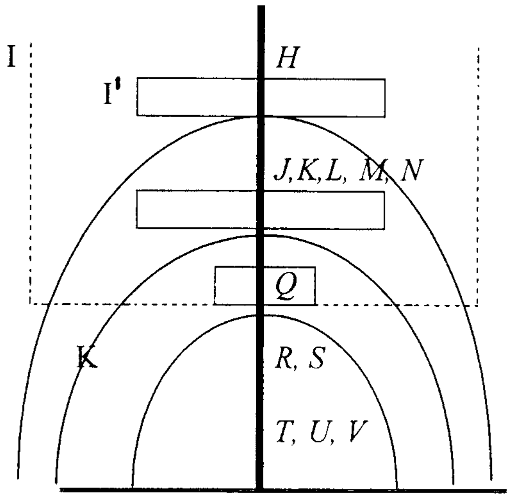
In this figure, K is the largest interval composed entirely of intervals some distance below I′. But K does not satisfy the subordinate clause (*exactly*

5 others are K-tall is false).[11] So the reference to the upper interval I will also be required to be the largest interval consisting entirely of I′ which satisfy the requirements imposed by the subordinate clause and the differential:

(86) $\mathrm{Mn}(\mu\mathrm{I}'[\mathrm{Sub}(\mu\mathrm{K}'[\mathrm{DIFF}(\mathrm{I}' - \mathrm{K}')])])$

Formula (86) requires that the main clause of the comparative be satisfied by the largest interval each non-empty part of which has the following property: the maximal interval consisting only of intervals separated from it by the differential verifies the subordinate clause. The following figure captures a moment in the verification of the sentence *H is taller than at least five of the others were*:

(87)



Three subintervals of I have been checked. In each case, the interval below it (indicated with a parabola) covers at least 5 individuals other than H. Since I covers H, the sentence is true. The process will continue covering the entire span from above R and S to the top of the scale, if there is one. From the formula in (86) and the figure in (87), it develops that an expression of the form "differential *more than* S" defines an interval. In (87), the interval I is the denotation of "some+er+than+at least 5 of the

[11] As an exercise the reader can also verify that although *H is no taller than Q is* is false for (80), the condition in (82) would be met for this case. (Hint: Let Z be the main clause interval.)

others were t-tall". This view of things is syntactically transparent in examples like the following:

(88) The boy is sick, much more than his father is.

(89) The boy is sick, somewhat more than his sister is.

(90) The boy is sick, (but) little more than his friend is.

Sentence (88), for example, asserts that the boy is I-sick, where I is the interval given by the comparative *much more than his father is (sick)*.

This brings us to the question of how the semantics we have proposed constrains syntactic analyses of the comparative. The answer is that it will depend very much on assumptions above how meanings are computed. We should therefore just say what the system needs to achieve. For (88), it seems reasonable to think of *more* as denoting a function which takes two arguments. One argument is provided by the *than*-clause and it corresponds to Sub in (86). The second argument is provided by the differential *much*. The function applies to these arguments to give an interval. That is the interval described in (86) as $\mu I'[Sub(\mu K'[DIFF(I' - K')]$. That interval serves as an argument for the function denoted by *sick* resulting in a one-place predicate over individuals, which applies to the boy. Unlike in (88)–(90), in run-of-the-mill comparatives of the form *John is much taller than George is* the interval-denoting expression appears to be wrapped around the scalar predicate. In order to "set things right" we have the usual choices. Either the syntax-semantics map involves more than just sisterhood interpreted as function-argument application or the meanings of the parts involve more than the functions and arguments just described, or most likely a combination of these last possibilities. Furthermore, although the semantic proposal made here was developed on the assumption that the subordinate clause contains an interval-type trace, this is not necessary. In particular, we are not committed to an ellipsis analysis of the missing material in the *than*-clause as opposed to an analysis in which the missing material is the type of a one-place predicate (see Kennedy 1997, 1999; Klein 1980).

According to the Persistence Principle ((62) in the previous section), an individual is covered by multiple ever-larger intervals of the scale. Since the comparative names an interval, this translates into the claim that multiple comparatives (based on the same adjective) will be true of a single individual. To illustrate this, we compute a collection of comparatives in the figure below. The interval pictured corresponds to the comparative expression below the scale; assume in each case that the letters correspond to the relevant boys quantified over in the example and that if a letter is used in two figures it represents the same boy:

(91)

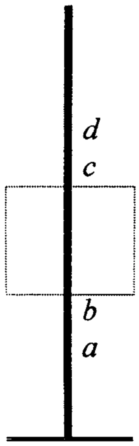


Figure 1. (Tall)er than exactly two of the boys are.

