# 6. Quantifiers in Natural Language

## 6.1. Introduction

In the previous chapter we were concerned with two kinds of noun phrases (or “determiner phrases”, in newer terminology:

(1) a. Names like *Molly*;
b. definite NPs like *the girl*

But there are many other types of noun phrases. Here are a few examples:

(2) a. *every girl* b. *a girl*
c. *some girl* d. *many girls*
e. *most girls* f. *few girls*
g. *three girls* h. *the three girls*

In this section we will turn to the semantics of such expressions. In general, they are called **quantifiers**, and the theory that investigated the properties of such quantifiers is called **Generalized Quantifier Theory**. We will leave our toy grammar for a while and investigate the semantic properties of such quantifiers.

## 6.2. Generalized Quantifiers

### *6.2.1. The General Format of Generalized Quantifiers*

The notion of Generalized Quantifiers (GQ) has its roots in Frege and arguably even farther back, in Aristotle; it has been developed more recently for mathematics by Mostowski (1957), and applied to linguistic phenomena first by Barwise & Cooper (1981).

What is the type of a quantifier? It is a noun phrase, and we have seen that one kind of noun phrases, names, are of the semantic type e, that is, they denote entities. But quantifiers like *every girl* and *no girl* cannot be of this type. For example, *no girl* cannot stand for any particular girl. One of the oldest recorded jokes makes precisely this point: When Odysseus (Ulysses) and his associates blinded the one-eyed giant (cyclop) Polyphemus, he asked for Odysseus’ name. Odysseus said that his name is “Nobody”. Polyphemus then asked his fellow cyclops to kill nobody, but of course they just laughed at him.

If a verb phrase is of type et, and the combination of a quantified NP with a verb phrase is of type t, but the quantified NP cannot of type e, then the only remaining option, if semantic combination is by functional application, is that the quantified NP is of type (et)t:

(3) $[_{S}[_{NP}\textit{Nobody}]\ [_{VP}\textit{laughed}]]$
$[\![\textit{Nobody}]\!]\ (\ [\![\textit{laughed}]\!]\ )$
$\dfrac{\text{(et)t} \qquad\qquad \text{et}}{\text{t}}$

If the quantificational noun phrase consists of a determiner and a noun, and the noun is of type et, then the determiner must be of type (et)(et)t:

(4) $[_S[_{NP}[_{Det}\text{No}]\ [_N \textit{girl}]]\ [_{VP}\ \textit{laughed}]]$
$\llbracket \textit{No} \rrbracket\ (\quad \llbracket \textit{girl} \rrbracket\ )\ (\ \llbracket \textit{laughed} \rrbracket)$
$\underline{\text{(et)(et)t} \qquad\qquad \text{et}\ }$
$\qquad \underline{\text{(et)t} \qquad\qquad\qquad\qquad \text{et}}$
$\qquad\qquad\qquad \text{t}$

In this view, determiners are **functions** from $D_{et}$ to functions from $D_{et}$ to $D_t$. Alternatively, we can consider determiners as relations between a noun meaning and a verb meaning:

(5) $\llbracket \textit{no} \rrbracket(\llbracket \textit{girl} \rrbracket, \llbracket \textit{laughed} \rrbracket)$

In this view, determiners are two-place **relations** between $D_{et}$ and $D_{et}$.

A note on terminology:

- You will sometimes find the term “quantifier” used for what we call here “determiner”.
- The first argument (here, the noun meaning) is called **restrictor**, and the second argument (here, the verb meaning) is called **matrix** or **nuclear scope**. For example, in *every girl smiled*, the noun *girl* is the restrictor, and the VP *smiled* is the matrix or nuclear scope.

The logical properties of quantifiers in natural language have been studied extensively (Barwise & Cooper, van Benthem, Keenan). This was done typically by assuming that determiners are two-place relations, with the general format: D(X,Y), or $D_A$(X)(Y), where A stands for the universe. We will switch back and forth between those two views whenever convenient.

### *6.2.2. Examples of Generalized Quantifiers*

Let us discuss a few natural-language examples of generalized quantifiers. In the functional format, a sentence like *every man walks* will be analyzed as follows.

(6) a. *every man walks:* $\llbracket \text{every} \rrbracket(\llbracket \text{man} \rrbracket)(\llbracket \text{walk} \rrbracket)$

b. $= \lambda P\quad D_{et}\lambda P'\quad D_{et}[P'\quad P](\llbracket \textit{man} \rrbracket)(\llbracket \textit{walk} \rrbracket)$
$\Leftrightarrow\ \lambda P[\llbracket \textit{man} \rrbracket\quad P](\llbracket \textit{walk} \rrbracket)$
$\Leftrightarrow\ \llbracket \textit{man} \rrbracket\quad \llbracket \textit{walk} \rrbracket$

That is, *every man walks* is true iff the set of men is a subset of the set of walkers.

Let us now look at a wider range of quantifiers, given in the same format as *every* above. I will generally assume that P, P′ are variables for elements in $D_{et}$ and leave that unexpressed.

(7) a. *all/every* N: $\lambda P[\ \llbracket N \rrbracket \cap P = \llbracket N \rrbracket]$, or $\lambda P[\ \llbracket N \rrbracket\quad P]$
b. *some* N: $\lambda P[\ \llbracket N \rrbracket \cap P \neq \varnothing]$
c. *no* N: $\lambda P[\ \llbracket N \rrbracket \cap P = \varnothing]$
d. *not all* N: $\lambda P[\ \llbracket N \rrbracket \cap P \neq \llbracket N \rrbracket]$, or $\lambda P[\ \llbracket N \rrbracket\ \_\ P]$
e. *at least two* N: $\lambda P[\#(\llbracket N \rrbracket \cap P) \geq 2]$
f. *exactly two* N: $\lambda P[\#(\llbracket N \rrbracket \cap P) = 2]$
g. *between two and five* N: $\lambda P[2 \leq \#(\llbracket N \rrbracket \cap P) \leq 5]$
h. *an odd number of* N: $\lambda P[\#(\llbracket N \rrbracket \cap P)$ is odd$]$
k. *more male than female* N: $\lambda P[\#(\llbracket N \rrbracket \cap \llbracket \textit{male} \rrbracket \cap P) > \#(\llbracket N \rrbracket \cap \llbracket \textit{female} \rrbracket \cap P)]$

## *6.2.3. Names, Definites and Indefinites*

The categories of noun phrases that we dealt with in the previous chapter can be rendered as Generalized Quantifiers as well. First of all, **names** can be seen as quantifiers, along the following lines:

(8) *Leopold*: $\lambda P[P(LB)]$

We can analyze a sentence like *Leopold smiled* as follows:

(9) a. $\llbracket \textit{Leopold} \rrbracket(\llbracket \textit{smiled} \rrbracket)$
b. $= \lambda P[P(LB)](\llbracket \textit{smiled} \rrbracket)$
c. $= \llbracket \textit{smiled} \rrbracket(LB)$

This analysis of names may seem a bit perverse at first. But there is one advantage: It allows us to treat names and true quantifiers as belonging to the same semantic type, (et)t.

We have analyzed **definite** NPs as involving a particular presupposition, a presupposition of existence and uniqueness. We can express definite NPs as follows:

(10) a. $\llbracket [_{NP} [_{Det} \textit{the}] [_{N} \textit{woman}]] \rrbracket(\llbracket \textit{smiled} \rrbracket)$
b. $= \llbracket \textit{the} \rrbracket(\llbracket \textit{woman} \rrbracket)(\llbracket \textit{smiled} \rrbracket)$
c. $= \lambda P \quad \{P \mid \#(P) = 1\}\lambda P'[P \quad P'](\llbracket \textit{woman} \rrbracket)(\llbracket \textit{smiled} \rrbracket)$
d. $= [\llbracket \textit{woman} \rrbracket \quad \llbracket \textit{smiled} \rrbracket]$, provided that $\#(\llbracket \textit{woman} \rrbracket) = 1$
e. = 1, if every woman smiled and there is exactly one woman,
= 0, if not every woman smiled and there is exactly one woman,
undefined else.

