Masses and Countables: Cognitive and Linguistic Factors

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Syntactic and semantic criteria

Jespersen (1924):

Differentiation Mass Words / Countables by syntactic and morphological criteria,

but also by nature of their denotata:

> There are a great many words which do not call up the idea of some definite thing with a certain shape or precise limits. I call these “mass-words”; they may be either material, in which case they denote some substance in itself independent of form, such as *silver, quicksilver, water, butter, gas, air*, etc., or else immaterial, such as *leisure, music, traffic, success, tact, commonsense*
>
> (p. 198)

Syntactic criteria:

(1)	Number distinction:	*bean / beans*
	Singulare tantum:	*rice / *rices*
	Plurale tantum:	**oat / oats*
(2)	Numerals:	*one bean / three beans*
		**one rice / three rice(s)*
(3)	Measure terms:	**one pound of bean*
		one pound of rice
		one pound of beans
(4)	Determiners:	*a bean* / *Ø beans*
		**a rice* / *Ø rice*
(5)	Quantifiers:	*every bean / all beans / most beans* / *many beans / *much beans*
		**every rice / all rice / most rice / *many rice(s) / much rice*
(6)	Anaphora:	*John ate a bean, and Mary ate one / *some, too.*
		*John ate rice, and Mary ate some / *one, too.*
		*John ate beans, and Mary ate some / *one, too.*

Semantic and cognitive criteria:

Mass nouns / count nous are not assigned randomly to entities

(7) Fluids and substances are mass: *water, milk, gold*

(8) Small objects tend to be mass: *rice – bean(s)*
sand, gravel – pebble(s), stone(s), rock(s)

(9) Entities high on the animacy scale tend to be count in many languages (Smith-Stark 1974), e.g. Manchu: nouns refering to people and 'horse'

Cognitive basis for this distinction:

- Substances: often no definite border, not cohesive, no obvious minimal parts – hence not countable
- Objects: definite border, cohesive, identifyable across time points, minimal parts – hence countable

Substances and objects as categories in cognition of infants: Cf. Spelke 1985 and others.

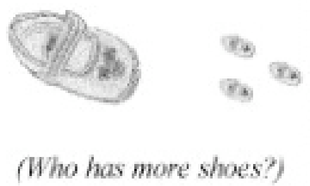
- Infants (3 months) stare at two cars bumping into each other, resulting in a bigger car, they don't stare if two drops of water bump into each other, resulting in a bigger drop.
- Infants stare at an object traversing a solid object, they don't stare at an object traversing a fluid substance. (Rhesus monkeys do the same, cf. Hauser 1996).

Explanation: It is advantageous to generalize the property of one instantiation of a substance to other instantiations, even if they are of different size, form, etc. This does not hold for objects.

To be expected: Cognitive differentiation of substances and objects bootstrap the acquisition of the mass/count-distinction; cf. Macnamara 1982, Soja, Carey & Spelke 1991.

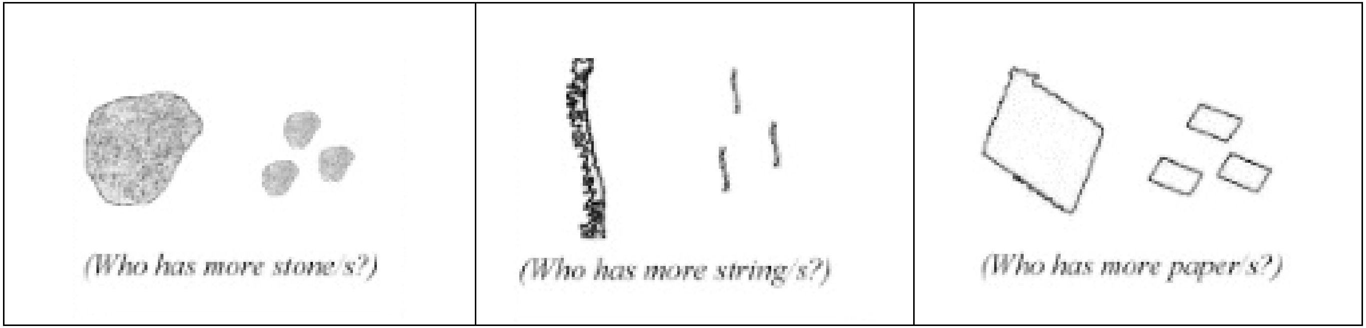
Barner & Snedeker (2005):

Dissociation between cognitive and linguistic criteria:
With comparisons, certain mass nouns behave like count nouns.

count nouns	mass nouns	
	object mass nouns	subject mass nouns
		
number-bassed judgement		quantity-based judgement

Object mass nouns investigated: *silverware, furniture, mail, jewelry, clothing*

The linguistic classification is relevant for comparisons:



In count-noun contexts (*stones, strings, papers*): number-based judgements

In mass-noun contexts (*stone, string, paper*): quantity-based judgements

Question: How to accomodate both the linguistic mass/count distinction
and the cognitive substance/object distinction?

Mereological structures and measure functions

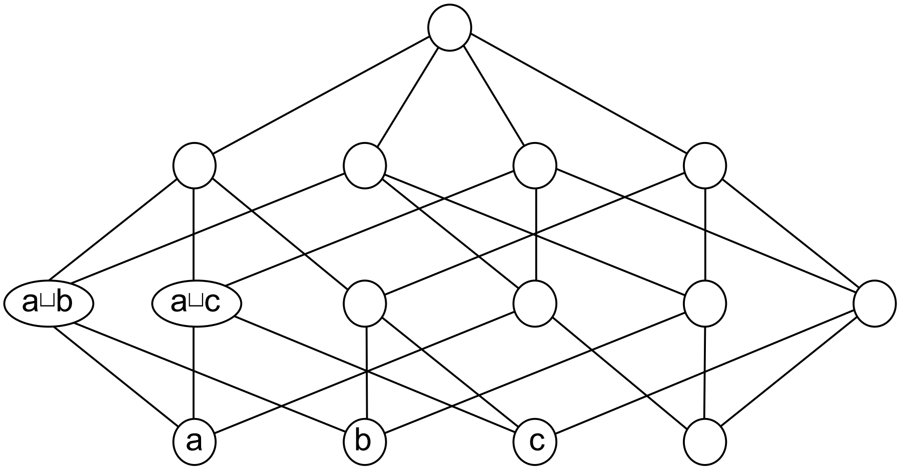
Operations and relations concerning entities:

(10) sum operation $\sqcup$, where $x \sqcup y$ = the sum of x and y;
an idempotent, commutative, associative relation.

part relation $\sqsubseteq$, where $x \sqsubseteq y :\Leftrightarrow x \sqcup y = x$

proper part relation $\sqsubset$, where $x \sqsubset y :\Leftrightarrow x \sqsubseteq y \wedge \neg\, y \sqsubseteq x$

overlap relation $x \text{ o } y :\Leftrightarrow \exists z[z \sqsubseteq x \wedge z \sqsubseteq y]$



Examples:

$a \sqsubseteq a \sqcup b$

$a \sqsubseteq a$

$a \sqsubset a \sqcup b$

$a \sqcup b \text{ o } a \sqcup c$

Cumulativity, Homogeneity, Quantization, Discreteness of sets of individuals P

We generally assume that CARD(P) > 2 (otherwise, classes or predicates coincide)

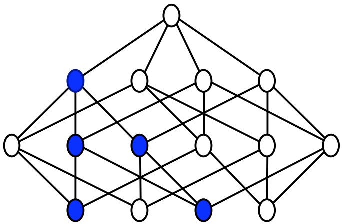
(11) $\text{CUM}(P) :\Leftrightarrow \forall x,y[P(x) \wedge P(y) \rightarrow P(x \sqcup y)]$

$\text{HOM}(P) :\Leftrightarrow \forall x[P(x) \rightarrow \forall y[y \sqsubseteq x \rightarrow P(y)]]$

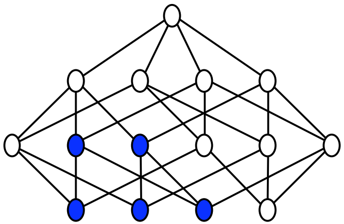
$\text{QUANT}(P) :\Leftrightarrow \forall x,y[P(x) \wedge x \sqsubset y \rightarrow \neg P(y)]$

$\text{DISCR}(P) :\Leftrightarrow \forall x,y[P(x) \wedge P(y) \rightarrow \neg\, x \circ y]$

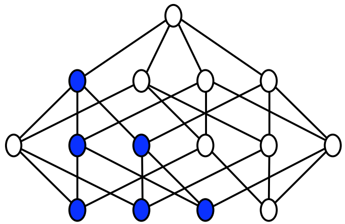
Notice: $\text{DISCR}(P) \rightarrow \text{QUANT}(P)$