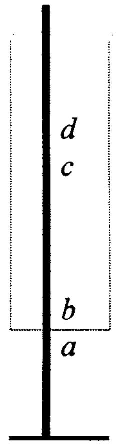


Figure 2. (Tall)er than one of the boys is.

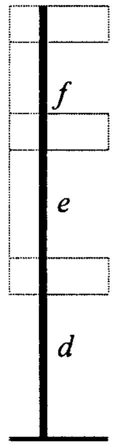


Figure 3. Two indices (tall)er than one of the boys is.

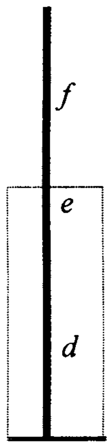


Figure 4. No (tall)er than e is.

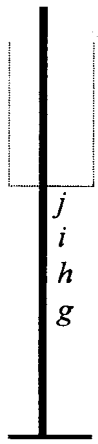


Figure 5. (Tall)er than all of the boys are.

Given that the interval in Figure 1 is a subinterval of the one in Figure 2, if the interval in Figure 1 covers Maxine, the one in Figure 2 will as well. This corresponds to the intuition that if Maxine is taller than exactly two of the boys then she is taller than one of the boys, an example of the upward entailing character of the comparative noted in section 2. The interval in Figure 3 is discontinuous. Assuming that a discontinuous interval can have a continuous one as a part, the Persistence Principle entails that scalar predicates will relate individuals to discontinuous intervals. Maxine is related to such an interval on the height scale in Figure 3, if she is two inches taller than one of the boys.[12] The interval in Figure 3 is not a subinterval of the one in Figure 4 but they do overlap. This means that if Maxine is two inches taller than one of the boys is and e is one of the boys, it is possible but not necessary that Maxine is no taller than e is.

We have made some use here of the zero interval. One might wonder then whether a comparative can even denote the zero. Such a comparative in conjunction with a degree adjective would yield a predicate that would be false of every individual in the domain of the adjective, assuming that no individual in the domain of a degree predicate is covered by the zero interval. Exactly this circumstance arises in the following example supplied by Maribel Romero.[13] Consider a scenario in which Tom, Uwe, and Victor are the same height and are the shortest people in question and where other than those three, Rhoda and Selma are the shortest and they are the same height. Such a scenario is depicted in the chart in (85). In that situation there is no-one who is *taller than exactly 4 other people*. To see how this works, consider the following expression:

(92) Exactly 4 people are $\mu K'[\text{SOME}(I' - K')]$-tall.

There are no I' that make this true. If we pick an I' above Rhoda, for example, $\mu K'[\text{SOME}(I' - K')]$ will give an interval that covers 5 people. If we have an interval at or below Rhoda, $\mu K'[\text{SOME}(I' - K')]$ is an interval that covers exactly 3 people. Since (92) is not true then for any I', it follows that 0 is an interval all of whose non-empty parts satisfy (92), vacuously, and 0 is the largest such interval (since it is the only such interval). Hence

(93) $\mu I'[\text{exactly 4 people are } \mu K'[\text{SOME}(I' - K')]\text{-tall}] = 0$

Expression (93) gives the meaning for *-er than exactly 4 people are tall* and combines with *tall* to give a predicate true of no-one.

[12] The limits of graphic depiction are stretched in this example. The thickness of the three boxes should be small, representing insignificant widths relative to 2 inches. Another expression that would denote a discontinuous interval is *exactly 2 degrees or exactly 8 degrees* (hott)*er than the white flask was*. And if the copier runs at three speeds, then the comparative in *The collator runs at* ***2 pages*** *fast****er than the copier runs*** would denote a discontinuous interval.

[13] The reader may have noticed that in defining the μ operator we included a non-zero clause which wasn't there in the conditions leading up to the μ operator. This change was in response to Romero's observation, for which we are grateful.

This concludes the development of an interval-based semantics for comparatives. While points on a scale may do the job where comparison between individuals is concerned (and one should in principle prove that the present account reduces to a point account in such cases), a study of the full range of cases shows that they are not at the heart of scalar predication.

