3.1 Preliminaries

3.2 DRS Construction

After we have seen how DRT is intended to work we will have a closer look at its precise formulation. In this section we will treat the way how DRSs are constructed from a given text. Previously, we have treated the interaction of syntactic structure and semantic interpretation by giving each syntactic rule like $X \rightarrow Y Z$ a corresponding interpretation rule, e.g. $[[X Y Z]] = [Y]([Z])$, where $[Y]$ and $[Z]$ were meanings, like objects, sets of objects, sets of possible worlds, or functions built up from such entities. Now we will have to develop a system in which syntactic rules are translated into rules that built up discourse representation structures. In the following section we will then be concerned with how these DRSs are interpreted.

3.2.1 Syntax of English Fragment

Kamp (1981) works with a very plain illustrative fragment of English. Things are considerably refined in Kamp & Reyle (1993). I will essentially follow the latter work here, which uses a GPSG syntax to specify the English fragment. See the attached copy for the syntactic rules.

3.2.2 The Basic Steps of DRS Construction

The DRS construction rules expect as input

- a discourse $D = S_1, S_2, \ldots, S_n$ ($= \text{a finite sequence of sentences}$)
- an initial DRS $K_0$. (We may assume that $K_0$ represents the background common knowledge of speaker and hearer, aspects of the situation, etc.; for our purposes we assume $K_0$ to be empty.)

and yield as output

- a DRS of the discourse $D$.

There are two recursion steps in the construction of DRSs:

- **Extrasentential**: The sentences of $D$ are covered sequentially, one after the other. This will lead to the well-known asymmetry for antecedents and anaphora: Antecedents have to precede their anaphora.
- **Intrasentential**: Each sentence $S_i$ leads to changes in the DRS according to its internal syntactic structure.

These steps can be spelled out as follows:

1. Given a discourse $D = S_1, \ldots, S_n$ and an initial DRS $K_0$, do the following:
   a) For $i = 1$ to $n$: Add sentence $S_i$ to the conditions of $K_{i-1}$; go to (b)
   b) Keep applying the DRS construction rules to each reducible condition of $K_{i-1}$ until a DRS is obtained that only contains irreducible conditions; call this DRS $K_i$ and go to (a).

Note that the extrasentential rules apply in the order of the sentences of the discourse, and the intrasentential rules apply in the order specified by the syntactic structure of each sentence (its “triggering configuration”). If a sentence is syntactically complex, this may lead to indeterminacy, that is, different construction orders are possible. We will see that this may be an attractive feature of DRT.
3.2.3 Names, Pronouns, and Indefinites

Assume the following discourse and an empty initial DRS $K_0$. (I will specify the syntactic structure only insofar as necessary.)

(2) a. $[_{[\text{NP Pedro}] [_{\text{VP} \text{has a donkey}]_{S1}} [_{[\text{NP He}] [_{\text{VP} \text{beats it}]_{S2}}.}}$

b. $K_0 = \[
\]

The extrasential recursion asks us to put the first sentence as a condition into the initial DRS. The DRS $K_0$ changes as follows:

$[_{[\text{NP Pedro}] [_{\text{VP} \text{has a donkey}]}}$

The first construction rule, CR.PN, the rule for proper names, applies to exactly this syntactic configuration.

CR.PN:
- Triggering configuration: $[_{[\text{NP}[\alpha]]_{[\text{VP} \text{ ]}]}}$ or $[_{[\text{VP}]\text{[NP}[\alpha]]}]$, as a condition of DRS K.
- Introduce into universe of main DRS of K a new discourse referent $d$
- Introduce into condition set of main DRS of K the condition $\alpha = d$
- Substitute $d$ for $[\text{NP}[\alpha]]$ in the triggering configuration.

In our case this will yield the following DRS:

$[_{u = \text{Pedro}} [_{\text{u} [_{\text{VP} [_{\text{has} [_{[\text{NP}[\alpha]] [_{\text{donkey}]}}}}}}]]$

Now the construction rule CR.ID for indefinite NPs can apply:

CR.ID:
- Trigger: $[_{[\text{NP}[\alpha]] [_{[\text{VP} \text{ ]}]}}$ or $[_{[\text{VP}]\text{[NP}[\alpha]]}]$, as a condition of a DRS K.
- Introduce into universe of K a new discourse referent $d$
- Introduce into condition set of K the condition $[_{\alpha}](d)$
- Substitute $d$ for $[\text{NP}[\alpha]]$ in the triggering configuration.