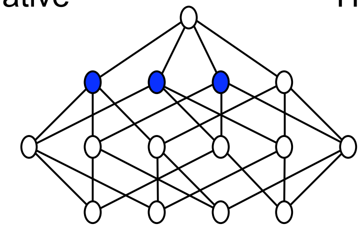
Cumulative



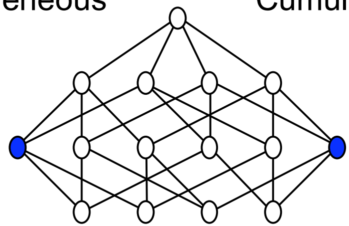
Homogeneous



Cumulative and Homogeneous



Quantized



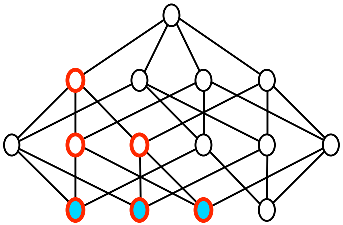
Discreet

Closure of predicates under sum formation, generalized join and maximal individual:

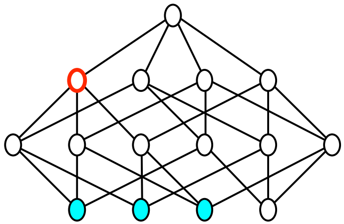
(12) ${}^{+}P$:= the smallest set such that $P \subseteq {}^{+}P \wedge \text{CUM}({}^{+}P)$

$\sqcup P$:= the smallest individual x such that $\forall y[P(y) \rightarrow y \sqsubseteq x]$

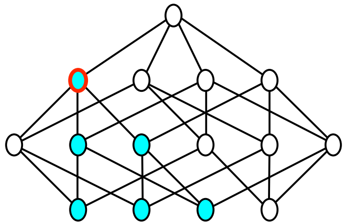
σP := the maximal individual in x, i.e. $\iota x[P(x) \wedge \forall y[P(y) \rightarrow y \sqsubseteq x]$



Closure under sum: ${}^{+}P$



Generalized join $\sqcup P$



maximal individual σP

Atomicity and Divisivity

Absolute atoms and P-atoms:

(13) $\mathrm{ATOM}(x) :\Leftrightarrow$ x is an atom, i.e. $\neg\exists y[y \sqsubset x]$

$\mathrm{ATOM}(P)(x) :\Leftrightarrow P(x) \wedge \neg\exists y[y \sqsubset x \wedge P(y)]$

Notice: $\mathrm{ATOM}(x)$ iff $\mathrm{ATOM}(x, U)$, where U: Universe

Atomic predicates:

(14) $\mathrm{ATOMIC}(P) :\Leftrightarrow \forall x[P(x) \rightarrow \exists P'\forall z[P'(z) \rightarrow \mathrm{AT}(P)(z) \wedge x = \sqcup P']]$

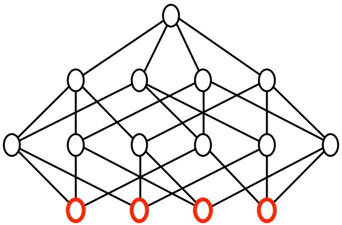
Notice: $\mathrm{ATOM}(P) \rightarrow \neg\mathrm{DIV}(P)$

$\mathrm{QUANT}(P) \rightarrow \mathrm{ATOM}(P)$

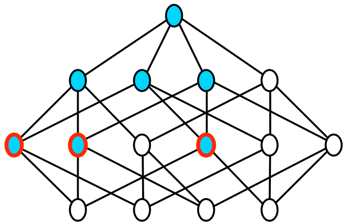
Divisivity:

(15) $\mathrm{DIV}(P) :\Leftrightarrow \forall x[P(x) \rightarrow \exists y[y \sqsubset x] \wedge \forall y[y \sqsubseteq x \rightarrow P(y)]]$

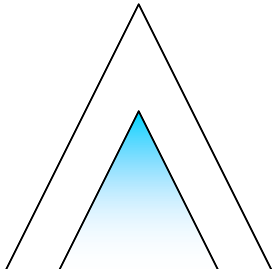
Notice: $\mathrm{DIV}(P) \rightarrow \neg\mathrm{ATOMIC}(P)$; $\mathrm{DIV}(P) \rightarrow \mathrm{HOM}(P)$



$\{x \mid \mathrm{ATOM}(x)\}$

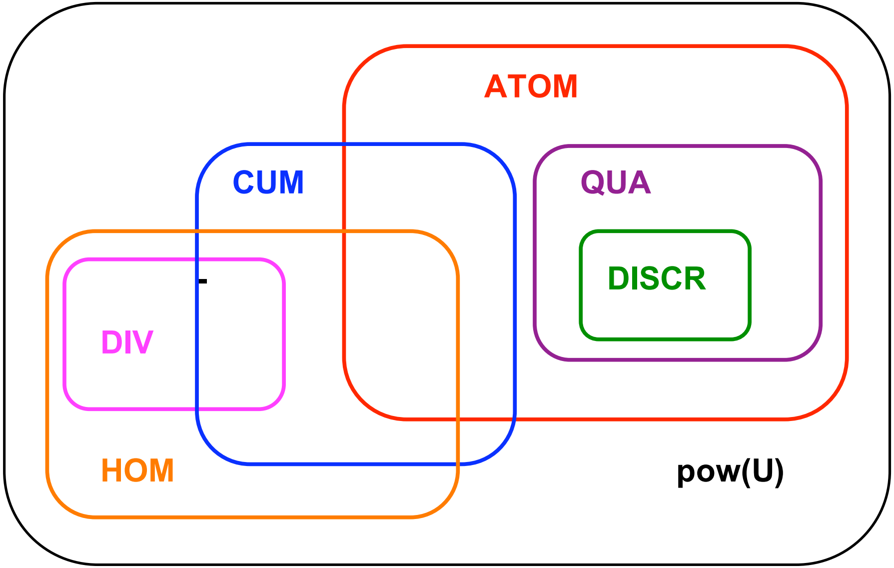


$\{x \mid \mathrm{ATOM}(P)(x)\}$



$\mathrm{CUM}(P) \wedge \mathrm{DIV}(P)$

Relation between predicate types, illustrated by Venn diagram on the powerset of the universe:



Additive measure functions:

(16) μ is an additive measure function with respect to $\sqcup$ iff it maps entities to numbers such that:
$\neg$ x o y $\rightarrow$ [$\mu(x \sqcup y) = \mu(x) + \mu(y)$]

Examples of additive measure functions: *kilograms, meters, cubic meters, calories, Euros, ...*

Examples of non-additive measure functions: *degrees Celsius, carats, IQ, ...*

Use of additive measure functions: Krifka 1989.

Schwarzschild 2002 uses monotonicity instead:

(17) μ is a monotonic measure function with respect to $\sqsubseteq$ iff:
if $x \sqsubset y$, then $\mu(x) < \mu(y)$

With two additional natural properties of the join operation and of measure functions, namely:

(18) Remainder principle: If $y \sqsubset x$, then there is a z, $\neg$ z o y, such that $y \sqcup z = x$

(19) Archimedian property: If $\mu(x) > 0$ and $y \sqsubseteq x$, then $\mu(y) > 0$

we can show:

(20) An additive, Archimedian measure function with respect to a join operation
for which the remainder principle holds is monotonic.

Atomic Count measure function:

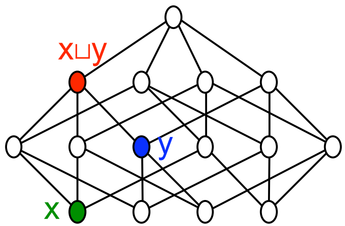
(21) AC is an additive measure function standardized by the following requirement:
AC(x) = 1 iff ATOM(x)

Example:
if ATOM(a) and ATOM(b) and ¬ a o b (i.e. for atoms, a ≠ b), then ATOM(a⊔b) = 2

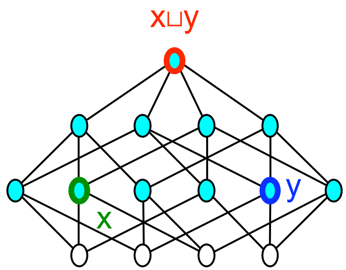
Atomic Count relativized to atomic sets:

(22) If ATOMIC(P), then:
AC(P) is an additive measure function standardized by the following requirement:
AC(P)(x) = 1 iff ATOM(P)(x) = 1

Notice: AC = AC(U), where U: Universe



AC(**x**) = 1, AC(**y**) = 2, ¬**x**o**y**
⇒ AC(**x⊔y**) = 3



AC(P)(**x**) = 1, AC(P)(**y**) = 1, ¬**x**o**y**
⇒ AC(P)(**x⊔y**) = 2

Theory I: Sortal distinction between objects and matter

Link (1983):

- Domain of individuals E
- Domain of portions of matter D
- Function h: E $\rightarrow$ D that maps individuals to the portions of matter they consist of, where h is a homomorphism: $h(x \sqcup y) = h(x) \sqcup h(y)$.