As before, we express the existence and uniqueness condition by a requirement for the domain of a function.

**Indefinite NPs**, like *a woman*, can be treated similarly to *some woman*:

(11) a. $\llbracket [_{NP} [_{Det} \textit{a}] [_{N} \textit{woman}]] \rrbracket(\llbracket \textit{smiled} \rrbracket)$
b. $= \llbracket \textit{a} \rrbracket(\llbracket \textit{woman} \rrbracket)(\llbracket \textit{smiled} \rrbracket)$
c. $= \lambda P\lambda P'[P \cap P' \neq \quad](\llbracket \textit{woman} \rrbracket)(\llbracket \textit{smiled} \rrbracket)$
d. $= [\llbracket \textit{woman} \rrbracket \cap \llbracket \textit{smiled} \rrbracket \neq \quad]$

## *6.2.4. Types of Quantifiers*

Many of the quantifiers listed above just impose a condition for the **intersection** $\llbracket N \rrbracket \cap P$, that is, the intersection of the noun meaning and the verb phrase meaning. For example, *at least two N* says that the number of elements in this set must be greater or equal than two. Such quantifiers are called **cardinal** or **intersective quantifiers**, as it depends solely on the cardinality of the intersection whether they hold or not.

But there are other quantifiers that are a bit more complex, like the following ones:

(12) a. *most* N: $\lambda P[\#(\llbracket N \rrbracket \cap P) / \#(\llbracket N \rrbracket) > 1/2]$,
b. *between 10 and 20 percent of the* N $\lambda P[0.1 \leq \#(\llbracket N \rrbracket \cap P) / \#(\llbracket N \rrbracket) \leq 0.2]$

In this case, the number of elements in $\llbracket N \rrbracket$ is important as well. More specifically, whether the quantifier obtains or doesn't obtain depends on the proportion of $\#(\llbracket N \rrbracket \cap P)$ and $\#(\llbracket N \rrbracket)$. Such quantifiers are called **proportional quantifiers**.

Quantifiers that come with a particular condition on their restrictor can be called **presuppositional**. Definite NPs like *the girl* are one type of example (see (10)), but there are others:

(13)a. *the* N: $\lambda P \;\; \{P \mid \#(\llbracket N \rrbracket) = 1\}[\,\llbracket N \rrbracket \quad P]$
b. *both* N: $\lambda P \;\; \{P \mid \#(\llbracket N \rrbracket) = 2\}[\,\llbracket N \rrbracket \quad P]$
c. *neither* N: $\lambda P \;\; \{P \mid \#(\llbracket N \rrbracket) = 2\}[\,\llbracket N \rrbracket \cap P = \emptyset]$
d. *the seven* N: $\lambda P \;\; \{P \mid \#(\llbracket N \rrbracket) = 7\}[\,\llbracket N \rrbracket \quad P]$

Another interesting class of quantifiers is the one based on the determiners *several*, *a lot of*, *many* or *few*. They are similar to cardinal quantifiers, as the intersection of $\llbracket N \rrbracket$ and P is all that matters, but we cannot give precise conditions for when a quantifier relation actually obtains. Such quantifiers are called **vague quantifiers**. The best we can do at the moment is to assume some threshold value n, a parameter that heavily depends on the context:

(14)a. *many* N: $\lambda P[\#(\llbracket N \rrbracket \cap P) > n]$, n: some context-dependent standard
b. *few* N: $\lambda P[\#(\llbracket N \rrbracket \cap P) < n]$

Vague quantifiers are of course only the tip of the iceberg when it comes to vagueness in natural language. For example, many adjectives are vague as well. When are you willing to say that someone is a *tall person*, or a *small person*? We have talked about such context-dependent adjectives in the preceding chapter.

Quantifiers like *many N* and *few N* are not only vague, they are in addition **ambiguous**. They also have a proportional reading. with a vaguely specified proportion:

(15)a. *many* N: $\lambda P[\, \#(\llbracket N \rrbracket \cap P) \,/\, \#(\llbracket N \rrbracket) > n]$
b. *few* N: $\lambda P[\, \#(\llbracket N \rrbracket \cap P) \,/\, \#(\llbracket N \rrbracket) < n]$

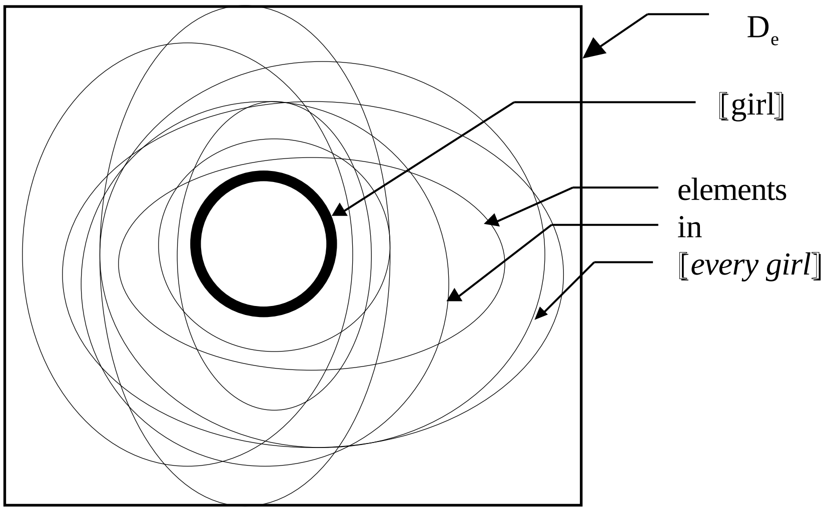
This reading is obvious in examples like the following:

(16)a. Many stars are red.
b. Few mosquitoes carry malaria.

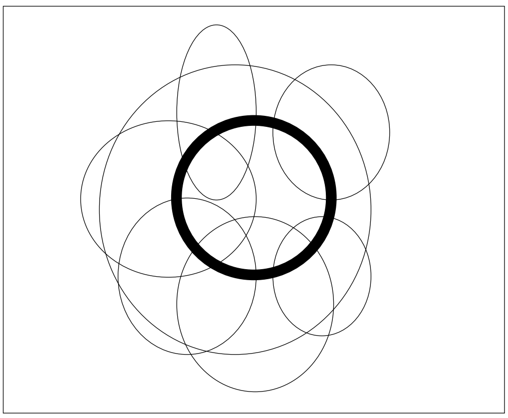
It is not the absolute number of stars that are red or the number of mosquitoes that carry malaria that is of interest, but the proportion. (Below we will discuss yet another possible interpretation of *many* and *few*, cf. (24)).

### *6.2.5. Representation of Quantifiers by Venn Diagrams*

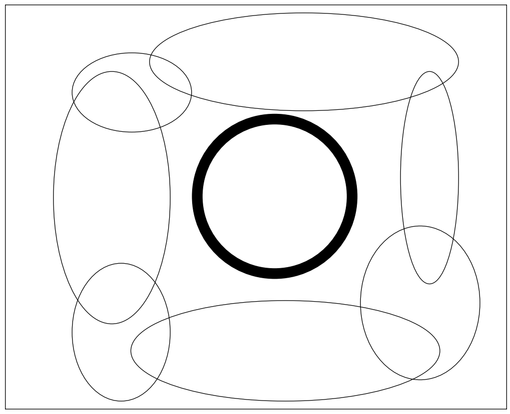
Quantifiers can be represented by Venn diagrams. For example, the quantifier *every girl* is the set of subsets of the universe $D_e$ such that the set $\llbracket$*girl*$\rrbracket$ is subset of them. We can illustrate this as follows:



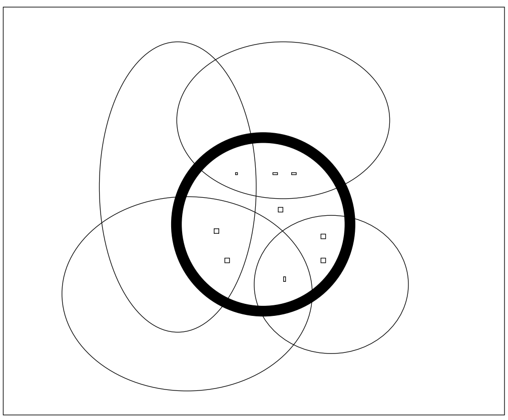

Other quantifiers can be represented similarly:



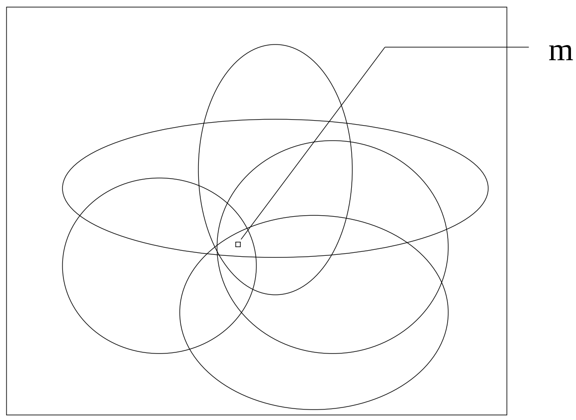
⟦*some girl*⟧



⟦*no girl*⟧



⟦*three girls*⟧




⟦*Mary*⟧

## 6.3. Universal Restrictions for Generalized Quantifiers

One important question in GQ research of the 1980's was: Which of the logically possible determiners and quantifiers are actually realized in natural language?

### *6.3.1. How Many Quantifiers (Determiners)?*

First, let us think about how many theoretically possible determiners there are — that is, how many determiners a universe of a given size can "support". We expect that natural languages actually have only a small fraction of the theoretically possible determiners. This is similar as with other expressions. For example, if we have a universe $D_e$, then every subset of $D_e$ represents the meaning of a potential one-place predicate. If the universe has n elements, we have $2^n$-many possible predicate meanings. But certainly, many of these possible predicate meanings cannot be expressed by any simple predicate in a given language. For example, there is no predicate that applies to this chair over here, that fly over there, the number 27, and nothing else.

It is convenient to analyze a **determiner** D as a **two-place relation between subsets** of $D_e$. Let us assume that $D_e$ has n elements. Then the calculation is a simple exercise:

- First, the number of subsets of $D_e$, #(pow($D_e$)), is $2^n$
- Second, the number of pairs of subsets of $D_e$, #(pow($D_e$) × pow($D_e$)), is $2^n \cdot 2^n$ , which is $4^n$
- A specific determiner, like *every*, is a particular relation between subsets of $D_e$, hence a **particular** subset of this set. And every subset of this set is the meaning of a possible determiner. The set of all possible determiners then is the power set of this set. We have:
- The number of sets of pairs of subsets of $D_e$, #(pow(pow($D_e$) × pow($D_e$))), is $2^{4^n}$

That's a lot of determiners! For example, when the universe $D_e$ consists of two elements only, that is, n = 2, then there are already $2^{16}$ = 65536 possible determiners.

You may wonder how we arrive at this huge number. Consider one example of a determiner meaning in this universe. I give as an example the meaning of the determiner *every* in a universe that consists only of the elements a and b:

(17)

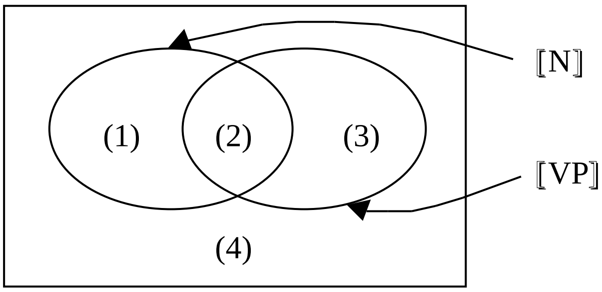
| pair of sets | | truth value | pair of sets | | truth value |
|---|---|---|---|---|---|
| , | → | 1 | , {b} | → | 1 |
| {a}, | → | 0 | {a}, {b} | → | 0 |
| {b}, | → | 0 | {b}, {b} | → | 1 |
| {a, b}, | → | 0 | {a, b}, {b} | → | 0 |
| , {a} | → | 1 | , {a, b} | → | 1 |
| {a}, {a} | → | 1 | {a}, {a, b} | → | 1 |
| {b}, {a} | → | 0 | {b}, {a, b} | → | 1 |
| {a, b}, {a} | → | 0 | {a, b}, {a, b} | → | 1 |

Notice that this characterizes just one determiner. Every distribution of truth values over these 16 pairs constitutes a distinct determiner meaning — which gives us $2^{16}$ determiner meanings alltogether.

### *6.3.2. Another Way of Expressing Determiners*

There is another perspective that allows us to express the meaning of determiners. We have seen that a determiner denotes a relation between two sets, ⟦N⟧ and ⟦VP⟧. We can depict this in the following generalized Venn diagram:

(18)




There are four "cells" that are relevant for a particular instance of a set ⟦N⟧ and ⟦VP⟧:

(19) (1): the set ⟦N⟧ ¾ ⟦VP⟧
(2): the set ⟦N⟧ ∩ ⟦VP⟧
(3): the set ⟦VP⟧ ¾ ⟦N⟧
(4): the set ($D_e$ ¾ ⟦N⟧) ¾ ⟦VP⟧.

We can characterize every determiner in terms of the four cells. For example:

(20)a. *every* N VP is true iff (1) = ,
b. *some* N VP is true iff (2) ≠ ,
c. *no* N VP is true iff (2) = ,
d. *an odd number of* N VP is true iff #(2) is odd.

We can compute the number of possible determiners under this perspective in the following way. A particular pair of two sets ⟦N⟧ and ⟦VP⟧, together with a particular universe $D_e$, can be described by determining, for each individual x in $D_e$, whether x belongs to (1), (2), (3), or (4). If the universe $D_e$ has n elements, we get matrices like the following:

(21)

| Elements of $D_e$ | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $x_1$ | √ | ¾ | ¾ | ¾ |
| $x_2$ | ¾ | ¾ | √ | ¾ |
| $x_3$ | ¾ | ¾ | ¾ | √ |
| .... | ... | ... | ... | ... |
| $x_n$ | ¾ | ¾ | √ | ¾ |

Each particular pair ⟦N⟧, ⟦VP⟧ corresponds to a particular matrix. We can calculate the number of all pairs of sets ⟦N⟧, ⟦VP⟧ according to this scheme, as follows: Each element $x_i$ might be in one of four sections, (1) to (4). As each element is independent of any other, we have 4 • 4 •...• 4 (n times) = $4^n$ many possible matrices. As each matrix corresponds to a pair of ⟦N⟧ and ⟦VP⟧, we have $4^n$ many possible pairs.

Now, a particular determiner is a particular **subset** of the set of all pairs of ⟦N⟧ and ⟦VP⟧. For example, the determiner *every* can be identified with the set of all matrices for which (1) is empty. We know that if a set has m elements, then there are $2^m$ subsets. A set with $4^n$ elements hence has

$24^n$ subsets. Hence there are $2^{4^n}$-many possible determiners in a universe with n elements. We see that this way of counting quantifiers gives us the same result as the one we used before.

An interesting question at this point is: Do we need, for every element x, the information whether x is in (1), (2), (3) or (4)? This depends on the specific determiner we are considering. A “worst-case” determiner for which we would indeed need information about all sets (1), (2), (3), (4) is D*, defined as follows:

(22) D*(⟦N⟧, ⟦VP⟧) is true in a universe $D_e$ iff #(1) = #(2) and #(3) < #(4).

For example, *D* boys smiled* would be true if the number of boys that smiled is equal to the number of boys that did not smile, and there are fewer smiling non-boys than entities that neither smile nor are boys. Such weird determiners do not occur in natural language. The quantifiers we have considered so far all can be expressed by using information present in the cells (1) and (2). It turns out that the cells (1) and (2) are by far the most important for quantifiers in general. Let us have a look at a number of constraints that have been discussed in quantifier research.