In our case this will lead to the following changes in the DRS $K_0$:

$[_{u = \text{Pedro}} [_{[\text{u} [_{\text{VP} [_{\text{has} [_{\text{NP}[\alpha]] [_{\text{donkey}]}}}}}}]]$

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Conditions like the last one often are abbreviated by \([u \text{ has } v]\).

Note that CR.PN and CR.ID differ insofar as the first one introduces a discourse referent in the **main** DRS to which K belongs, whereas the second one introduces a DRS in K itself. In the present example this doesn’t make a difference, but it will be crucial in later examples.

There are no more construction rules that could be applied. Hence we rename the DRS \(K_1\), go back to the extrasentential recursion, and add the second sentence as a condition. After this step, \(K_1\) looks as follows:

\[
\begin{align*}
   & u, v \\
   & u = \text{Pedro} \\
   & [\text{\textsc{n}} \text{donkey}](v) \\
   & [\text{\textsc{s}} u [\text{\textsc{vp}}[\text{\textsc{v}}\text{\textit{has}}][v]]] \\
   & [\text{\textsc{s}}[\text{\textsc{np}}[\text{\textsc{pro}}\text{\textit{he}}]][\text{\textsc{vp}}[\text{\textsc{v}}\text{\textit{beats}}][\text{\textsc{np}}[\text{\textsc{pro}}\text{\textit{it}}]]]]
\end{align*}
\]

Now we have to apply a rule for pronouns. Here it is:

**CR.PRO:**

- Trigger: \([\text{\textsc{s}}[\text{\textsc{np}}[\text{\textsc{pro}}\alpha]][\text{\textsc{vp}}]]\) or \([\text{\textsc{vp}}[\text{\textsc{v}}][\text{\textsc{np}}[\text{\textsc{pro}}\alpha]]]\), as condition of DRS K.
- Choose a suitable antecedent discourse referent d accessible from K.
- Substitute d for \([\text{\textsc{np}}[\text{\textsc{pro}}\alpha]]\) in the triggering configuration.

The notion of “accessible” is crucial. For now note that any discourse referent in the universe of K is accessible. Given that *he* denotes a male person and *it* may denote an animal, we get the following result after two applications of CR.PRO:

\[
\begin{align*}
   & u, v \\
   & u = \text{Pedro} \\
   & [\text{\textsc{n}} \text{donkey}](v) \\
   & [\text{\textsc{s}} u [\text{\textsc{vp}}[\text{\textsc{v}}\text{\textit{has}}][v]]] \\
   & [\text{\textsc{s}} u [\text{\textsc{vp}}[\text{\textsc{v}}\text{\textit{beats}}][v]]]
\end{align*}
\]

No other construction rule can be applied at this point; we rename \(K_1\) to \(K_2\) and go back to the extrasentential recursion. We find out that there is no further sentence in the discourse, hence \(K_2\) (= \(K_1\)) is the final DRS.

A comment is in order: The DRS construction rules we have considered so far do not follow nicely the Fregean ideal that the meaning (the representation) of a complex expression \([\alpha \beta]\) is given in terms of the meaning (the representation) of the immediate parts \(\alpha\) and \(\beta\). Certain properties of the DRS construction rule prevent an easy formulation of construction rules in that format. For example, an indefinite NP like *a donkey* is associated with a discourse referent \(d\), the requirement that \(d\) be new, and the condition \(\text{donkey}(d)\). Hence it is not simply an entity of a certain type.

### 3.2.4 Relative Clauses and Indeterminacy

Let us add here one construction for a complex syntactic construction, relative clauses. The syntactic form of relative clauses can be illustrated as follows:

(3)  Pedro owns \([\text{\textsc{np}}[\text{\textsc{det}}][\text{\textsc{n}} \text{\textsc{donkey}}]][\text{\textsc{rc}}[\text{\textsc{vp}}[\text{\textsc{v}}\text{\textit{beats}}][\text{\textsc{np}}[\text{\textsc{pro}}\text{\textit{he}}]]]]])

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Note that the relative pronoun which corresponds to a gap e within the relative clause; the pronoun and the gap share features, e.g. +/- human.