8. Equatives and Numerals

It is commonplace to treat equatives alongside comparatives. The purpose of this section is to sketch an analysis of the equative and of numerals that does justice to similarities as well as differences with the comparative.

The semantics of comparatives proposed here relies crucially on the meaning of the particular differential that has been chosen. Equatives, though syntactically similar to comparatives in English, do not allow for differentials.

(94) # He is 2 inches as short as his brother was.

(95) # He isn't that much as short as his brother was.

Moreover, as Klein (1991, pp. 676–677) points out, crosslinguistically these two constructions are fairly sharply distinguished. Besides syntactic distinctions in various languages, Klein, citing Ultan (1972), notes that comparative and superlative markers are similar to each other and generally dissimilar to equative markers. Suppletive paradigms join comparatives with superlatives distinguishing both from the equative and the positive: *better* ~ *best* versus *good* ~ *as good*. It is plausible then that the correct analysis of equatives would not parallel that of the comparative too closely.

Equatives are context dependent. Consider (96):

(96) Alice is as rich as her mother was.

Alice's wealth could be close enough to her mother's so that (96) would count as true if wealth is measured in dollars but not once we start counting pennies. (96) equates an interval that marks Alice's wealth with one marking her mother's wealth, taking into consideration the level of detail in the conversation. Using the concept of a ruler as our guide, we associate with a context a partition of the scale into equal-sized, contiguous intervals. We will call these intervals "unit intervals." Alice is as rich as her mother is if her unit interval of richness is equivalent to her mother's. Moving from a fine-grained context to a rougher-grained one increases the size of the unit interval and the truth of an equative is preserved.[14] Moving in the opposite direction, however, may lead to reversal of the truth of an equative, as we saw with Alice when we started counting pennies. The general idea is stated in (97):

(97) For any given context, the equative is true if the unit interval satisfying the subordinate clause also satisfies the main clause, where the context determines what counts as a unit interval.

According to (97), if an equative is true, there must be a contextually specified unit interval which satisfies the subordinate clause. Our experience with degree analyses of comparatives tells us we had better check what happens when quantifiers are involved, as in the following examples:

(98) Alice is as rich as everybody else is.

(99) The subject shouted "bird" as quickly as he shouted only one other word.

(100) Seymour was as smart as one of the girls was.

The analysis we have given so far predicts that (98) is true just in case we are in a context where everybody has the same unit interval of wealth: otherwise no unit interval will satisfy the embedded clause. This prediction is correct on the 'exactly' reading, but not on the 'at least' reading (Seuren 1984; Rullmann 1995). This difference is spelled out as follows:

(101) Alice is exactly as rich as everybody else is.

(102) Alice is at least as rich as everybody else is.

Only (101) intuitively entails that for all intents and purposes, everybody has the same unit interval of wealth. (102) does not carry this entailment. It could be true in a context where wealth is measured in the number of watermelons a person has and where no two people have the same number of watermelons. It is sufficient that Alice has more than any of the others. One might try to argue that (101) is, in fact, the only reading we get for (98). But even in that case, we still need an analysis of (102) that doesn't commit us to a unique unit interval that everybody shares. Another problem with our initial proposal in (97) is that it makes reference to **the** unit interval satisfying the subordinate clause. But (99) could be true even if there are

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[14] This assumes that new boundaries are not drawn in the process. The picture is one of moving from a ruler where centimeters are indicated to one where meters are the smallest unit indicated.

many speeds s such that the subject shouted only one word at speed s. And (100) could be true even if there are many IQ's, such that one of the girls is that smart. The unit interval idea might be on the right track, but the implementation in (97) needs immediate revision.

The use of the modifiers "exactly" and "at least" is reminiscent of discussion of numeral modifiers. There too we find that *bare* numerals can have readings spelled out with these modifiers. The chart below shows various contexts where these different readings arise both for numerals and for equatives:

Modifier	Numeral	Equative
Exactly	(How many children does John have?) He has three children.	(How tall is Bill?) He is as tall as Peter.
At least	You must have \$4 to enter.	You must be as strong as Bill to be accepted.
	He ate four cookies, if not more.	He went as far as Billy went, if not further.
At most	You can eat four cookies (but no more).	You can go as far as Billy went (but not further).