### *6.3.3. Extensionality*

One constraint that seems to be obvious for natural-language determiners is that cell (4) (the elements in the rest of the universe) does not matter. For example, to assess whether *most girls smiled* is true, we are not concerned at all about rocks, or cars, or whatever the universe contains besides the girls and the sleepers.

Determiners that have this property are called **extensional**. We can define extensionality as follows:

(23) A determiner D is extensional iff for all P,P′ with P, P′   $D_e$ and all $D_e$* with $D_e$   $D_e$* the following holds:
D(P, P′) is true in $D_e$ iff D(P, P′) is true in $D_e$*.

The only possible exception to extensionality that has been discussed (by Dag Westerståhl, 1985, *Linguistics & Philosophy*) is a certain use of *many* and *few*. Assume that we are talking about human beings, hence the universe $D_e$ is the set of all people. A sentence like

(24) Many Scandinavians are blond

may be paraphrased as: The incidence of blondness among Scandinavians is greater than the incidence of blondness among people (= the universe) in general. Hence:

*many* N VP is true in $D_e$ iff $\dfrac{\#(⟦N⟧ \cap ⟦VP⟧)}{\#(⟦N⟧)} > \dfrac{\#(⟦VP⟧)}{\#(D_e)}$

Or, in terms of cells:

$$\frac{\#(2)}{\#(1) + \#(2)} > \frac{\#(2) + \#(3)}{\#(1) + \#(2) + \#(3) + \#(4)}$$

Obviously, for this interpretation of *many* the cell (4) counts — adding more entities to the universe might make the sentence false. Hence *many* is not an extensional quantifier in this interpretation. It is, however, an open issue whether this is an appropriate semantic analysis of *many*.

## *6.3.4. Conservativity*

One particularly important restriction for natural-language determiners is that they have the property of being **conservative**, in the following sense:

(25) D is conservative iff for all P, P′: D(P, P′) iff D(P, P ∩ P′) (that is, the extension of P′ counts only insofar as it overlaps with P).

Examples:

(26) a. *Every* N VP: ⟦N⟧ ⟦VP⟧ iff ⟦N⟧ ⟦N⟧ ∩ ⟦VP⟧
b. *No* N VP: ⟦N⟧ ∩ ⟦VP⟧ = Ø iff ⟦N⟧ ∩ ⟦N⟧ ∩ ⟦VP⟧ = Ø

Test for conservativity: A determiner D is conservative iff it supports the following logical equivalence, for all N and VP:

D N VP ⇔ D N *are* N *that* VP.

Examples:

(27) a. *All girls smiled.* ⇔ *All girls are girls that smiled.*
b. *Some girls smiled.* ⇔ *Some girls are girls that smiled.*
c. *Most girls smiled.* ⇔ *Most girls are girls that smiled.*
d. *No girl smiled* ⇔ *No girl is a girl that smiled.*

If all natural language determiners are conservative, then the question is: How many conservative determiners are there? We can find out about this by simply looking at the relevant cells in the representation of quantifier meanings in (17). For conservative determiners we don't care for cell (3) (the elements of the VP meaning that are not also elements of the N meaning), and we can combine cell (3) and (4). Hence we have only three sections that are relevant. The number of determiners drops to $2^{3^n}$, for a universe with n elements. In a universe with only two elements, we will only have $2^9$, that is, 512 conservative determiners.

Why is it that natural-language determiners are conservative and extensional? One answer might be in the way how we process statements with quantifiers. It turns out that conservative and extensional quantifiers are conceptualy simpler than other quantifiers. To check whether a statement of the form D(P,Q) is true or false, we can concentrate on the set P (the extension of the noun); other entities are irrelevant. This means, effectively, that we can shrink the universe to the set P, for the purpose of evaluating the statement.

There is a potential counterexample to the claim that all natural language quantifiers are conservativity, namely *only*, analyzed as a determiner, as in *Only girls sleep*: This sentence is true iff there is no sleeper that is not a girl. Hence *only* imposes a restriction on cell (3): (3) must be empty.

(28) *Only* N: λP[P ⟦N⟧]

But *only* is quite different from other determiners: It can be used with names, as in (a), or as an adverbial, as in (b) and (c). Also, in order to interpret sentences containing *only* we must identify the constituent that is stressed (the constituent in **focus**).

(29) a. Only JOHN kissed Mary.
b. John only kissed MARY.
c. John only KISSED Mary.

The position of the focus has crucial influence on the interpretation. For example, (b) means 'the only person John kissed was Mary', and (c) means 'the only thing that John did to Mary was that he kissed her'. Hence it seems that it is focus that determines the readings of sentences with *only*. Very roughly, we have:

(30)[...*only*...F...] means: The only P such that [...P...] is F.
(or: For all P such that [...P...] it holds that F = P)

But this is a radically different case from nominal determiners. Hence we can assume that *only* isn't a determiner after all.

Note that even if we treat *only* as a determiner, it has one property that is related to conservativity. It holds for D = *only* that

(31)For all P, P′: $D(P, P') \Leftrightarrow D(P \cap P', P')$.

This property is sometimes called **anti-conservativity**. For example, we have that *Only girls smiled* is logically equivalent to *Only girls that smiled smiled*. In terms of cells, only (2) and (3) matter for anti-conservative determiners.

## *6.3.5. Intersectivity*

Intersective determiners are those for which the truth conditions of a sentence of the form D(P, P′) can be stated solely in terms of the intersection of P and P′. For example, *some*, *more than two*, *less than seven*, *an odd number of*, or *no* are intersective determiners. Clearly, determiners like *every*, *the* or *most* are not intersective. Of course, all intersective determiners are also extensional and conservative. But intersective determiners are even more restricted, as only the content of cell (2) matters.

How many intersective determiners are there, in a universe with n elements? For intersective quantifiers it is sufficient to know whether an individual is in cell (2) or not. Following the reasoning above, we will have only $2^{2^n}$ intersective determiners. This means, for example, that a universe with 2 elements will have just 16 intersective determiners.

Intersective determiners are a linguistically interesting class, because it is precisely these determiners that occur easily in **existential constructions** (cf. Keenan 1987).

(32)(i) There was a student at the party.
There were more than seven students at the party.
There were more male than female students at the party.
There were few students at the party.
There was no student at the party.

(ii) *There was every student at the party.
*There were most students at the party.
*There was the student at the party.
*There were both students at the party.
*There was John at the party.

The following explanation has been proposed why intersectivity may lead to this distribution in grammaticality. Existential sentences express the existence of entities in the universe. For example, the first sentence in (i) says that the universe of discourse (which is restricted to the people at the party, by the phrase *at the party*) contains a student. The second sentence says that it contains more than seven students, and the last sentence says that it contains no student. Notice that these are all informative sentences — they could be false.

Contrast this with the sentences in (ii). If we interpret these sentences in the same way, as a statement about the universe restricted to people at the party, then the first sentence says that the universe contains every student. But this is trivially true, a tautology. The second sentence says that the universe contains most students — again a tautology. The third sentence has a presuppositional determiner, and can be evaluated only in case the universe contains exactly one student. But in all universes that satisfy this requirement, the sentence again expresses a triviality. Similar with the other sentences. In particular, the last sentence can be interpreted only if the name *John* refers to some individual. If it can be evaluated, it says that this individual is in the universe, again something that is trivially true. Hence the explanation for the distribution that we find with (i) and (ii) is that the sentences (ii) express tautologies and hence cannot be used to communicate anything meaningful.

Notice that they contrast with the following non-existential sentences that do express something meaningful:

(iii) Every student was at the party.
Most students were at the party.
The student was at the party.
Both students were at the party.
John was at the party.

But in this case, *at the party* does not specify the domain of discourse, but is rather the VP argument of the determiner.