Assume that (3) is a one-sentence discourse. After rules CR.PN and CR.ID have applied, we get the following DRS:

\[
\begin{align*}
\text{The rule for relative clauses, CR.RC, and its result in the case at hand, is as follows:} \\
\text{CR.RC:} \\
\text{• Triggering configuration: } [N[N[α][RPRO which][S[he][VP[beats][NP e]..]]]](d) \text{ in DRS K.} \\
\text{• Introduce new condition } [S[α])(d) \\
\text{• Replace the trigger configuration by } [S...d...], \text{the sentence in which the gap occurs, with the gap replaced by } d. \\
\end{align*}
\]

Application of CR.PRO (with u as the antecedent discourse referent) will yield the following final (abbreviated) DRS:

\[
\begin{align*}
\text{We can now illustrate an important property of intrasentential DRS-construction with the construction rule for relative clauses: The construction process may be } \text{indeterministic}. \text{ We need this feature to account for the anaphoric possibilities with examples like the following one:} \\
(4) \quad \text{a. A farmer who saw Chiquita liked her.} \\
\quad \text{b. A farmer who saw her kicked Chiquita.} \\
\end{align*}
\]

In both cases her can refer to Chiquita. To get this we must first apply CR.PN to Chiquita and then CR.PRO to her (otherwise the discourse referent for her is not available).

Now, if we would always first reduce the relative clause and then the main clause, we could get (4.a) but not (4.b). And if we would always first reduce the main clause and then the relative clause, we would get (b) but not (a). In order to get both (a) and (b), the order of reduction must stay undetermined. That means we may first reduce one clause as far as possible, then go to the other, and perhaps come back to the first one. But this is allowed only within a clause, not for separate sentences in a discourse — otherwise, we would be able to construct a decent DRS for a text like Pedro owns her. He kicked Chiquita.
3.2.5 Negation, Conditionals, and Universal Quantifiers

We turn to syntactic constructions that introduce complex conditions. First, negation, illustrated with the following example:

\( (5) \quad \text{[a farmer [VP\{AUX did\] not [VP own a donkey]]} \)

**CR.NEG:**
- Triggering configuration: \([s\text{ did [AUX ] not [VP a]}] \)
- Replace triggering configuration by the condition
- Replace the triggering configuration by the condition \( \neg [s \text{ d [VP [VP a]]]} \)

Example: We start out with the following DRS:

\[ \text{[a farmer [VP\{AUX did\] not [VP own a donkey]]} \]

**CR.ID** will yield the following DRS:

\[
\begin{align*}
\text{u} & \quad \text{[farmer](u)} \\
\text{[s\text{ u [VP\{AUX did\] not [VP own a donkey]}]} \\
\end{align*}
\]

This is the input for **CR.NEG**, which will yield the following DRS:

\[
\begin{align*}
\text{u} & \quad \text{[farmer](u)} \\
\neg \quad [\text{s\text{ u [VP [VP owns a donkey]]}}] \\
\end{align*}
\]

Further reductions via **CR.ID** will yield the following final DRS. Note that the condition in **CR.ID** that a new discourse referent must be introduced within the **local** DRS \( K \) (in contrast to names) makes a difference here!

\[
\begin{align*}
\text{u} & \quad \text{[farmer](u)} \\

\neg \quad \text{v} & \quad \text{[donkey](v)} \\
\text{[s\text{ u [VP [VP owns v]]}] } \\
\end{align*}
\]

The fact that \( v \) is introduced in the local DRS should make it inaccessible for pronouns outside of the local box, but accessible inside (see (6.a, b). Hence by determining in which DRS a discourse referent is introduced we specify the “life span” of this discourse referent, in Karttunen’s sense. Names behave different (see c); their discourse referent is always introduced in the main DRS.
Let us now turn to conditional sentences:

(7) \[ \text{if } [\text{Pedro owns a donkey}] \text{ then } [\text{she beats it}] \]

**CR.COND:**

- **Triggering configuration:** \([s \text{ if } [s \alpha] \text{ then } [s \beta]]\)

Replace triggering configuration by the condition \([s \alpha] \Rightarrow [s \beta]\)

This yields the following result for example (7):

\[
\begin{array}{c}
[s_{\text{NP}[\text{PN Pedro}]][\text{VP}[\text{VP owns } [\text{NP[DET a]}[\text{N donkey}]]]]]) \Rightarrow [s_{\text{NP}[\text{PRO he}]][\text{VP}[\text{VP beats } [\text{NP[PRO it]]]]]])
\end{array}
\]