The parallel with numerals suggests a line of analysis that follows the treatment of numerals in Kadmon (1987). Like Kadmon, we want a semantics for the simple equative that can be modified by 'exactly' or 'at least' and we ultimately need a pragmatic account of how the meanings of these modifiers show up in the bare case. To pursue this line, we should first get straight what the readings are for the modified cases and then go about deciding whether we can successfully extract the modifiers to get a reasonable story for the bare case. Keeping in mind the trouble we had above by assuming uniqueness, we start with the following basic intuition, ignoring 'at most' for the moment:

(103) *Alice is exactly as rich as Bill is.*

IS TRUE IF there is some wealth interval that covers Bill that also covers Alice and that interval is a contextually specified unit interval.

Alice is at least as rich as Bill is.

IS TRUE IF there is some wealth interval I_{Bill} that covers Bill and there is some wealth interval I_{Alice} that covers Alice, and

(i) the intersection of I_{Bill} and I_{Alice} is a contextually specified unit interval

(ii) every part of I_{Bill} is at or below I_{Alice} ($\exists K[K \sqsubseteq I_{Bill} \rightarrow K - I_{Alice} = 0]$)
(it follows that 'exactly' entails 'at least' but not vice versa).

The truth conditions for the equative that are reflected in the intuitions on the right in (103) now make correct predictions for the cases considered. (101) but not (102) requires a unit interval that covers everybody, meaning that in the relevant context everybody has the same unit interval of wealth. And since we have existential truth conditions, (99) and (100) no longer break down for lack of uniqueness. On the 'exactly' reading of (99) for example, we require some unit interval of speed with which the subject shouted "bird" and with which he shouted one other word. This allows that he shouted many words, with different unit intervals of alacrity.

Next, we attempt to extract the modifiers from the equatives. In the following sketch we make similar assumptions (or non-assumptions) about the syntax-semantics of equatives that we made with comparatives (E.g. "subordinate clause" means the clause under *as* and the equative is interpreted without the higher adjective; though see the brief remarks at the end of section 7.). The interpretation of the bare equative is as follows:

(104) *As-as-S* denotes the following set of intervals:
{I: $\exists$J, J satisfies S, the intersection of J and I is a contextually specified unit interval and it includes a bound for I}

Now we outline the action of the relevant modifiers when they apply to an expression denoting a set of intervals like the one defined in (104):

(105) *Exactly*: limits the set to contextually specified unit intervals.

(106) *At least*: limits the set to intervals which include the top of the scale if there is one, or else have no upper bound.

(l07) *At most*: limits the set to intervals which include the bottom of the scale if there is one, or else have no lower bound.

The figures in (108) below illustrate how things are meant to work:

(108)

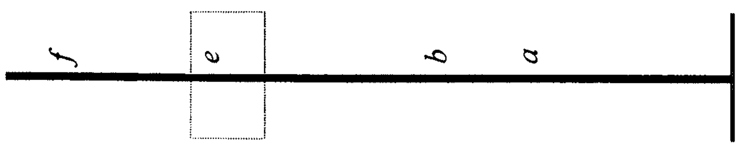

Figure 6. Exactly as tall as e is.

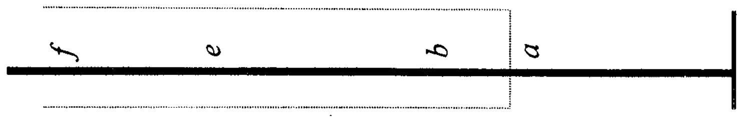

Figure 7. At least as all as b is.

Figure 8. CONTEXTUALLY SPECIFIED UNIT INTERVALS.

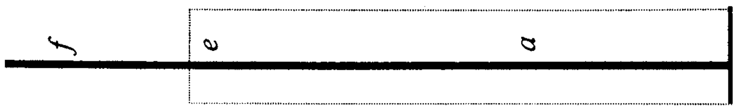

Figure 9. At most as tall as e is.

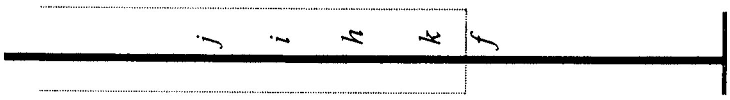

Figure 10. #At least as tall as k is.

The unit intervals are indicated in the middle in Figure 8. In Figures 6, 7, and 9, we have indicated an interval in the set denoted by the expressions in italics below. Note that since the subordinate clause contains no quantifiers, there is just one such interval in each case. Finally, the interval in Figure 10 does not fit the specifications. Call the interval depicted in Figure 10, I. There is no interval J such that I ∩ J is a contextually specified degree AND I ∩ J contains a bound for I. To make things right, the interval must extend down to include f. This corresponds to the fact that in this context, anyone who is at least as tall as k is at least as tall as f and vice versa.