This explanation is perhaps not the final one, because we sometimes indeed find that sentences of type (ii) can be quite good:

(33)a. There is every reason to believe that Bill is the thief.
b. I turned the corner, and what do I see? There was John standing at a lamppost.

But in these cases we don't seem to talk about existing, pre-specified universes and their members. Example (a) expresses something like: Every potential reason to believe that Bill is the thief exists in the universe under consideration, that is, can be regarded as a good reason. And (b) says John, whose existence in a larger universe of discourse is given, actually existed in the smaller universe that appeared when the speaker turned around the corner.

### *6.3.6. Quantitativity*

Another universal restriction for natural-language determiners is the following: In order to determine whether D(P, Q) is true, it is sufficient to check the cardinality of the sets P and P ∩ Q. That is, we don't have to know the **identity** of those elements; it suffices to know **how many** elements there are. Examples:

(34)a. ⟦*every*⟧(P, P′): $\#(P) = \#(P \cap P')$
b. ⟦*no*⟧(P, P′): $\#(P \cap P') = 0$
c. ⟦*some*⟧(P, P′): $\#(P \cap P') \geq 1$
d. ⟦*most*⟧(P, P′): $\#(P \cap P')/\#(P) > 1/2$.

An obvious counterexample are genitive NPs, like *John's* in *John's books*. If we paraphrase *John's* by "the objects that belong to John", we get the following interpretation, which cannot be rephrased as a simple statement about cardinalities.

(35) ⟦*John's*⟧(P, P′): [P ∩ {y| y belongs to John}] P′.

Quantitative determiners are particularly simple, as all that counts is the number of elements in the cells (1), (2), (3), (4) of the diagram in (17). If we have a universe with n elements, then a quantitative determiner can be characterized by four numbers $n_1$, $n_2$, $n_3$ and $n_4$ (one number for each cell); the numbers add up to n, the number of elements in the universe. For conservative and/or extensive determiners it may be sufficient to know one or two of these numbers. For example:

(36)a. *no*: $n_2 = 0$
b. *every*: $n_1 = 0$
c. *some*: $n_2 \geq 1$
d. *most*: $n_2 > 1/2\ n_1$

## 6.4. Monotonicity

So far we were interested in general properties of all natural-language quantifiers. Now, there are interesting logical differences between quantifiers that allow us to classify them in semantically interesting subclasses.

### *6.4.1. Monotonicity of Quantifiers*

First let us have a look at some inference patterns that relate to the VP-element of a quantifier (we use the symbol to express that if the right-hand sentence is true, then the left-hand sentence is true).

(37)a. *All men walked rapidly.* *All men walked.*
b. *A girl smoked a cigar.* *A girl smoked.*
c. *Most students slept and snored.* *Most students slept.*
d. *Both tourists ate spaghetti.* *Both tourists ate.*

In (37) we replaced the VP of the first sentence by a VP with a **more general** meaning; note that the opposite pattern ( ) does not hold.

(38)a. *No man walked.* *No man walked rapidly.*
b. *Few girls smoked.* *Few girls smoked a cigar.*
c. *Less than 10% of the students slept.* *Less than 10% of the students slept and snored.*
d. *Neither tourist ate.* *Neither tourist ate spaghetti.*

In (38) we replaced the VP of the first sentence by a VP with a **more specific** meaning; note that the opposite pattern ( ) does not hold.

The quantifiers in (37) are called **upward monotone** or **increasing**, a property that is defined in the following way:

(39)a. Q is upward monotone iff it holds for all interpretations:
If P Q and P P′, then P′ Q.
(that is, upward monotone quantifiers are closed under extension.)

Quantifiers in (38) are called **downward monotone** or **decreasing**:

(39)b. Q is downward monotone iff it holds for all interpretations:
If P  Q and P′  P, then P′  Q.
Downward monotone quantifiers are closed under contraction.

Other formulations of monotonicity that are equivalent with the ones given above:

(39)c. Q is upward monotone iff for all P, P′: If P ∩ P′  Q, then P  Q and P′  Q.
d. Q is downward monotone iff for all P, P′: If P  P′  Q, then P  Q and P′  Q.

Consider the following examples that make use of the schematas in (39.c,d):

(40)a. *All girls were smoking and drinking.* *All girls were smoking.*
*All girls were drinking.*

b. *Few boys were singing and dancing.* / *Few boys were singing.*
/ *Few boys were dancing.*

c. *All girls were smoking or drinking.* / *All girls were smoking.*
/ *All girls were drinking.*

d. *Few boys were singing or dancing.* *Few boys were singing.*
*Few boys were dancing.*

There are quantifiers that are neither upward monotone nor downward monotone. Examples are *exactly three boys* and *between three and five boys.*

(41) *Exactly three boys were smoking and drinking.* / *Exactly three boys were smoking.*
/ *Exactly three boys were drinking.*

*Exactly three boys were smoking or drinking.* / *Exactly three boys were smoking.*
/ *Exactly three boys were drinking.*

## *6.4.2. Monotonicity and Count Complexity*

Montonic quantifiers are particularly simple to evaluate when compared to non-monotone ones. Imagine the following task:

| |
|---|
| Check whether a quantified statement D(N)(VP) is true or false, where N applies to n elements! |

(Note that the number of elements in VP or the universe doesn't matter, as we assume conservativity and extensionality)

Let us now compute the minimal number of elements in the domain we have to check in order to prove to an incredulous opponent whether a particular statement is true or false; the sum of these numbers can be taken as a kind of complexity measure for the semantic computation. We call this number the **count complexity** of a quantifier.

(42)a. *All children are asleep* — Verify: n elements, Falsify: 1 element,
Count complexity: n+1 elements.

b. *Some children are asleep*: — Verify: 1 element, Falsify: n elements
Count complexity: n+1

c. *No child is asleep:* — Verify: n elements, Falsify: 1 element.
Count complexity: n+1

d. *At least k children are asleep* — Verify: k elements, , Falsify: n-k+1 elements,
Count complexity: n+1

e. *At most k children are asleep*: — Verify: n-k elements., Falsify: k+1 elements.
Count complexity: n+1

These were all monotone quantifiers. Now take a non-monotone one, for comparison:

*Exactly k children are asleep*: — Verify: n elements, Falsify: k+1 elements.
Count complexity: n+k+1 (!)

In general, n+1 is the minimal count complexity for quantifiers. It can be shown that all quantifiers that have this “minimal count complexity” are upward or downward monotone.

Interestingly, we find that monotone quantifiers are special in their syntactic form, too: We find that every non-compound quantifier in natural language is (upward or downward) monotone (cf. a). Non-monotone quantifiers, are expressed by complex expressions (cf. b):

(43)a. *John, everybody, something, nobody,*
b. *exactly three children, between three and seven children, an odd number of children.*

Hence we find that quantifiers that are most simple syntactically are also most simple semantically.

## *6.4.3. Monotonicity of Determiners*

We have seen that determiners like *every, most,* or *no* may be treated as two-place relations between sets. As with quantifiers like *everybody* or *nothing*, we can investigate the semantic properties of certain classes of determiners. For example, we can investigate the monotonicity properties of the noun argument of determiners.

One set of determiners that includes *some*, *at least three*, *not all,* and many others exhibit the following inference patterns:

(44)a. *Some lions roared* / / *Some animals roared.*
b. *At least three girls smoked* / / *At least three persons smoked.*
c. *Not all boys drank.* / / *Not all persons drank.*

Determiners with that property are upward-monotone in their noun argument, or **persistent**:

(45)A determiner D is persistent iff for all P, P′, P″, if D(P, P″) and P  P′, then D(P′,P″).

Another set of determiners that includes *all* and *every, at most*, and *no,* exhibits the following pattern of inference:

(46)a. *All animals roared.* / / *All lions roared.*
b. *At most three persons smoked* / / *At most three girls smoked*
c. *No person drank* / / *No boy drank.*

Such determiners are downward-monotone in their noun argument, or **anti-persistent**:

(45)A determiner D is anti-persistent iff for all P, P′, P″,
if D(P, P″) and P′ P, then D(P′, P″).