Applying rules CR.PN and CR.ID on the antecedent box we will get the following: note that the discourse referent for the name and the condition is introduced in the principal DRS, whereas the discourse referent for the indefinite NP is introduced in the local DRS.

\[
\begin{array}{c}
u \\
\text{u = Pedro} \\
\begin{array}{c}
v \\
[\text{donkey}] (v) \\
[\text{u owns v}]
\end{array} \\
\Rightarrow [s_{\text{NP[PRO he}]][\text{VP}[\text{VP beats } [\text{NP[PRO it]]]]]]
\end{array}
\]

We now have to apply rule CR.PRO for the consequent. Now it is time to define the notion of accessible discourse referents. It is defined in terms of the relation of one DRS being subordinated to another one. [The definition below differs slightly from the one in K&R 1993]

(8) **Def:** A DRS \(K_1\) is **immediately subordinate** to a DRS \(K_2\) iff either

a) \(K_2\) contains the condition \(\neg K_1\), or

b) \(K_2\) contains the condition of the form \(K_1 \Rightarrow K_3\) or \(K_3 \Rightarrow K_1\), for some DRS \(K_3\),

c) \(K_1\) occurs in a condition \(K_2 \Rightarrow K_1\).

(9) **Def:** A DRS \(K_1\) is **subordinate** to a DRS \(K_2\), \(K_1 < K_2\), iff either

a) \(K_1\) is immediately subordinate to \(K_2\), or

b) There is a \(K_3\) such that \(K_3\) is subordinate to \(K_2\) and \(K_1\) is immediately subordinate to \(K_3\).

We write \(K_1 \leq K_2\) iff \(K_1 < K_2\) or \(K_1 = K_2\).

(10) **Def:** A discourse referent \(d\) is **accessible** from a DRS-condition \(c\) in a DRS \(K\) iff \(d\) belongs to a DRS \(K'\), and \(K \leq K'\).
Note that for the condition in the consequent DRS in our example u and v are accessible in this sense

- u is accessible because of clause (c.i).
- v is accessible because of clause (c.ii)

Using CR.PRO twice we get the following result:

\[
\begin{align*}
\text{u} & = \text{Pedro} \\
\text{v} & \Rightarrow \left[ [\text{NP} \ u \ V] \left[ \text{VP}' \ [\text{VP} \ \text{beats} \ [\text{NP} \ v]] \right] \right]
\end{align*}
\]

Universally quantified NPs are treated like conditionals:

**CR.EVERY:**

- Triggering condition: \( [\text{NP} \left[ \text{DET every} \ [\text{N} \ \beta] \right] \ [\text{VP}'] \] or \( [\text{VP} \left[ \text{NP} \left[ \text{DET every} \ [\text{N} \ \beta] \right] \right] \].

- Introduce into conditions of \( K \) the new condition \( K_1 \Rightarrow K_2 \), where \( K_1, K_2 \) empty.

- Introduce into \( K_1 \) a new discourse referent \( d \)

- Introduce into \( K_1 \) the condition \( [\text{N} \ \beta](d) \).

- Introduce into \( K_2 \) a condition like the sentence of the triggering condition, but with \( d \) for \( [\text{NP} \left[ \text{DET every} \ [\text{N} \ \beta] \right] \).

- Delete the triggering condition.

**Example:**

(11) \( [\text{NP} \left[ \text{DET every} \ [\text{N} \ \text{farmer} \left[ \text{RC who owns a donkey} \right] \right] \ [\text{VP} \ \text{beats it}]] \]

Applying CR.EVERY we get the following DRS:

\[
\begin{align*}
\text{u} & \Rightarrow \left[ [\text{NP} \ u \ V] \left[ \text{VP}' \ [\text{VP} \ \text{beats} \ [\text{NP} \ v]] \right] \right]
\end{align*}
\]

And after application of CR.RC, CR.ID and CR.PRO we arrive at:

\[
\begin{align*}
\text{u} & \Rightarrow \left[ [\text{NP} \ u \ V] \left[ \text{VP}' \ [\text{VP} \ \text{beats} \ [\text{NP} \ v]] \right] \right]
\end{align*}
\]

\[
\begin{align*}
\text{u} & \Rightarrow [\text{u beats v}]
\end{align*}
\]
3.2.6 Antecedents for Anaphora

Anaphora resolution (i.e. the task of finding an antecedent for an anaphoric element, like a pronoun) is of central concern to DRT. DRT mainly covers the logical accessibility of antecedents. However, there is a lot more to anaphora resolution:

- Grammatical gender information. Many languages use grammatical gender information to restrict suitable antecedents. Every noun belongs conventionally to a gender class, and if a pronoun is to pick up a discourse referent, it must agree with the gender class of the head noun that introduced this discourse referent.