We now assume that the grammar conspires to make it true that x is Ω-tall, where Ω denotes a set intervals, just in case **there is an I** in Ω and x is I-tall. In other words, we need to assume that somehow an existential comes in from the outside, applying to all equatives after modification. Kadmon faced a similar challenge in her account of the numerals, but in her case she needed no explanation because she worked in a DRT framework where the idea of external existential quantification is central.

It should be noted that the bare equative is analyzed here in such a way that it says next to nothing. To be as tall as Jane, for example, is just to be covered by some interval on the height scale that has a unit interval overlapping with an interval covering Jane. This would be true for any heighted person. This is pretty much the right outcome, if we truly believe that the bare equative is somehow vague or ambiguous between an 'at least', an 'exactly', and an 'at most' reading. As on Kadmon's account, we should like eventually to have a pragmatic story explaining how we routinely rescue this violation of the Maxim of Quantity.

While paralleling an account of numerals, the story told here is about scalar intervals. As such it still looks quite different from accounts of numerals, unless we adopt a view like that of Nerbonne (1995) where numerals are understood as relating to points on a scale. The intuition very roughly would be that a predicate like '4 boys' would be true of x if x is an object that is at the "4-portion" of the boy scale. We might modify things in such a way that 4 denotes a set of intervals around a certain point with the modifiers working roughly as defined in (105)–(107).

Hopefully, this sketch makes plain how equatives are like comparatives and how they differ. The differences can be appreciated by considering the two sentences below, which are truth-conditionally equivalent; however, if we are right, the procedure for arriving at those truth conditions is very different:

(109) Quentin is no taller than Hank is.

(110) Hank is at least as tall as Quentin is.

9. Other Degree Constructions

It is to be expected that quantifiers in the scopes of other kinds of degree expressions will lead to issues similar to the ones addressed here. We have in mind examples such as the following:

(111) Frawley is ***too*** far from most of the hospitals to qualify.

(112) John treated most of the children fairly ***enough*** to make us believe he had a good heart.

(113) The clos***est*** station to almost all of the units is Starfish I.

(114) ***How*** old are most of the members?

For (111) it will not do to talk about a point on the distance scale which corresponds to how far Frawley is from most of the hospitals, since there may not be such a point. One might suppose that this problem can be solved by removing the quantifier from the scope of *too*. But imagine that in order to qualify, Frawley has to be within one mile of at least 3 hospitals. Suppose there are 10 hospitals and Frawley is within a mile of just one of them. In that case, (111) is clearly true, as compared with (115):

(115) [For most of the hospitals m] Frawley is too far from m to qualify.

For any given hospital, it is neither the case that if Frawley qualified it would be closer to that hospital nor that if Frawley was closer to that hospital it would qualify. Similar reasoning applies in (112). (See Meier, to appear, for analysis of *too*/*enough*.)

As for (114), if the group in question includes mostly individuals in the 20–30 age range with a few exceptional 50-year-olds, the answer to the question is "between 20 and 30." We seem to be looking for the smallest interval that is included in every other interval I that satisfies the statement 'Most members are I-old'. In some cases, there will not be such an interval and (114) becomes infelicitous.

10. Positives (Adjectives with No Degree Modification)

In the sentence *The monkey is fat*, the degree argument of *fat* is left free. The standard view is that in some not yet fully understood manner, the context determines a value for the implicit degree argument. In accounts that deal in points on the scale this is not enough. For it is possible in one and the same context to say *Monkey A is fat* and *Monkey B is fat* and *Monkey A is fatter than monkey B*. Spelling these assertions out in point terms makes the difficulty clear:

(116) Monkey A is d_1-fat.

(117) Monkey B is d_2-fat.

(118) $\exists d_1 \exists d_2$ monkey A is d_1-fat and monkey B is d_2-fat and $d_1 > d_2$.

Assuming that the context supplies a salient point on the obesity scale, (116)–(118) should be impossible. By (118) d_1 and d_2 are different, but if they are both the contextually supplied point, they must be identical. The solution that is normally proposed (at least as early as Cresswell 1976) is that not only does the context supply a value for the free degree variable, but the meaning of the adjective in *The monkey is fat* is more than what you find in the comparative. In these cases, the adjective undergoes a meaning change by which it takes on a comparative sense, something along the lines of 'more Adj than X' where X is supplied by context. As Klein (1980) has pointed out, this is somewhat surprising, given that it is rare or impossible to find a language where the relevant operation is morphologically realized. This operation becomes unnecessary once we include intervals of the scale as possible values for free degree arguments, given that two individuals can be of different weights but associated with some of the same intervals on the weight scale.