Motivation: Individuals and the portions of matter they consist of may have distinct properties:

(23) *The ring is new, but the gold (that this ring consists of) is not new.*

Proposal:

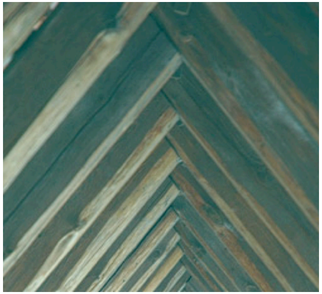
- count nouns and object mass nouns denote individuals, e.g. ⟦*letter*⟧ ⊆ E, ⟦*mail*⟧ ⊆ E
- substance mass nouns denote portions of matter, e.g. ⟦*paper*⟧ ⊆ D

Problems:

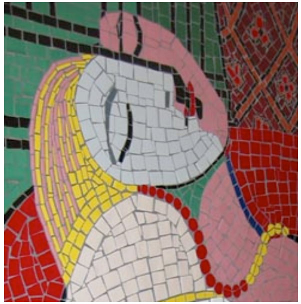
- Not feasible for mass/count noun pairs
  like *rice* / *grains of rice* — *change / coins* — *Gebälk / Balken* 'beams' (German)
  gold / *nuggets* — *drapery / curtains* — *Gewölk / Wolken* 'clouds'
- The matter objects consist of may be grainy:
  The mosaic
  The face consists of carrots, cucumbers, pumpkins, and lemons.

rice / grains of rice

change / coins



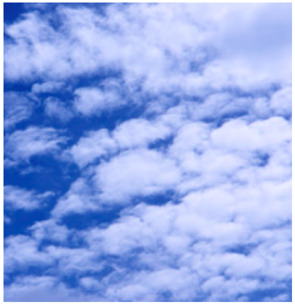
Gebälk / Balken



The face consists of white tiles / tiling.

gold / nuggets

drapery / curtains



Gewölk / Wolke

The face consists of cucumbers, carrots and pumpkins.

Theory II: Number words count atoms

<u>Basic assumptions:</u>

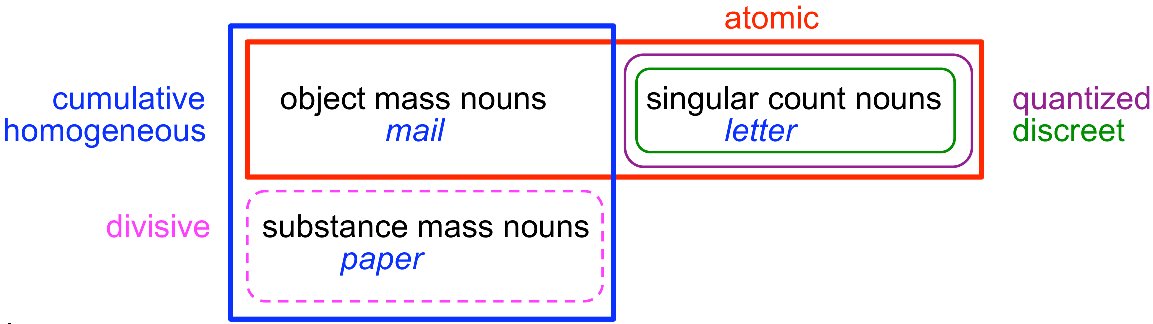
(24) substance and object mass nouns are cumulative: CUM(⟦*paper*⟧), CUM(⟦*mail*⟧)

substance mass nouns are also divisive: DIV(⟦*paper*⟧), hence homogeneous HOM(⟦*paper*⟧)

substance mass nouns are non-atomic, or at least not guaranteed to be atomic

singular count nouns are atomic, ATOM(⟦*letter*⟧),
quantized: QUANT([*letter*⟧), and discrete: DISC(⟦*letter*⟧)

object mass nouns are atomic, ATOM(⟦*mail*⟧)



Notice:

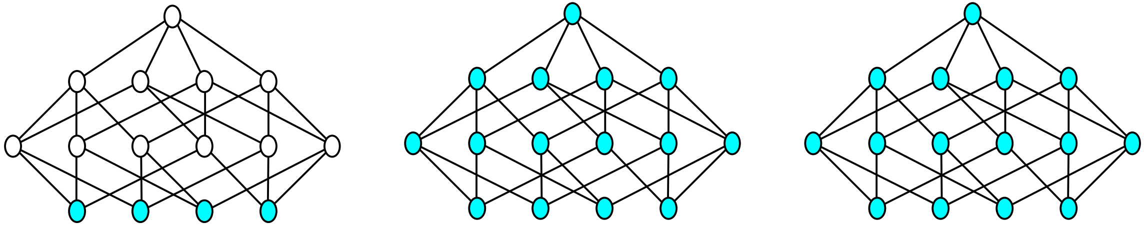
- ➢ singular count nouns and object mass nouns share Atomicity;
  this may be the basis for their similarity in comparison tasks.
- ➢ object mass nouns and substance mass nouns share Cumulativity;
  this may be the basis for their linguistic similarity.

Plural formation

Closure under sum operation:

(25) ⟦*letter-s*⟧ = $^{+}$⟦*letter*⟧

Hence: ⟦*mail*⟧ = ⟦*letter-s*⟧



⟦*letter*⟧ ⟦*letters*⟧ ⟦*mail*⟧

We can motivate: **mail-s*, **paper-s*,
as pluralization is superfluous: $^{+}$⟦*mail*⟧ = ⟦*mail*⟧, $^{+}$⟦*paper*⟧ = ⟦*paper*⟧
(cf. Gillon 1992, Chierchia 1998a)

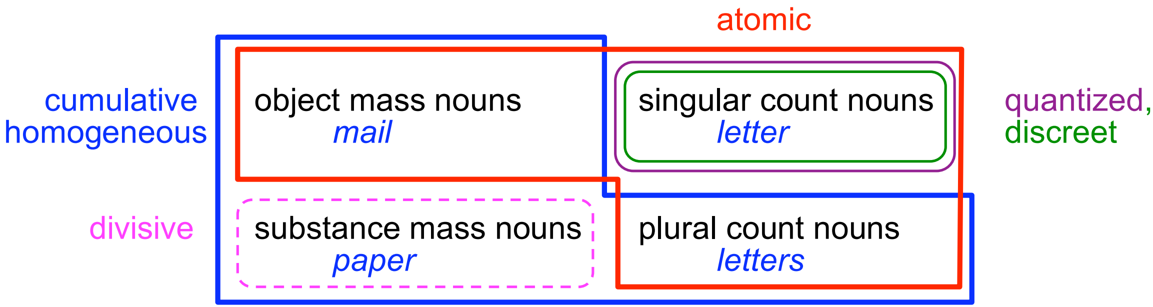
But Tsoulas (2007) argues that Modern Greek generally allows for plurals of mass nouns (not in the taxonomic meaning or portion-of-stuff meaning):

(26) *Epesan nera sto kefali mu*
fell.PL water.PL on head mine

Plural formation results in cumulative atomic predicate: CUM($^{+}$⟦*letter*⟧), ATOMIC($^{+}$⟦*letter*⟧);
this predicts linguistic similarities with mass nouns, e.g. combination with *all*:

(27) *all mail, all paper, all letters, *all letter*

Suggested relations between nominal predicates:



Comparison constructions:

(28) $[\![$NP$_1$ *is more* N *than* NP$_2]\!]$ iff $\mu[\![N]\!]([\![NP_1]\!]) > \mu[\![N]\!]([\![NP_2]\!])$

where $\mu[\![N]\!]$ is a measure function dependent on $[\![N]\!]$:

-- If $[\![N]\!]$ is atomic, then $\mu[\![N]\!] = \textsc{ac}([\![N]\!])$, the atom count based on $[\![N]\!]$, if the atoms are absolute atoms, then $\mu[\![N]\!] = \textsc{ac}$.

-- if $[\![N]\!]$ is not atomic, then $\mu[\![N]\!]$ is a measure function based on weight, volume, or length, depending on the prominent dimension of the entities in $[\![N]\!]$, potentially ambiguous:

In case of *mail, letters*: Predicate is atomic, hence measure function based on (absolute) atoms; in case of *paper*: predicate is not atomic, hence measure function based on weight, or area.