Many determiners are neither persistent nor anti-persistent, e.g. *most*, *the*:

(47)a. *most girls smoked* / / / *most persons smoked*.
b. *the girl smoked* / / / *the person smoked.*

Marking upward monotonicity by ↑ and downward monotonicity by ↓, and if we distinguish between the left argument of a determiner (= the noun argument) and the right argument (= the VP argument), then we can characterize the monotonicity properties of the four basic determiners in the following way:

(48) *all*: ↓mon↑ *no*: ↓mon↓
*some*: ↑mon↑ *not all*: ↑mon↓

## 6.5. Quantifiers and Negation

### *6.5.1. Negation Ambiguities*

Let us now consider how quantifiers interact with negation. We have discussed the semantics of negation in chapter 3, where we discussed sentences like the following:

(49)a. It is not the case that Molly snores.
b. Molly doesn’t snore.

These sentences are unambiguous. But consider now a sentence with a quantified subject:

(50)Every student doesn’t snore.

Arguably, this sentence has two meanings:

- ‘It is not the case that every student snores’, which includes that some students might snore;
- ‘For every student holds: he or she doesn’t snore’, which excludes this possibility.

We could have expressed the second reading also by *No student snores*. The two meanings appear to correlate with particular intonation patterns. We have ecountered it already with examples like the following:

(51)All that glitters isn’t gold.

The idiomatic interpretation corresponds to (i); ‘it is not the case that all that glitters is gold’. But the sentence also has another possible interpretation, corresponding to (ii): ‘For all that glitters the following holds: It isn’t gold.’

Why do these ambiguities arise in the case of quantifiers, but not in the case of names? This is a consequence of the fact that quantifiers of type (et)t are semantically more complex than names of type e. We can discuss this in the framework of the Toy Grammar that we have developed so far.

Notice that the way how we paraphrased the reading (i) suggests the use of a sentential negation, *it is not the case that*, whereas the way we paraphrased reading (ii) suggests the use of an auxiliary negation. Let us first recall the derivation of the two sentences in (64):

(52)a. $\llbracket[_S[_{Mod}\textit{it-is-not-the-case-that}][_S\textit{Molly snores}]]\rrbracket$
   b. $= \llbracket[_{Mod}\textit{it-is-not-the-case-that}]\rrbracket(\llbracket[_S\textit{Molly snores}]\rrbracket)$
   c. $= \lambda t\ \ D_t[1—t]([\text{MB snores}])$
   d. $= [1 — [\text{MB snores}]]$
   e. $= 1$, if $[\text{MB snores}] = 0$,
      $= 0$, if $[\text{MB snores}] = 1$

(53)a. $\llbracket[_S\ [_{NP}\ \textit{Molly}]\ [_{VP}\ [_{Aux}\ \textit{doesn't}\ ]\ [_{VPinf}\ [_{Vinf}\ \textit{snore}]]]]\rrbracket$
   b. $= \llbracket[_{Aux}\ \textit{doesn't}\ ]\ [_{VPinf}\ [_{Vinf}\ \textit{snore}]]]]]\rrbracket(\llbracket[_{NP}\ \textit{Molly}]\rrbracket)$
   c. $= \llbracket \textit{doesn't}\rrbracket(\llbracket \textit{snore}\rrbracket)(\llbracket \textit{Molly}\rrbracket)$
   d. $= \lambda P\ \ D_{et}\lambda y\ \ D_e[1—P(y)](\lambda x\ \ D_e[\text{x snores}])(\text{MB})$
   e. $= \lambda y\ \ D_e[1—\lambda x\ \ D_e[\text{x snores}](y)](\text{MB})$
   f. $= \lambda y\ \ D_e[1—[\text{y snores}]](\text{MB})$
   g. $= [1—[\text{MB snores}]]$
   h. $= 1$, if $[\text{MB snores}] = 0$,
      $= 0$, if $[\text{MB snores}] = 1$

We arrive at the same result, though on different paths. Now look what happens with a quantified sentence:

(54)a. $\llbracket[_S[_{Mod}\textit{it-is-not-the-case-that}][_S\textit{every student snores}]\rrbracket$
   b. $= \llbracket[_{Mod}\textit{it-is-not-the-case-that}]\rrbracket(\llbracket[_S\textit{every student snores}]\rrbracket)$
   c. $= \lambda t\ \ D_t[1—t]([\llbracket \textit{student}\rrbracket\quad \llbracket \textit{snores}\rrbracket])$
   d. $= [1 — [\llbracket \textit{student}\rrbracket\quad \llbracket \textit{snores}\rrbracket]]$
   e. $= 1$, if $[\llbracket \textit{student}\rrbracket\quad \llbracket \textit{snores}\rrbracket] = 0$,
      $= 0$, if $[\llbracket \textit{student}\rrbracket\quad \llbracket \textit{snores}\rrbracket] = 1$.

We get the truth value 0 if all the students are snorers, otherwise the truth value 1. That is compatible with situations in which a few students snore, provided that it's not all of them.

(55)a. $\llbracket[_S\ [_{NP}\ \textit{every student}]\ [_{VP}\ [_{Aux}\ \textit{doesn't}\ ]\ [_{VPinf}\ [_{Vinf}\ \textit{snore}]]]]\rrbracket$
   b. $= \llbracket[_{Aux}\ \textit{doesn't}\ ]\ [_{VPinf}\ [_{Vinf}\ \textit{snore}]]]]]\rrbracket(\llbracket[_{NP}\ \textit{every student}]\rrbracket)$
   c. $= \lambda P\ \ D_{et}[\llbracket \textit{student}\rrbracket\quad P](\lambda y\ \ D_e[1— \llbracket \textit{snores}\rrbracket(y)])$
   d. $= [\llbracket \textit{student}\rrbracket\quad \lambda y\ \ D_e[1— \llbracket \textit{snores}\rrbracket(y)]$
   e. $= 1$, if $[\llbracket \textit{student}\rrbracket\quad \lambda y\ \ D_e[1— \llbracket \textit{snores}\rrbracket(y)]$,
      $= 0$, else.

This sentence is true if the students are a subset of the non-snores, that is, if no student snores, and true otherwise. Notice that this reading is clearly distinct from the one we have obtained in the first case.

The two readings differ in what semanticists call the **scope** of the negation and the quantifier. In (69), the negation has wide scope over the quantifier, and in (71), the quantifier has wide scope over negation.

We have seen that the English sentence *Every student doesn't snore* may have also the reading we have derived in (69) for *It is not the case that every student snores*, that is, a reading with wide-scope negation. How can we derive this reading, then, for *Every student doesn't snore*? One way is to type-lift negation in an ingenious way so that it takes the subject NP as an argument again, but now a subject NP of the type of quantifiers:

(56) $[\![not]\!] = \lambda P \quad D_{et} \lambda Q \quad D_{(et)t}[1 — Q(P)]$

We then get the following derivation:

(57)a. $[\![[_S\ [_{NP}\ \textit{every student}]\ [_{VP}\ [_{Aux}\ \textit{doesn't}\ ]\ [_{VPinf}\ [_{Vinf}\ \textit{snore}]]]]]\!]$
b. $= [\![[_{Aux}\ \textit{doesn't}\ ]\ [_{VPinf}\ [_{Vinf}\ \textit{snore}]]]]]\!]([\![[_{NP}\ \textit{every student}]]\!])$
c. $= [\![\textit{doesn't}]\!]([\![\textit{snore}]\!])([\![[_{NP}\ \textit{every student}]]\!])$
d. $= \lambda P \quad D_{et} \lambda Q \quad D_{(et)t}[1 — Q(P)]([\![\textit{snore}]\!])(\lambda P \quad D_{et}[[\![\textit{student}]\!] \quad P])$
e. $= \lambda Q \quad D_{(et)t}[1 — Q([\![\textit{snore}]\!])](\lambda P \quad D_{et}[[\![\textit{student}]\!] \quad P])$
f. $= [1 — \lambda P \quad D_{et}[[\![\textit{student}]\!] \quad P]([\![\textit{snore}]\!])]$
g. $= [1 — [[\![\textit{student}]\!] \quad [\![\textit{snore}]\!]]]$

Notice that this is the same result as the one we got for the derivation (69).