11. Connectedness

It is a common assumption (Klein 1982; van Benthem 1983; Sánchez-Valencia 1994) that the relation given by a compared adjective is connected: for any adjective and entities x,y in the domain of the adjective, either x is more Adj than y or y is more Adj than x or they are equally Adj. Intuitively, unless John and Mary are the same height, either John is taller than Mary or Mary is taller than John. It is clear how connectedness comes about in a system where scalar predicates like *tall* relate individuals to unique points on the scale and where the points themselves are connected. But once we move to a system in which individuals are related to intervals on the scale, the picture changes. The greater-than relation on intervals is not connected. If one interval overlaps another, they are neither identical nor is one higher than the other. This suggests that one should encounter situations in which a scalar predicate does not induce a connected relation among individuals in its domain. Such cases exist. Imagine a scarf that ranges in color from white to grayish white. Take a patch of pure white cloth, and a patch of the grayish white. Both (119) and (120) hold true.

(119) The first patch is whiter than the second.

(120) That fresh snow over there is whiter than the whole scarf is.

Taken together, (119)–(120) entail that the scarf and the two patches are associated with intervals on the whiteness scale. Nevertheless, we cannot say that the first patch is whiter than the whole scarf is, nor that the whole scarf is whiter than the first patch. In effect, the smallest intervals of the whiteness scale associated with the scarf include the smallest ones associated with the first patch. Here's another example from a different domain. The temperature in the living room varies across the parts of the room from 60° to 70°. The living room is warmer than the inside of the refrigerator and the cup of lukewarm water (69°) is warmer than the inside of the refrigerator, but the living room is not warmer than the cup of water, nor is the reverse true. Moving to yet another domain, even though one talks about how far Los Angeles is from New York City or from New York State, one cannot answer which of the two is closer to Los Angeles.

This type of disconnectedness is familiar from the temporal domain. If one's gaze is limited to moment-sized events one finds temporal connectedness. However, once one includes intervals, connectedness is not predicted nor is it found. If the interval at which the house was built includes the interval at which the President was elected, then the election occurred neither before nor after the house building. In effect, we are drawing a parallel between accomplishments which are essentially interval-sized events and other entities, like the scarf, which relate to intervals but not points on a scale.

This type of parallelism suggests other possible connections between these various scalar domains. We will briefly mention two such connections drawing on Bennett and Partee (1972), a paper devoted to establishing the need for an interval-based tense semantics. Bennett and Partee discuss the verbs *begin* and *finish*, which serve to locate an event **e** in terms of a temporal relation that **e** bears to a large (potential) event E. In the case of *begin*, **e** occurs in the initial part of the interval in which E occurs. In the case of *finish*, **e** occurs in the final part of the interval in which E occurs. A parallel diagnosis seems plausible for the superlative. Suppose I is the smallest interval on the fluffiness scale that covers the ducks. Then the fluffiest ducks are covered by a final subinterval of I and the least fluffy ducks are covered by an initial subinterval of I.

Another parallel comes from Bennett and Partee's discussion of *resume*. In the figures in (91), at the end of section 7, we made use of discontinuous (i.e., non-convex) intervals. One might suppose that in tense semantics we only encounter convex intervals, so that this would be where tense and scalar semantics part ways. This suspicion is dispelled by Bennett and Partee's (1972) discussion of the semantics of *resume. John resumes building a house* is true at I_1 just in case *John build a house* is true at $(I_0 \cup I_1)$ for some I_0 strictly before I_1. This allows a significant gap between I_0 and I_1. A relevant intuition is that the same house has to be under construction at I_0 as at I_1, hence the need to refer to a non-convex interval.

12. Conclusion

Scalar predicates relate individuals to parts of a scale. The discovery that it is intervals which form the basis for this relation paves the way for understanding this as a relation that is monotonic with respect to the part-of ordering. Monotonicity is key in the analysis of expressions in which individual quantifiers and scalar predicates are mixed. Once this much is in place, we gain a new perspective on other aspects of scalar predication and a bridge is thrown up between the semantics of tense and the semantics of scalar predicates.

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