(29) *These are more letters than those.* *This is more mail than that*. — Based on number of atoms, as $[\![$*mail*$]\!]$, $[\![$*letter-s*$]\!]$ is atomic.

$\textsc{ac}(\sqcup[\![\textit{this}]\!]) > \textsc{ac}(\sqcup[\![\textit{that}]\!])$

(30) *This is more paper than that.* — Based on other measure function, $[\![$*paper*$]\!]$ is not atomic

$\mu(\sqcup[\![\textit{this}]\!]) > \mu(\sqcup[\![\textit{that}]\!])$

Possible ambiguity of choice of measure function μ for non-atomic predicates:

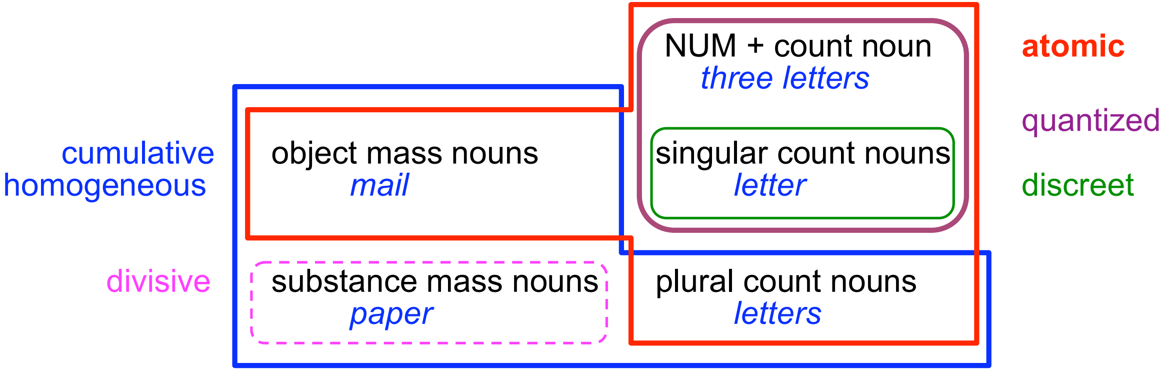
(31) *This is more wool than that.*

May be based on weight, volume, or length.

Number word constructions:

(32) $\llbracket$*three*$\rrbracket$ = $\lambda P \lambda x[P(x) \wedge \mathrm{AC}(x) = 3]$

$\llbracket$*three letters*$\rrbracket$ = $\llbracket$*three*$\rrbracket$($\llbracket$*letter-s*$\rrbracket$) = $\llbracket$*three*$\rrbracket$($^{+}\llbracket$*letter*$\rrbracket$) = $\lambda x[^{+}\llbracket$*letter*$\rrbracket(x) \wedge \mathrm{AC}(x) = 3]$,
a quantized predicate referring to sum individuals of three letters.



We predict:

(33) **three paper(-s)*, as ⟦*paper*⟧ is not atomic, hence AC(⟦*paper*⟧) is not defined.
three letter-s, as hence AC(⟦*letter-s*⟧) is defined.

But we **wrongly** predict, of course:

(34) *three mail*, or *one mail*, as ⟦*mail*⟧ is atomic, hence AC(⟦*mail*⟧) is defined.

Hence **three mail*, **one mail* has to be excluded by non-semantic means,
e.g. by syntactic features:

(35) *letter(-s)* [+count], *mail* [–count], *paper* [–count]
NP $\rightarrow$ NUM N [+count]

This is unsatisfying:
The difference between *letter-s* and *mail* reduces to a purely formal difference,
reminiscent of gender differences:

(36) *ein schöne-s Buch*
a nice-NEUTR book.NEUTR

**ein schöne-r Buch*
a nice-MASC book.NEUTR

ein schöne-r Brief
a nice-MASC letter.MASC

**ein schöne-s Brief*
a nice-NEUTR letter.MASC

But mass/count otherwise does not appear to be a gender-like feature.

Theory III: Number words presuppose atomicity

Perhaps we should assume:

- Denotation of plural count nouns is guaranteed to be atomic, by their linguistic construction:

  $\llbracket$*letter*$\rrbracket$ is atomic, applies to atomic entities,

  $\llbracket$*letter-s*$\rrbracket$ $=$ $^{+}\llbracket$*letter*$\rrbracket$ is atomic, as it is the closure under sum formation.

- We assume that $\llbracket$*mail*$\rrbracket$ is not guaranteed to be atomic.

Denotation of number words:

(37) $\llbracket$*three*$\rrbracket$ $= \lambda P.\textsc{atomic}(P)\ \lambda x[P(x) \wedge \textsc{ac}(x) = 3]$ (atomicity presupposition)

Reason for presupposing atomicity: Atomicity guarantees countability (applicability of AC).

Example derivations:

(38) $\llbracket$*three letter-s*$\rrbracket$ $=$ $\llbracket$*three*$\rrbracket$($\llbracket$*letter-s*$\rrbracket$)

$= \lambda P.\textsc{atomic}(P)\ \lambda x[P(x) \wedge \textsc{ac}(x) = 3](^{+}\llbracket$*letter*$\rrbracket)$ (atomicity presupposition satisfied)

$= \lambda x[^{+}\llbracket$*letter*$\rrbracket(x)\ \ \textsc{ac}(x) = 3]$

(39) $\llbracket$*three mail(-s)*$\rrbracket$ $=$ $\llbracket$*three*$\rrbracket$($\llbracket$*mail*$\rrbracket$)

$= \lambda P.\textsc{atomic}(P)\ \lambda x[P(x) \wedge \textsc{ac}(x) = 3](\llbracket$*mail*$\rrbracket)$ (atomicity **not** guaranteed)

Problem: Even if atomicity is not guaranteed by a linguistic process like pluralization, atomicity could be accomodated easily, which is typically for presupposition triggers.

Theory IV: Plurals do not include atoms

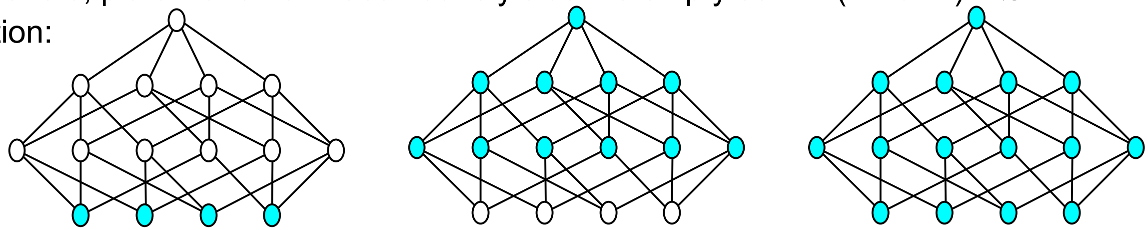
Chierchia (1998a, b) proposes a structual criterion that distinguishes between plural count nouns and mass nouns, including object mass nouns:

- ➢ The denotation of (objectt) mass nouns contains non-overlapping atoms
- ➢ The denotation of plural count nouns is derived from the denotation of singular count nouns, but does not include them, and hence does not have non-overlapping (discrete) atoms

(40) □*letter-s*□ = PL(□*letter*□) = $\lambda x[\neg$□*letter*□$(x) \wedge \forall y[y \sqsubseteq x \wedge \mathrm{ATOM}(y) \rightarrow$ □*letter*□$(x)]$

With this rule, pluralization of mass nouns yields the empty set: PL(□*mail*□) = $\varnothing$

Illustration:



□*letter*□ □*letters*□ □*mail*□

Number words can be applied only to sets of atoms or sets created by pluralization.

Problem: exluding atoms from plurals is problematic (cf. Krifka 1989, Sauerland e.a. 2005):

(41) A: *Do you have children?*
B: *Yes, one. / *No, (just) one.*

A: *Do you have more than one child?*
B: **Yes, one. / No, (just) one.*

(42) *Basket A contains more letters than that Basket B.*
(Assume that A contains two letters and basket B one letter).

Theory V: Measured Atomicity

Rothstein (2007) proposes that count nouns are derived from (object) mass nouns.

- N_{root}, Root Nouns: Sets of minimal elements in N closed under sum formation
- N_{count} derived from N_{count} by a semantic operation M-ATOM, deriving a predicate based on "measured atoms"

Definition of M-ATOM:

(43) a M-ATOM(P) = $\lambda x\,[P(x) \wedge \mu(x) = 1]$, where μ is a suitable additive measure function

b. $\mu(x) = 1 \wedge \mu(y) = 1 \wedge x \neq y \;\rightarrow\; \neg\, x \circ y$, i.e. entities in the domain of μ are discrete

Difference between object mass nouns and singular count nouns:

(44) a ⟦*mail*⟧ is an atomic, cumulative predicate

b ⟦*letter*⟧ = $\lambda x[$⟦*mail*⟧$(x) \wedge \mu(x) = 1]$, i.e. ⟦*mail*⟧ restricted to atomic, non-overlapping entites.

c. ⟦*letter-s*⟧ = $^{+}$⟦*letter*⟧

Number words like *three* somehow "recognize" whether predicate atoms are measured.