### *6.5.2. Other Quantifiers and Negation*

So far we have just looked at negation patterns with one type of quantifier, namely, universal quantifiers like *every student*. What about other quantifiers? Consider the following examples:

(58)a. It is not the case that three arrows hit the target.
b. Three arrows didn't hit the target.
(i) 'For three arrows it holds: they did not hit the target.'
(ii) 'It is not the case that three arrows hit the target.'

Assume that there were 10 arrows shot at the target, and 7 of those hit the target. Then (i) is true but (ii) is false. The truth conditions are different, hence the sentence has different readings.

Sometimes we have the feeling that a sentence is ambiguous even though there is no difference in truth conditions. Consider the following:

(59)a. It is not the case that most of the arrows hi the target.
b. Most of the arrows didn't hit the target.
(i) 'For most of the arrows it holds: they did not hit the target.
(ii) 'It is not the case that most of the arrows hit the target.'

Where "most of the arrows" is interpreted as: "more than half of the arrows". Assume as before 10 arrows shot at the target. You can experiment with different numbers, and you will see that (i) and (ii) are either both true or both false. So, should we assume an ambiguity in the first place? Yes, because it seems that the syntactic structure of those sentences in general allows for two interpretations schemes that just happen to have the same truth conditions. (This is similar to $1\cdot 2+2$, which is either $(1\cdot 2)+2$, or $1\cdot (2+2)$, in both cases, 4.)

Another case of a quantifier that allows for an ambiguous reading is the following:

(60) a. It is not the case that a student sit on the bank.
b. A student didn't sit on the bench.
(i) 'It is not the case that a student sat on the bench.'
(ii) 'There is a student for which the following holds: He/she sat on the bench.'

But notice that not every quantifier can be easily combined with negation. Contrast (63) with the following:

(61)a. It is not the case that some student sat on the bench.
b. Some student didn't sit on the bench.

Here, (b), and even (a), is rather interpreted with narrow-scope negation: 'There is some student for which it holds that he/she sat on the bench', at least if *some* is not reduced to something that linguists sometimes write *sm*. In order to enforce the other interpretation, we should replace *some* by *any*:

(62)It is not the case that any student came to the party.

Another case in point is is the negative quantifier *no* :

(63)a. It is not the case that no student sat on the beach.
b. No student didn't sit on the bench.

While (a) is fine, with a meaning as predicted ('At least some student sat on the bench'), (b) sounds quite odd, except for speakers of English dialects that allow for multiple negation. But even then it doesn't mean what it is supposed to, but rather the same as 'No student sat on the bench'. In these dialects, one of the negations is superfluous; it's there just for the sake of grammar.

### *6.5.3. Negation of Quantifiers*

Certain quantifiers correspond to each other — one being the "upward monotone" counterpart, the other the "downward monotone" counterpart. Let's discuss a few examples:

| (64)Upward monotone: | Downward monotone: |
|---|---|
| *all* N | *not all* N |
| *some* N | *no* N |
| *many* N | *few* N |
| *at least n* N | *at most n* N. |
| *more than half of the* N | *less than half of the* N |

In general: Whenever Q is an upward (downward) monotone quantifier, then its negation is downward (upward) monotone. The negation of a quantifier Q, for which I will write ¬Q, is defined as the complement of Q in its domain:

(65) ⟦¬Q⟧ = {P| P ⟦Q⟧}, which in turn is pow(A) ¾ ⟦Q⟧.

Examples:

(66)a. ⟦*not all* N⟧,
= ⟦¬*all* N⟧,
= {P| P ⟦*all* N⟧},
= {P| P {P| ⟦N⟧ P]},
= {P| ⟦N⟧ _ *P*}

b. ⟦*not some* N⟧
  ⟦¬*some* N⟧,
  = {P| P  ⟦*some* N⟧}
  = {P| P  {P| ⟦N⟧ ∩ P≠Ø]},
  = {P| P ∩ ⟦N⟧ = Ø}
  = ⟦*no* N⟧

Actually, this is just one type of negation that we can define for quantifiers — the **external negation**. In addition, we we can define an **internal negation**, for which I will write Q¬:

(67) ⟦Q¬⟧ = {P| (A — P)  Q} (take for every element of Q its complement).

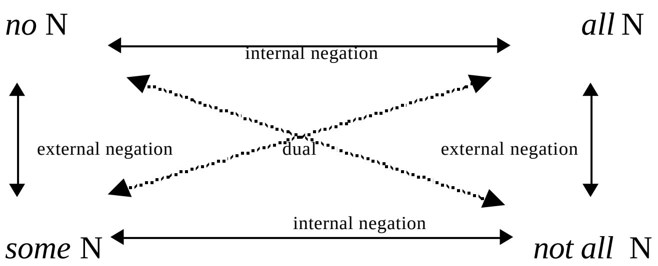
Examples:

(68)a. ⟦*all* N *not*⟧
  = ⟦*all* N ¬⟧,
  = {P | (A ¾ P)  ⟦*all* N⟧}
  = {P| (A ¾ P)  {P| ⟦N⟧  P]},
  = {P| ⟦N⟧  (A¾P)},
  = {P| ⟦N⟧ ∩ P = Ø}
  (= ⟦*no* N⟧)

*b.* ⟦*some* N *not*⟧
  = {P| (A ¾ P)  {P| ⟦N⟧ ∩ P ≠ Ø}},
  = {P| ⟦N⟧ ∩ (A¾P) ≠ Ø},
  = = {P| ⟦N⟧  P)
  (= ⟦*not all* N⟧)

The basic quantifiers *some*, *all*, *no* and *not all* can be arranged in the following way:

(69)




Internal and external negation reverse monotonicity:

· If Q is upward monotone, then ¬Q, Q¬ are downward monotone.

· If Q is downward monotic, then ¬Q, Q¬ are upward monotone.

Let us consider a proof of the first clause. The following proof is of the following structure: We try to prove the negation of what we actually want to prove, and it turns out that this leads to a contradiction.

(70)a. Assume that Q is u.m.,
Take arbitrary P, P′ such that P ¬Q and P′ P.
P ¬Q means: P Q.
Assume to the contrary that P′ Q, i.e. P′ ¬Q, then P Q, because Q is u.m..
That’s a contradiction, hence P′ Q, i.e. P′ ¬Q.
Hence ¬Q is d.m.

b).Assume Q is u.m.,
take arbitrary P, P′ such that P Q¬, P′ P.
That is, (A¾P) Q,
Since P′ P, it holds that (A¾P) (A¾P′),
hence (A¾P′) Q, as Q is u.m.
hence P′ Q¬.
Hence Q¬ is d.m.

There is an interesting observation due to Barwise & Cooper (1981) with regard to monotonicity and negation. They observe that if a language has a syntactic construction whose semantic interpretation is to negate a quantifier, then this construction is unacceptable with monotone decreasing quantifiers. Examples[1]:

(71) *not all students*, *not many students*, but **not no student*, **not few students*

Another important notion that should be mentioned here is the **dual** of a quantifier:

(72)The dual of Q, is defined as: {P| (A¾P) Q}

Note that the dual of a quantifier Q is ¬Q¬. Examples:

(73)a. *all N* and *some N* are duals of each other;
b. *no N* and *not all N* are duals of each other;
c. *John* is the dual of itself.

## 6.6. Monotonicity and Negative Polarity Items

We have seen that we can classify determiners according to their monotonicity properties. It turns out that monotonicity plays an important role for a certain class of expressions in English and in many, perhaps all, other languages, namely, so-called **negative polarity items** (NPIs).

### *6.6.1. Negative Polarity Items*

NPIs were first identified as expressions that have to occur in a “negative” context, like in the scope of a negation (hence their name). Frequent polarity item in English is the determiner *any* and NPs formed with it, like *any student*, *anybody*, *anything*, as in the following examples:

(74)a. Mary didn’t talk to any students.
*Mary talked to any students.