- Natural gender information. A pronoun has to correspond to the semantic sort, e.g. the sex, of the referent of the discourse referent that is to be picked up. Example German, which sometimes can use either strategy:

\[(12) \text{ Das Mädchen trat herein. } \{\text{Sie/es}\} \text{ setzte sich.}\]
\[
\text{the girl.NEAT entered. She/it sat down.}
\]

Gender information may be added, using special conditions with the introduction of discourse referents, like “Feminine(d)” (see K&R 1993).

- Recency. Pronouns tend to pick up the antecedent that is closest.

\[(13) \text{ a. A farmer had a donkey. He also had a dog. He beat it.}\]
\[
\text{b. A farmer had a donkey. If he also has a dog, he beat it.}\]

Recency information requires us to keep track of the order in which discourse referents are introduced, or at least to be able to identify the discourse referents introduced most recently. (Cf. the notion of “focus” in Grosz & Sidner 1986).

3.3 DRS Interpretation

After having seen how DRS construction works we turn to the interpretation of DRSs.

3.3.1 Models

DRSs are interpreted with respect to a model, just like a formal language like predicate logic. A model \( M = \langle A, F \rangle \) consists of a universe \( A \) and a function \( F \), where \( F \) has the following property:

- for every proper name \( \alpha \), \( F(\alpha) \in A \)
- for every intransitive verb and common noun \( \alpha \), \( F(\alpha) \subseteq A \)
- for every transitive verb \( \alpha \), \( F(\alpha) \subseteq AxA \).

Note: In K&R models are given, somewhat unconventionally, as triples \( \langle U_M, \text{Name}_M, \text{Pred}_M \rangle \), where \( U_M \) is the universe, where \( \text{Name}_M(\alpha) \) gives the extension of names \( \alpha \) and \( \text{Pred}_M(\alpha) \) gives the extension of predicates \( \alpha \).

3.3.2 Truth for DRSs

We now come to a definition what it means for a DRS to be true with respect to a model \( M = \langle A, F \rangle \).

We first define some auxiliary concepts:

- If \( g \) is a function, then \( \text{Dom}(g) \) and \( \text{Ran}(g) \) give the domain and range of \( g \).
- \( g' \) is called an extension of \( g \) iff \( g \subseteq g' \). That is, \( g \) and \( g' \) agree on the values for \( \text{Dom}(g) \), but \( g' \) might have a larger domain.
- \( U(K) \) and \( C(K) \) denote the universe and the set of conditions of a DRS \( K \).
A central notion for the truth definition is that a function \( g \) verifies a DRS \( K \), or a DRS condition, with respect to a model \( M \).

(14) \( g \) verifies the DRS \( K \) in \( M \) iff \( g \) verifies every element of \( C(K) \) in \( M \).

(ii) \( g \) verifies the condition \( \gamma \) in \( M \) iff:

a) \( \gamma \) is of the form \( d = \alpha \), and \( g(d) = F(\alpha) \).

b) \( \gamma \) is of the form \( \alpha(d) \), and \( g(d) \in F(\alpha) \).

c) \( \gamma \) is of the form \( [d \alpha] \), and \( g(d) \in F(\alpha) \).

d) \( \gamma \) is of the form \( [d \alpha d'] \) and \( (g(d), g(d')) \in F(\alpha) \).

e) \( \gamma \) is of the form \( \neg K' \), and there is no extension \( g' \) of \( g \) with \( \text{Dom}(g') = \text{Dom}(g) \cup U(K') \) such that \( g' \) verifies \( K' \) in \( M \).

g) \( \gamma \) is of the form \( K_1 \Rightarrow K_2 \), and for every extension \( g' \) of \( g \) with \( \text{Dom}(g') = \text{Dom}(g) \cup U(K_1) \) that verifies \( K_1 \) in \( M \), there is an extension \( g'' \) of \( g' \) with \( \text{Dom}(g'') = \text{Dom}(g') \cup U(K_2) \) that verifies \( K_2 \) in \( M \).