Problem: It is not spelled out how this is done, and it is not obvious how to do it compositionally:

E.g., what is the difference between ⟦*letter*$_{root}$⟧ = LETTER and ⟦*letter*$_{count}$⟧ = $\lambda x[\text{LETTER}(x) \wedge \mu(x)=1]$, as both are predicates that are atomic and discreet?

Theory VI: Number words trigger closure

Basic assumptions as before,
but now we assume that the number word construction expresses closure under sum formation:

(45) a $[\![\textit{three}]\!] = \lambda P.\textsc{quant}(P)\ \lambda x[^{+}P(x) \wedge \textsc{at}(x) = 3]$ (presupposition of quantization)

b $[\![\textit{three letter-s}]\!] = [\![\textit{three}]\!]([\![\textit{letter}]\!]) =$
$= \lambda P.\textsc{quant}(P)\ \lambda x[^{+}P(x) \wedge \textsc{at}(x) = 3]([\![\textit{letter}]\!])$ (quantization presupposition satisfied)
$= \lambda x[^{+}[\![\textit{letter}]\!](x) \wedge \textsc{at}(x) = 3]$

c $[\![\textit{three mail}]\!] = [\![\textit{three}]\!]([\![\textit{mail}]\!]) =$
$= \lambda P.\textsc{quant}(P)\ \lambda x[^{+}P(x) \wedge \textsc{at}(x) = 3]([\![\textit{letter}]\!])$ (quantization presupp. not satisfied)

Why *_three letter_? – Violation of number agreement:

(46) _one_ [– PL], _three_ [+ PL], _letter_ [– PL], _letters_ [+ PL]
NP $\rightarrow$ NUM [α PL] N [α PL]

Notice that we need number agreement anyway: Theory I/II did not exclude *_one letters_,
and we do not want to exclude atoms from the extension of _letters_ (cf. above):

(47) A: _Do you have children?_ B: _Yes,one._ / *_No, just one_.

What about bare plurals, e.g. _letters_? Semantic plural formation, not agreement (cf. Krifka 1989)

(48) $[\![[_{\text{NP}}\ \textit{letter-s}]]\!] = {}^{+}[\![\textit{letter}]\!]$

Difference between agreement plural, semantic plural can be found e.g. in Turkish:

(49) _çokuk_ ‘child’, _çokuk-lar_ ‘children’, _dört çokuk_ ‘five children’, *_dört çokuk-lar_

Quantity comparison as before based on atoms if predicates are atomic,
hence: atom-based comparisons with _letters_ and _mail_, measure-based comparison with _paper._

Within this theory, we now have a look at:

Measure constructions

(50) *three* {*kilograms / baskets*} *of paper / mail / letters / *letter*

Assume: *kilogram, basket* etc. denote additive measure functions,

plural forms in *three kilograms / baskets*: agreement plural

Different in German:
No plural agreement with measure terms, but semantic plural in non-measure term use:

(51) *drei Pfund / *Pfunde Papier*
three pound / pound.PL paper

Peter hat viele Pfunde verloren.
Peter has many pound.PL lost

Assume: Measure phrases must apply to a cumulative predicate,
as measure constructions should single out entities of a particular size.

This is possible if the type of measure function is additive (cf. Krifka 1989),
or monotonic, a consequence of additivity (cf. Schwarzschild 2002).

(52) $[\![\textit{three kilograms}]\!] = \lambda P.\textsc{cum}(P)\ \lambda x[P(x) \wedge \textsc{kg}(x) = 3]$

$[\![\textit{three kilograms of letters}]\!]$
$= \lambda P.\textsc{cum}(P)\ \lambda x[P(x) \wedge \textsc{kg}(x) = 3]({}^{+}[\![\textit{letter}]\!])$ Cumulativity satisfied
$= \lambda x[{}^{+}[\![\textit{letter}]\!](x) \wedge \textsc{kg}(x) = 3]$

$[\![\textit{three kilograms of letter}]\!]$
$= \lambda P.\textsc{cum}(P)\ \lambda x[P(x) \wedge \textsc{kg}(x) = 3]({}^{+}[\![\textit{letter}]\!])$ Cumulativity not satisfied

But then: Why no cumulativity requirement, but quantization requirement for number words?

<u>Answer: Measure constructions provide the requirements for number words</u>

Alternative parsing for measure constructions (cf. Borer 2005):

(53) $[\![\textit{kilogram}]\!] = \lambda P.\textsc{cum}(P)\ \lambda x\ [P(x) \wedge \textsc{kg}(x) = 1]$

$[\![\textit{kilogram of letters}]\!] = [\![\textit{kilograms}]\!](^{+}[\![\textit{letter}]\!])$
$= \lambda x\ [^{+}[\![\textit{letter}]\!](x) \wedge \textsc{kg}(x) = 1]$ (cumulativity is satisfied)

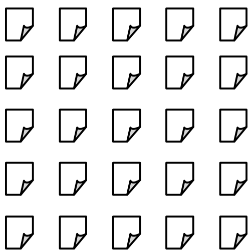
Alternative meaning of number words:

(54) $[\![\textit{three}]\!] = \lambda P.\textsc{quant}(P)\ \lambda x[^{+}P(x) \wedge \textsc{ac}(P)(x) = 3]$

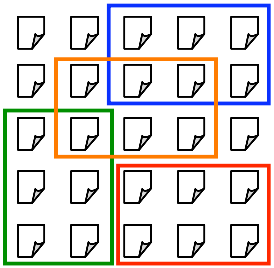
Notice: The number words measures P-atoms; measuring of atoms is just a subcase.

The number word is applied to the modified nominal predicate:

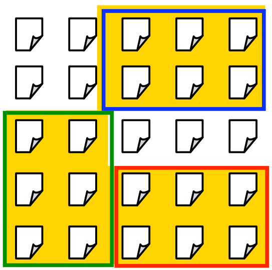
(55) $[\![\textit{three}\ [\textit{kilograms of letters}]]\!] = [\![\textit{three}]\!]([\![\textit{kilograms of letters}]\!])$
$= \lambda P.\textsc{quant}(P)\ \lambda x[^{+}P(x) \wedge \textsc{ac}(P)(x) = 3]([\![\textit{kilograms of letters}]\!])$ (quantization is satisfied)
$= \lambda x[^{+}[\![\textit{kilograms of letters}]\!](x) \wedge \textsc{ac}([\![\textit{kilogram of letters}]\!])(x) = 3]$



⟦*letter*⟧



⟦*kilogram of letters*⟧
(overlapping quantities)



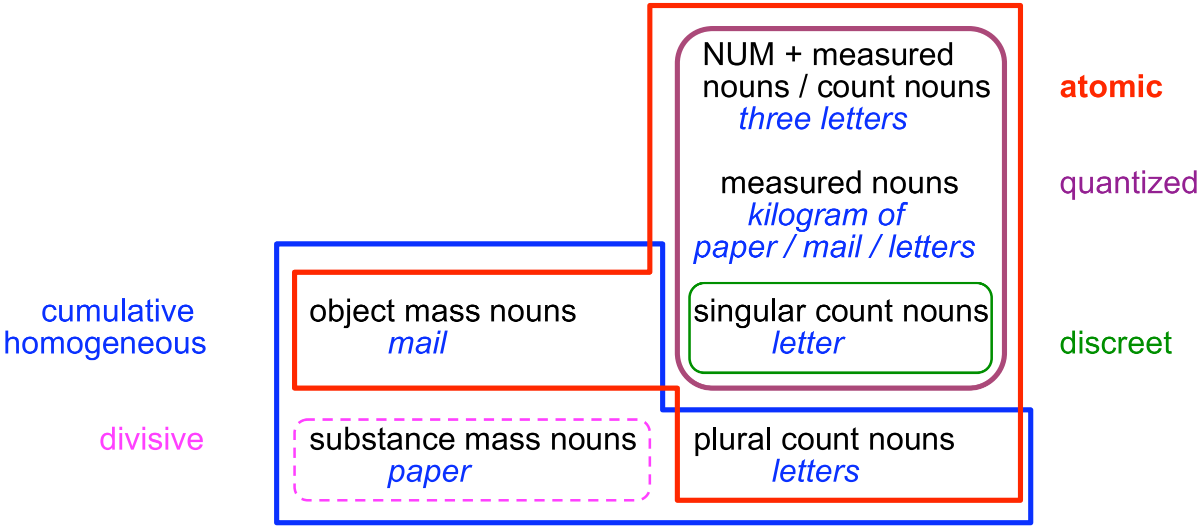
⟦*three kilograms of letters*⟧
(overlapping quantities; one illustrated)

The emerging picture:

- Measure terms, like *kilogram* require a cumulative predicate, create a quantized predicate.
- Number word, like *three*, require a quantized predicate, create a quantized predicate.
- Count nouns, like *letter*
  don't need a measure term, as it is already quantized in its lexical meaning;
  as measure terms require cumulative predicates, they cannot be applied to count nouns.
- Object mass nouns, like *mail*, and substance mass nouns, like *paper*,
  need measure terms, as they are not quantized in their lexical meaning

The essential feature is quantization, not atomicity,
as object mass nouns are atomic, yet cannot be combined with number words.