[1] But notice that focussing negation is possible, e.g. *not FEW, but MANY students*. Also, languages may have idiomatic constructions that seem to violate this generalization, for example German *nicht wenige Studenten*, literally ‘not few students’, meaning ‘quite a few students’. But idiomatic constructions are not interpreted compositionally, and hence this does not constitute a counterexample to the generalization by Barwise and Cooper.

b. John didn't say anything.
*John said anything.

We see that *any*-NPs cannot occur in the non-negated sentences. In these cases they have to be replaced by *some*-NPs. In turn, *some*-NPs in negated sentences typically have a different interpretation than *any*-NPs (the wide-scope interpretation):

(75)a. *Mary talked to some student.*
'There is a student such that Mary talked to him.'

b. *Mary didn't talk to any student.*
'It is not the case that there is a student such that Mary talked to him.'

c. *Mary didn't talk to some student.*
'There is a student such that Mary didn't talk to him.'

The last sentence is preferably interpreted as: There is some student that Mary did not talk to. In this interpretation the indefinite NP *some student* is not in the scope of the negation. — Another example of a negative polarity item is *ever.* The corresponding positive forms for *ever* are expressions like *once* or *at some time*

(76)a. *John didn't ever go to graduate school.*
**John ever went to graduate school.*
b. *John once / at some time went to graduate school.*

There are many more NPIs in English. Many of them are idiomatic, such as *lift a finger* or *bat an eye* In their idiomatic meaning they show a similar distribution as *any* and *ever*:

(77)a. *John didn't lift a finger to help Mary.*
**John lifted a finger to help Mary.* (o.k. in the literal interpretation).
b. *Mary didn't bat an eye when she heard about the bad news.*
**Mary bat an eye when she heard about the bad news.* (o.k. in the literal interpretation).

Early accounts for NPIs (E. Klima, 1964) have suggested that their distribution can be explained by a transformation triggered by negation. For example, the negation of the sentence *Mary talked to some student* was supposed to trigger a change from *some* to *any*. However, this does not explain the distribution of idiomatic NPIs like *lift a finger*. And it does not explain instances in which we find NPIs in other contexts than negation, a subject we turn to next.

### *6.6.2. NPI's in Downward-Entailing Contexts*

Consider the following examples of quantified sentences:

(78)a. **Every student lifted a finger / has ever been to China.*
b. **Some*/**Most*/**Many*/**More than three students have ever been to China.*
c. *No*/*Few*/*Less than three students have ever been to China*

The quantified NPs *no student, few students* and *less than three* students have in common that they are downward-entailing (monotone decreasing) in their VP argument, in contrast to NPs like *every student, some students, most students, many students* etc.

(79)a. *Every student who has ever been to China enjoyed it.*
b. **Some/*Many/*More than three students who have ever been to China enjoyed it.*
c. *No/Few/Less than three students who have ever been to China enjoyed it.*

The determiners *every, no, few, less than three* have in common that they are downward entailing (monotone decreasing) in their N argument, in contrast to determiners like *some*, *many, more than three.*

Notice that the scope of negation is downward entailing as well, as illustrated with the following example:

(80) *John did not go to university* *John did not go to graduate school*

So when we assume that NPIs occur **only** in downward-entailing contexts, we have a common explanation for their distribution in negated and quantified sentences. This was actually suggested by Bill Ladusaw (1979), in a UT dissertation.

There are other instances of downward-entailing contexts that support Ladusaw's generalization. For example, the scope of *before* is downward-entailing:

(81) *Mary spoke Chinese before she had been to China*
*Mary spoke Chinese before she had been to Bejing*

We find NPIs in this context:

(82) *Mary spoke Chinese before she had ever been to China.*
*Mary spoke Chinese before she had been to any Chinese city.*

Contrast this with *after.* This expression does not create a downward-entailing context, and it does not house NPIs:

(83)a. *Mary spoke Chinese after she had been to China*
/ *Mary spoke Chinese after she had been to Bejing.*

b. **Mary spoke Chinese after she had ever been to China.*
**Mary spoke Chinese after she had been to any Chinese city.*

It seems that Ladusaw's Generalization holds up in these cases as well.

### *6.6.3. Why are NPI's restricted to Downward-Entailing Contexts?*

But we should ask now: Why is Ladusaw's Generalization true? Why are NPIs restricted to downward-entailing contexts?

One answer that has been suggested is the following: The meaning of an NPI invokes a set of **alternative meanings**. It is not always easy to describe these alternative meanings, but the following examples should at least give a rough idea:

(84)a. *any student*: Alternative meanings are John, Mary, graduate students, foreign students, and other students or subsets of students.
b. *ever*: Alternatives are 1985, 1986, September 1993, and other times.
c. *lift a finger*: Alternatives are carry the suitcase, cleaning the dishes, and other acts of labor.
d. *bat an eye*: Alternatives are cry for help, start sweating, faint, and other types of reactions to negative stimuli.

Among these meanings, the NPI itself denotes an extreme value. For example, the meaning of *any student* is related to the set of all students, the meaning of *ever* is related to the set of all relevant times, *lift a finger* denotes the act of labor that involves the absolutely minimal effort, *bat an eye* denotes the absolute minimal reaction to a negative stimulus, and so on. (This has been observed already in 1975 by Gilles Fauconnier).

Now observe the semantic relation between an expression with an NPI and a similar expression in which the NPI was replaced by some other expression that denotes an alternative to the NPI meaning. For the sake of this argument we have to pretend that the sentences in which the NPI occurs in an upward entailing environment are actually allright.

(85)a. *Mary did not talk to any student* / / *Mary did not talk to a graduate student.*
*Mary talked to any student* (= *a student*) / / *Mary talked to a graduate student.*

b. *John did not lift a finger.* / / *John did not do the dishes.*
*John lifted a finger* (= *did at least something minimal*) / / *John did the dishes.*

We find that in the good cases, the sentence with the NPI expresses something that is semantically stronger than the alternative sentences (it entails the alternative sentences). In the bad cases, the sentence with the NPI expresses something weaker than alternative sentences (it is entailed by the alternative sentences). Hence we can suggest the following reason for the distribution of negative polarity items:

(86)a. Sentences with NPIs have the function to express "strong" statements.
They work under the assumption that the sentence with the NPI is semantically stronger than all the alternatives.

b. This relation between the strength of the sentence with the NPI and the strength of the alternative sentences holds only if the NPI occurs in a downward-entailing context.

For the specific class of NPIs that are formed with *any* there is another plausible motivation (one that was suggested by Victor Sanchez Valencia in a 1991 dissertation). Such NPIs indicate the type of inferences that can be drawn. We can observe the following:

(87)Every declarative sentence Φ[...[*any* α]..] that contains an NP [*any* α],
and for every noun phrase β that is semantically stronger than α (that is, ⟦β⟧ ⟦α⟧)
it holds that Φ[...[*any* α]..] Φ[.. [*any* β]...]

For example,

(88)a. *Mary did not talk to any student* *Mary did not talk to any graduate student.*
b. *Every tourist who has visited any Asian country was impressed*
*Every tourist who has visited any East Asian country was impressed.*

In contrast, NPs like *some* α or *an* α occur in contexts where the opposite inference pattern holds:

(89)a. *Mary talked to a graduate student* *Mary talked to a student.*
b. *A tourist who has visited an East Asian country was impressed.*
*A tourist who has visited an Asian country was impressed.*

In this sense, the near-complementary distribuition of *any*-phrases and *a*/*some*-phrases indicates the type of inference that we can draw. In a case like *Mary didn't talk to some student* we know that we have to switch to the wide-scope interpretation of *some student*, because only then the inference pattern that is typical for *some student* holds. The presence of negative polarity items allows us to draw such inferences in a rather automatic and "cheap" way. That is, natural languages have a device to facilitate such semantic inferences.