Note that in the definition of what it means that \( g \) verifies a DRS \( K \) we call up the definition of what it means that \( g \) verifies a DRS condition \( \gamma \) (in \( K \)), and in the definition of what it means that \( g \) verifies a condition \( \gamma \) we call up the definition of what it means that \( g \) verifies a DRS \( K' \) (in \( e \) and \( g \)). But this is not a *circulus vitiosus*, as \( K' \) is contained in \( K \) and hence will always be smaller than \( K \). We have the form of a double recursive definition.

With the help of the notion of verification we can define the notion of truth:

(15) A DRS \( K \) is true in a model \( M = \langle A, F \rangle \) iff there is a function \( g: U(K) \rightarrow A \) that verifies \( K \) in \( M \).

That is, for a DRS to be true with respect to a model we must find some embedding of the universe of the DRS into the universe of the model that verifies the DRS with respect to the model.

3.3.3 Example

Let us discuss the following discourse:

(16) Pedro owns a donkey. He loves it. If he owns a donkey, a widow beats it.

The DRS construction rules will give us the following DRS \( K \):

(17) \[
\begin{array}{c}
\text{u} & \text{v} \\
\text{u = Pedro} \\
\text{[donkey](v)} \\
\text{[u owns v]} \\
\text{[u loves v]} \\
\text{w} \\
\text{[donkey](w)} \\
\text{[u owns w]} \\
\Rightarrow \\
\text{x} \\
\text{[widow](x)} \\
\text{[x beats w]}
\end{array}
\]

Assume the following model \( M \):

- (18) \( A = \{p, j, m, s, d1, d2, d3\} \),
  \( F(\text{Pedro}) = p, F(\text{farmer}) = \{p, j\}, F(\text{donkey}) = \{d1, d2, d3\}, F(\text{widow}) = \{m, s\} \),
F(owns) = \{(p, d1), (p, d2), (j, d3)\}, F(loves) = \{(p, d1)\}, F(beats) = \{(m, d1), (s, d2)\}

According to the definition, K is true in M iff there is a function from U(K) = \{u, v\} to A which verifies K in M. The function g = \{(u, p), (v, d1)\} has the required property, that is, it verifies all the conditions of K. In particular, we have

- g verifies “u = Pedro”, as g(u) = F(Pedro) = p;
- g verifies “donkey(v)”, as g(v) \in F(donkey);
- g verifies “[u owns v]”, as \langle g(u), g(v) \rangle \in F(owns);
- g verifies “[u loves v]”, as \langle g(u), g(v) \rangle \in F(loves);
- g verifies “[w | donkey(w), [u owns w]] ⇒ [x | widow(x), [x beats w]]”, as we have:
  — for every g’: \{u, v\} \cup \{w\} \rightarrow A, with g \subseteq g’ that verifies K_1 in M
  — there is a g”: \{u, v, w\} \cup \{x\} \rightarrow A, with g’ \subseteq g” that verifies K_2 in M.

To check the last claim:

- Take g’ = g_1 = \{(u, p), (v, d1), (w, d1)\}; we have that g_1 verifies [w | donkey(w), [u owns w]] in M, as — g_1 verifies “donkey(w)”, as g_1(w) \in F(donkey); — g_1 verifies “[u owns w]”, as \langle g_1(u), g_1(w) \rangle \in F(owns); and there is a g : \{u, v, w\} \cup \{x\} \rightarrow A, with g \subseteq g”, namely h_1 = \{(u, p), (v, d1), (w, d1), (x, m)\} that verifies [x | widow(x), [x beats w]] in M, as — h_1 verifies “widow(x)”, as h_1(x) \in F(widow); — h_1 verifies “[x beats w]”, as \langle h_1(x), h_1(w) \rangle \in F(beats).

- Take g’ = g_2 = \{(u, p), (v, d1), (w, d2)\}; we have that g_2 verifies [w | donkey(w), [u owns w]] in M (as above), and there is a g” : \{u, v, w\} \cup \{x\} \rightarrow A, with g_2 \subseteq g””, namely h_2 = \{(u, p), (v, d1), (w, d2), (x, s)\} that verifies K_2 in M, as above.

- Take other versions of g, e.g. g_3 = \{(u, p), (v, d1), (w, m)\}; they verify the condition as well, as they fail to verify the antecedent condition [w | donkey(w), [u owns w]]

Hence we have shown that K is true in M.