Measured nouns like *kilogram of paper / mail / letters* are similar to singular count nouns, except that they typically are not discreet (kilograms can overlap):



Theory VII: Number and measure create countability

Borer (2005): Grammatical number, classifiers and measure terms create the conditions for number words to apply.

One way to implement this idea in semantics:

Measure construction are treated as in Theory VI:

(56) ⟦*kilogram*⟧ = $\lambda P.\text{CUM}(P)\ \lambda x\,[P(x) \wedge \text{KG}(x) = 1]$

⟦*kilogram of honey*⟧ = $\lambda x[$⟦*honey*⟧$(x) \wedge \text{KG}(x) = 1]$

⟦*three*⟧ = $\lambda P.\text{QUANT}(P)\ \lambda x[P(x) \wedge \text{AC}(P)(x) = 3]$

⟦*three* [*kilograms of honey*]⟧
= $\lambda x[$⟦*kilogram of honey*⟧$(x) \wedge \text{AC}($⟦*kilogram of honey*⟧$)(x) = 3]$

Classifier constructions: Reference to a Natural Unit, specific for a type

Typical for classifier languages like Chinese; here an English example.

(57) ⟦*head*⟧ = $\lambda P.\text{CUM}(P)$, P:headed animal $\lambda x\,[P(x) \wedge \text{NU}(P)(x) = 1]$

⟦*head of cattle*⟧ = $\lambda x\,[$⟦*cattle*⟧$(x) \wedge \text{NU}($⟦*cattle*⟧$)(x) = 1]$

⟦*three* [*head of cattle*]⟧ = $\lambda x[$⟦*head of cattle*⟧$(x) \wedge\ \text{AC}($⟦*head of cattle*⟧$)(x) = 3$

Number constructions: Reference to a Natural Unit present in Pluralization / Number head

(58) ⟦*apple*⟧ = $\lambda x[x \text{ is apple}]$

⟦*apple(-s)*$_{\text{CN}}$⟧ = $\lambda x\,[$⟦*apple*⟧$(x) \wedge \text{NU}($⟦*apple*⟧$)(x) = 1]$

⟦*three apples*⟧ = $\lambda x[$⟦*apple(-s)*$_{\text{CN}}$⟧$(x) \wedge \text{AC}($⟦*apple(-s)*$_{\text{CN}}$⟧$)(x) = 3]$

Theory VIII: Numbers as arguments

<u>A problem with previous theories: Non-integer number words</u>

(59) a *three and a half kilograms of apples*
b *three and a half apples*

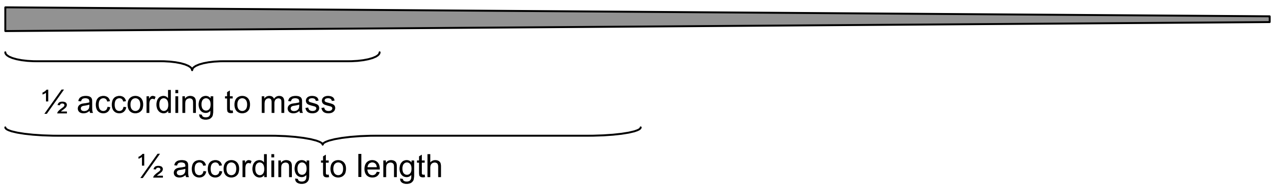
(60) ⟦*three and a half* [*kilograms of apples*]⟧
$\lambda P.\text{QUANT}(P)\ \lambda x[{}^{+}P(x) \wedge \underline{\text{AC}(P)(x) = 3.5}]$(⟦*kilograms of apples*⟧)
ill-formed, as AC only has integer values

Can AC(P) be extended to non-integer values?

(61) *three and a half kilograms of wire*:
$\lambda x[{}^{+}$⟦*kilogram of wire*⟧$(x) \wedge \exists y,z[x = y \sqcup z \wedge \neg y \circ z \wedge \text{AC}($⟦*kilogram of wire*⟧$)(y) = 3$
$\wedge$ z is ½ of an individual u such that $\text{AC}($⟦*kilogram of wire*⟧$)(u) = 1]]$

What does "½ of an individual" mean? Depends on how we measure!

Take wire that weighs one kilogramm, and is thicker on one end:



In (61) mass counts, not length, as we use the measure term *kilogram.*
But the ½-operation does not have access to the measure term!

An alternative approach (Krifka 1989, 1995, 2004):
Number words are arguments for measure functions.

Measure constructions

(62) $[\![$*kilograms*$]\!]$ = $\lambda P.\textsc{cum}(P)\ \lambda\mathbf{n}\ \lambda x\ [P(x) \wedge \textsc{kg}(x) = \mathbf{n}]$ — notice: number argument n

$[\![$*kilograms of paper*$]\!]$ = $[\![$*kilograms*$]\!]$($[\![$*paper*$]\!]$)
= $\lambda P.\textsc{cum}(P)\ \lambda n\ \lambda x\ [P(x) \wedge \textsc{kg}(x) = n]$($[\![$*paper*$]\!]$) — cumulativity presupp. satisfied
= $\lambda n\ \lambda x[$$[\![$*paper*$]\!]$$(x) \wedge \textsc{kg}(x) = n]$

$[\![$*three and a half* [*kilograms of paper*]$]\!]$ — number word specifies number argument of measure term
= $[\![$*kilograms of paper*$]\!]$($[\![$*three and a half*$]\!]$)
= $\lambda n\ \lambda x[$$[\![$*paper*$]\!]$$(x) \wedge \textsc{kg}(x) = n](3.5)$
= $\lambda x[$$[\![$*paper*$]\!]$$(x) \wedge \textsc{kg}(x) = 3.5]$

We assume real-valued measure functions like KG, as usual.

We can also support an alternatrive parsing strategy:

(63) $[\![$*kilograms*$]\!]$ = $\lambda n\ \lambda P.\textsc{cum}(P)\ \lambda x\ [P(x) \wedge \textsc{kg}(x) = n]$

$[\![$*three and a half kilograms*$]\!]$
= $[\![$*kilograms*$]\!]$($[\![$*three and a half*$]\!]$)
$\lambda n\ \lambda P.\textsc{cum}(P)\ \lambda x\ [P(x) \wedge \textsc{kg}(x) = n](3.5)$
= $\lambda P.\textsc{cum}(P)\ \lambda x\ [P(x) \wedge \textsc{kg}(x) = 3.5]$

$[\![$*three and a half kilograms of paper*$]\!]$
= $[\![$*three and a half kilograms*$]\!]$($[\![$*paper*$]\!]$)
= $\lambda P.\textsc{cum}(P)\ \lambda x\ [P(x) \wedge \textsc{kg}(x) = 3.5]$($[\![$*paper*$]\!]$) — cumulativity presupp. satisfied
= $\lambda x[$$[\![$*paper*$]\!]$$(x) \wedge \textsc{kg}(x) = 3]$

Count noun constructions:

Count denote are additive measure functions, just like measure terms:

(64) ⟦*letter*⟧ = λnλx[x is letter ∧ Natural Unit(⟦*letter*⟧)(x) = n]

⟦*three letters*⟧ = ⟦*letters*⟧(⟦*three*⟧) =
= λnλx[x is letter ∧ NU(⟦*letter*⟧)(x) = n](3)
= λx[x is letter ∧ NU(⟦*letter*⟧)(x) = 3]

NU(P) is an additive measure function related to the meaning of P,
it maps an individual x that counts as one P to the number word 1.

NU(P) can be extended to non-integer based functions by determination what counts as a part (a half, a quarter etc.) of an entity of a particular sort, for *three and a half letters*

For singular/plural forms of the noun, we assume agreement (absent, e.g., in Turkish).

For semantic plural (bare plural NPs): binding of number argument.

(65) ⟦*letter-s*⟧ = λx∃n[⟦*letter*⟧(n)(x)] = λx∃n[x is letter ∧ NU(⟦*letter*⟧)(x) = n]

Why **three mail(-s)*? Because *mail* does not come as a measure function, hence semantic plural is not applicable.

Somewhat idealized:

(66) Lexical entry for meaning of *mail*: λx[x is mail], an atomic, cumulative predicate

Lexical entry for meaning of *letter*: $\lambda n \lambda x$[x is mail $\wedge$ NU(mail)(x) = n]

Letter has a 'built-in' criterion of counting that *mail* lacks.

Cf. Quine (1960): To learn a term like *apple*, it is not sufficient to learn "how much of what goes on counts as an apple", we must also "learn how much counts as **an** apple, and how much as another (...) Such terms possess built-in modes (...) of dividing their reference."

Quine's notion of "built-in modes of dividing reference" cannot be atomicity of predicates, as this would not distinguish between count nouns like *letter* and object mass nouns like *mail*.

But this notion can be modelled by measure functions.

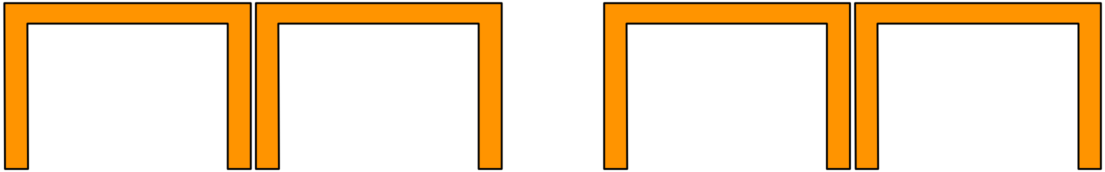
Difference between cound nouns and measure terms:

- Count nouns have a qualitative and a quantitative criterion of application:
  ⟦*apple*⟧ = $\lambda n \lambda x$[x is apple $\wedge$ NU(apple)(x) = n]
- Measure terms only have a quantitative criterion of application:
  ⟦*kilogram*⟧ = $\lambda n \lambda x$[KG(x) = 1],
  conjunctive combination with qualitative noun by type-lifting to $\lambda n \lambda P \lambda x$[P(x) $\wedge$ KG(x) = n]

Context-sensitivity of measure function

Rothstein (2007) points out: Measure function for count nouns is context-sensitive.
Assume there are 4 tables, placed in two groups of two adjacent tables.

(67) a *These are four tables.* (as there are four physical tables)
b *These are two tables.* (as there are two objects that can be used as one table)



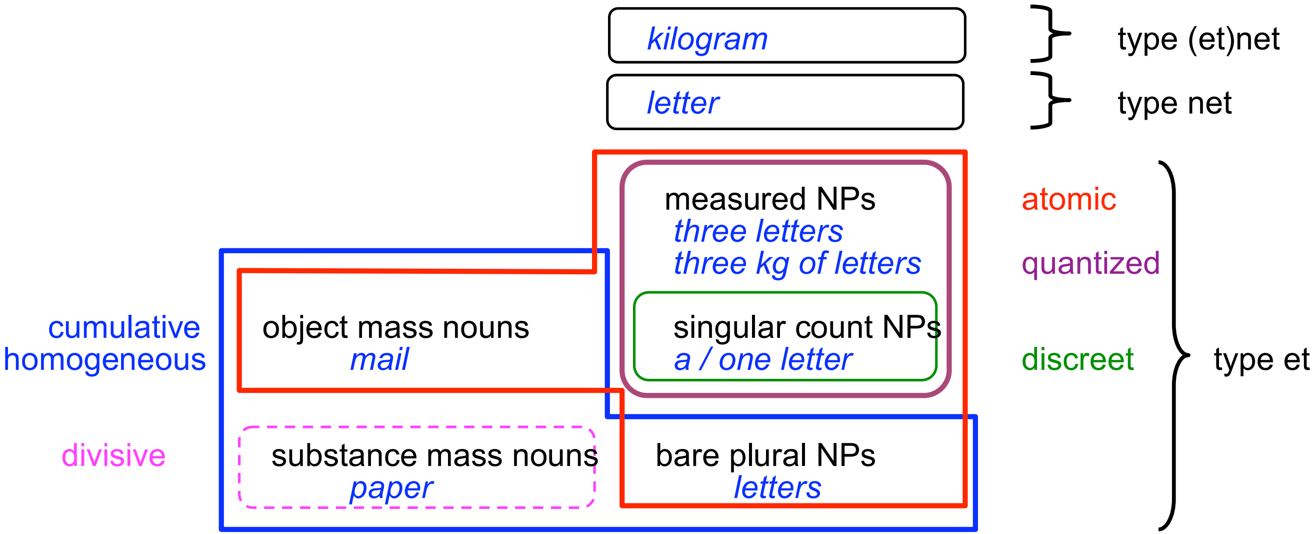
Acknowledging this context-dependence of count nouns explains the apparent non-quantiziation of count nouns like *line, plane* (Mittwoch 1988), *sequence, twig* (Krifka 1992), *fence, wall, hedge, bouquet* (Rothstein 2004), cf. also Zucchi & White (2001).

It can be incorporated into the current theory by assuming that the NU function is context-dependent, for example:

(68) ⟦*table*⟧ = $\lambda x[\text{TABLE}(x) \wedge \text{NU}_{\mathbf{c}}(\text{TABLE})(x) = 1]$

The context c can select for physical objects (4 tables) or for functional objects (2 tables).

Overview of nominal types, according to Theory VIII:



<u>Comparison task:</u>

As above:

(69) $[\![$NP$_1$ *is more* N *than* NP$_2]\!]$ iff $\mu[\![$N$]\!]([\![$NP$_1]\!]) > \mu[\![$N$]\!]([\![$NP$_2]\!])$
where $\mu[\![$N$]\!]$ is a measure function dependent on $[\![$N$]\!]$:
-- If $[\![$N$]\!]$ is atomic, then $\mu[\![$N$]\!] = \textsc{ac}([\![$N$]\!])$, the atom count based on $[\![$N$]\!]$,
if the atoms are absolute atoms, then $\mu[\![$N$]\!] = \textsc{ac}$.
-- if $[\![$N$]\!]$ is not atomic, then $\mu[\![$N$]\!]$ is a measure function based on weight, volume, or length, depending on the prominent dimension of the entities in $[\![$N$]\!]$.

Examples:

(70) *This is more mail than that*.
$[\![$*mail*$]\!]$ is atomic, comparison is preferentially based on atoms.

(71) *These are more letters than that*.
$[\![$*letters*$]\!]$ is atomic, comparison is preferentially based on atoms.

(72) *This is more paper than that.*
As before, $[\![$*paper*$]\!]$ is not atomic, hence a different measure function is used for comparison.

Classifier constructions

An English example; examples like that are dominant in classifier languages.

(73) *fifty head of cattle*

Assume: Classifier selects a measure function depending on the head noun:

(74) $[\![$*head*$]\!] = \lambda n\ \lambda P.\textsc{cum}(P), \textsc{animal}(P)\ \lambda x[P(x) \wedge NU(\textsc{animal})(x) = n]$

$[\![$*fifty head*$]\!] = \lambda P.\textsc{cum}(P), \textsc{animal}(P)\ \lambda x[P(x) \wedge NU(\textsc{animal})(x) = 50]$

$[\![$*fifty head of cattle*$]\!] = [\![$*fifty head*$]\!]([\![$*cattle*$]\!])$
$= \lambda P.\textsc{cum}(P), \textsc{animal}(P)\ \lambda x[P(x) \wedge NU(\textsc{animal})(x) = 50]([\![$*cattle*$]\!])$ (presupp. satisfied)
$= \lambda x[[\![$*cattle*$]\!](x) \wedge NU(\textsc{animal})(x) = 50]$

Lack of Count Construction for Verbal Predicates

Verbal measure construction occur in many languages:

(75) *Mary coughed three times.*

(76) a ⟦*cough*⟧ = $\lambda e[\text{COUGH}(e)]$ an event predicate, atomic

b ⟦*three times*⟧ = $\lambda P.\text{ATOMIC}(P)\ \lambda e[P(e) \wedge \text{AC}(e) = 3]$

Verbal count constructions do not occur in any language:

(77) **Mary coughed three.*

This bears on the analysis of number words:

- If *three* is a modifier in constructions like *three letters*, then (77) is expected to be possible, as in both cases, *three* would be applied to a predicate, cf. *fast runner* and *run fast*
- If *three* is an argument in constructions like *three letters*, then (77) is expected not to be possible, as verbs project thematic role arguments, not number arguments.

Theory VIII-a: Two distinct sum operations

We have to enrich Theory VIII to account for

Gillon's problem (Gillon 1992, Chierchia 1998a, Rothstein 2007):

(78) a *The curtains and the carpets resemble each other*.
a. The curtains resemble the carpets (and vice versa)
b. The curtains resemble each other, and the carpets resemble each other.

b. *The curtaining and the carpeting resemble each other.*
Only a: The curtains resemble the carpets (and vice versa)

Similar case: Reciprocal interpretations.

(79) *The letters are similar (to each other)*
**The mail is similar (to each other)*

Rothstein 2007 suggests:

$[\![$*the curtaining*$]\!] \neq [\![$*the curtains*$]\!]$,
as *curtains* denotes a set based on measured atoms, whereas *curtaining* does not.

Problem: According to standard definitions of definite article as referring to the maximal element, these meanings are the same:

(80) $[\![\textit{the curtaining}]\!] = \sigma\, [\![\textit{curtaining}]\!]$

$[\![\textit{the curtain-s}]\!] = \sigma({}^{+}[\![\textit{curtain-s}]\!])$

$\sigma\, [\![\textit{curtaining}]\!] = \sigma({}^{+}[\![\textit{curtain-s}]\!])$

Even if the atoms of *curtains, curtaining* are different, their sum or maximal elements is the same.

Proposal: We assume another sum operation (cf. Link 1983: individual sum formation):

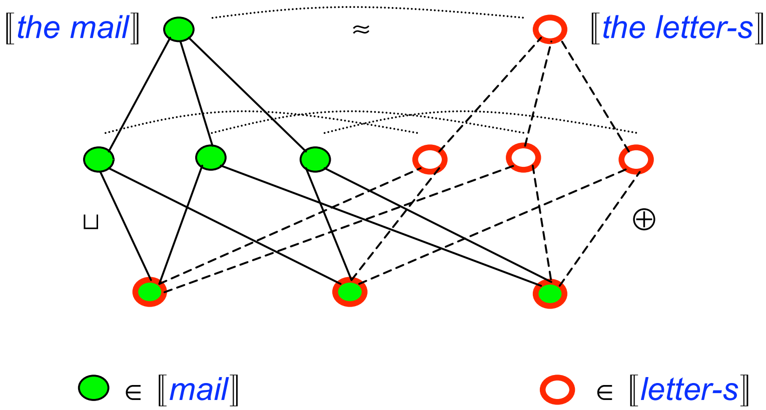
- There are two distinct sum operations, $\sqcup$ and $\oplus$
- $x \sqcup y$ and $x \oplus y$ are distinct, but materially equivalent ($\approx$)
- $\sqsubseteq_{\oplus}$ is atomic, and the atoms are discrete (do not overlap) with respect to $\sqcup$

(81) $[\![$*mail*$]\!]$, $[\![$*paper*$]\!]$: as before, only contains individuals formed by $\sqcup$

(82) $[\![$*letter-s*$]\!]$ = closure of $[\![$*letter*$]\!]$(1) under $\oplus$,
i.e. $^{\oplus}{}_{[}[[\![letter]\!](1)]$ is the smallest predicate such that:

a. $[\![letter]\!](1) \subseteq {}^{\oplus}{}_{[}[[\![letter]\!](1)]$

b. $[\![letter\text{-}s]\!] = {}^{\oplus}{}_{[}[[\![letter]\!](1)](x) \wedge {}^{\oplus}{}_{[}[[\![letter]\!](1)](y) \rightarrow {}^{\oplus}{}_{[}[[\![letter]\!](1)](x \oplus y)$



<u>Reciprocals</u> make use of part relation $\sqsubseteq_{\oplus}$ related to the join operation $\oplus$:

(83) $[\![$*the letters are similar (to each other)*$]\!]$ =

$\forall x,y \in A[x,y \sqsubseteq_{\oplus} \sigma[\![$*letter-s*$]\!] \wedge x \neq y \rightarrow$ x is similar to y]

Prediction:

(84) **The mail is similar (to each other)*,
as $\sqsubseteq_{\oplus}$ is not defined for the sum individuals in $[\![$*mail*$]\!]$

Many other predicates disregard the difference between entities that are materially equivalent:

(85) $\forall x,y[x \approx y \rightarrow [[\![$*be on the table*$]\!](x) \leftrightarrow [\![$*be on the table*$]\!](y)]]$

<u>Distributives</u> also make use of $\sqsubseteq_{\oplus}$:

(86) a *The letters weigh 50 grams each.*
b *The mail weighs 50 grams (*each / a letter)*.

Contrast this with lexically collective predicates like *numbered*:

(87) a *The letters are numbered.* ‘Each letter has a distinct number.’
b *The mail is numbered.* ‘Each atomic part of the mail has a distinct number’.

Non-syntactic distributives do not necessarily make us of $\sqsubseteq_{\oplus}$:

(88) a. *The letters are numbered consecutively.*
b. *The mail is numbered consecutively.*

<u>Pluralization of mass nouns</u>

Tsoulas (2007) points out that mass nouns in Greek pluralize.
One possibility: Plural is formed over discrete, locally disconnected entities falling under the singular mass noun.

Partitive constructions

(89) a *three of the letters* b **three of the mail* c **every of the letter*

Traditional proposal, Jackendoff (1977): *three ~~letters~~ of the letters*

Within Theory IX, a more surface-oriented approach is possible
(with AC the atomic count function for $\oplus$):

(90) $[\![\textit{three}_{D}]\!] = \lambda P.[\text{CUM}(P) \wedge \text{ATOMIC}(P)]\ \lambda x\ [P(x) \wedge \text{AC}(x) = 3]$

$[\![\textit{the letters}]\!] = \sigma([\![\textit{letter-s}]\!])$

$[\![\textit{of}]\!] = \lambda y \lambda x[x \sqsubseteq_{\oplus} y]$

$[\![\textit{of}\ [\textit{the letters}]]\!] = \lambda x[x \sqsubseteq_{\oplus} \sigma([\![\textit{letter-s}]\!])]$

$[\![\textit{three}\ [\textit{of}\ [\textit{the letters}]]]\!] = \lambda x[x \sqsubseteq_{\oplus} \sigma([\![\textit{letter-s}]\!]) \wedge \text{AC}(x) = 3]$

Contrast with:

(91) $[\![\textit{three letters}]\!] = [\![\textit{letter}]\!](3)$

This predicts a difference in acceptability between:

(92) a. $[\![\textit{three and a half apples}]\!] = [\![\textit{apple}]\!](3.5)$

b $[\![\textit{three and a half of the apples}]\!] = \lambda x[x \sqsubseteq_{\oplus} \sigma([\![\textit{apple-s}]\!]) \wedge \underline{\text{AC}(x) = 3.5}]$

AC only has integer values, hence (92.b) is not defined.
But it can be extended to non-integer values by applying a relevant dimension (size or mass).

Measure partitive constructions:

(93) $[\![[\textit{three kilograms}\ [\textit{of the apples}]]]\!]$
$= \lambda P.\text{CUM}(P)\ \lambda x[P(x) \wedge \text{KG}(x) = 3](\lambda x[x \sqsubseteq_{\oplus} \sigma([\![\textit{apple-s}]\!])])$
$= \lambda x[x \sqsubseteq_{\oplus} \sigma([\![\textit{apple-s}]\!]) \wedge \text{KG}(x) = 3]$, as cumulativity is satisfied

Theory VIII-b: Discourse-level sum operations

Hoeksema (1983):

(94) [$_{\text{DP}}$ *Napoleon and* [*Wellington and Blücher*]] *fought each other in the battle of Waterloo.*
‘Napoleon fought Wellington and Blücher, and Wellington and Blücher fought Napoleon.’

[$_{\text{DP}}$ *Napoleon, Wellington and Blücher*] *fought each other in the battle of Waterloo.*
‘Each two of N, W and B fought against each other.’

Link (1984), Landman (1989) propose an join operation yielding groups, different from sum individuals.

Krifka (1990) presents arguments that this operation joins discourse referents:

➢ The joined elements have to be mentioned explicitly, hence introduce discourse referents:

(95)

$$
\begin{array}{|l|}
\hline
d_1\ d_2\ d_3\ D_4\ D_5 \\
\hline
\begin{array}{ll}
\begin{array}{l}
d_1 = \text{Napoleon} \\
d_2 = \text{Wellington} \\
d_3 = \text{Blücher} \\
D_4 = d_2 \oplus d_3 \\
D_5 = d_1 \oplus D_4
\end{array}
&
\begin{array}{|l|}
\hline
d,\ d' \\
\hline
d \sqsubseteq D_5 \\
d' \sqsubseteq D_5 \\
\hline
\end{array}
\Rightarrow
\begin{array}{|l|}
\hline
\\
\hline
d \text{ fought } d' \\
\hline
\end{array}
\end{array} \\
\hline
\end{array}
$$

➢ Plurals cannot denote complex individuals like D_5. The following example may be vague, but cannot express specifically a reading like (95).

(96) *The three generals fought against each other*.

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