# Superlative Quantifiers as Meta Speech Acts

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# Superlative quantifiers as meta speech acts: Outline of talk

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# Introduction: Common intuitions about SQs

Superlative quantifiers (SQs):

*at least, at most.*

Commonly assumed:

truth-conditional equivalences with comparative quantifiers:

(1) *John petted at least three rabbits*

⇔ *John petted more than two rabbits*

(2) *John petted at most three rabbits*

⇔ *John petted fewer than four rabbits*

# The generalized quantifier theory of superlative quantifiers

Keenan and Stavi (1986):

SQs are just like comparative quantifiers

(1) $= |\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(\mathbf{j},x)| \geq 3$

(2) $= |\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(\mathbf{j},x)| \leq 3$

Advantages:

- Gets the truth conditions right.
- Is extensional:

Suppose rabbits are the animals that Mary likes.

Then (1) and (3) have the same truth value

(1) *John petted at least three rabbits.*

(3) *John petted at least three animals that Mary likes.*

# The generalized quantifier theory: Problem #1: distribution

The distribution of comparative quantifiers is more restricted than that of superlative quantifiers:

(4) a. *John petted three rabbits at least/*more than.*

b. *John petted at least/*more than Bugs Bunny.*

c. *John petted at least/*more than two rabbits, namely Bugs Bunny and Peter.*

# The generalized quantifier theory: Problem #2: Certainty

Suppose John petted exactly three rabbits, and we know this.

Then (1) is strange, though (5) is fine:

(1) *John petted at least three rabbits.*

(5) *John petted more than two rabbits*.

# The epistemic theory of superlative quantifiers

Geurts and Nouwen (2007):

A. Comparative quantifiers are focus sensitive NP modifiers.

B. SQs are complex epistemic operators:

(1) *John petted at least three rabbits.*
‘It is epistemically **necessary** that John petted three rabbits, and it is epistemically **possible** that he petted more.’

(2) *John petted at most three rabbits.*
‘It is epistemically **possible** that John petted three rabbits, but it is epistemically **impossible** that he petted more.’

We accept (A), which solves the distribution problem.

We will argue against (B) and propose an alternative.

# The epistemic theory: Problem #1: Superlative Morphology

The superlative morphology (*at least, at most*) of superlative quantifiers is ignored.

Why is it that

- the linguistic form *at least three* means $\Box 3 \wedge \Diamond >3$
- the linguistic form *at most three* means $\Diamond 3 \wedge \neg \Diamond >3$

# The epistemic theory
# Problem #2: Truth Conditions

Truth conditions are predicted to be subjective, depending on epistemic states.

But suppose John petted four rabbits in the actual world

Then (1) would be true and (2) would be false, regardless of any belief worlds.

(1) *John petted at least three rabbits.*

(2) *John petted at most three rabbits.*

# The epistemic theory: Problem #3: Intensionality

Since superlative quantifiers are claimed to be modal,
they are predicted to be intensional.

Prediction: Expressions with the same extension but different intensions cannot be replaced by each other salva veritate.

For example: If it is not known that rabbits are the animals that Mary likes, then (1) and (3) are predicted to have different truth values.

(1) *John petted at least three rabbits.*

(3) *John petted at least three animals that Mary likes.*

This prediction is not borne out.

# The epistemic theory: Problem #4: NPIs

*At most* licenses negative polarity items:

(8) *At most three people have* ***ever*** *been in this cave*.

But *at least* does not:

(9) **At least three people have* ***ever*** *been in this cave*.

Not explained by Geurts and Nouwen's account.

# The epistemic theory: Problem #5: Asymmetry

If John petted exactly three rabbits and the speaker knows this, then:

(1) *John petted at least three rabbits*

Predicted **false**,
because it is not epistemically possible for John to pet more.

(2) *John petted at most three rabbits*

Predicted **infelicitous**,
since it is not only epistemically possible, but in fact necessary, that John petted three rabbits.

This predicted asymmetry between *at least* and *at most* is unintuitive.

Moreover, it is not supported by empirical evidence (Geurts *et al* 2010)

# A new explanation: SQs as illocutionary operators

- We would like to propose that superlative quantifiers are **illocutionary operators.**
- To make this view explicit, we need to look more closely at **speech acts.**

# The nature of speech acts: Propositions or actions?

What is the **type** of speech acts? There are two proposals.

(10) *Mary petted a rabbit.*

- Speech acts are **propositions**, well-known objects for semanticists. (Lewis 1970, Vanderveken 1990)
  'Speaker asserts to addressee that Mary petted a rabbit.'
- Speech acts are **actions**, an entirely different ball game than semantics. (Stenius 1967, Lewis 1969).
  The speaker follows the report game, which requires the speaker to utter a declarative sentence only if the speaker considers the proposition expressed by its sentence radical to be true. Hence the addressee can infer that the speaker considers the proposition 'Mary petted a rabbit' true.

# The nature of speech acts: Attitudes or commitments?

If speech acts are actions – how can they be modelled?

Two perspectives, cf. Harnish (2005):

- **Expressed Attitude Theory** (Grice 1967, Bach & Harnish 1979): Speaker expresses propositional attitudes like beliefs, desires.

  -- Speaker expresses belief in the proposition 'Mary petted a rabbit'

  -- Speaker wants addressee to believe that proposition.

- **Normative Theory** (Aliston 2000, Searle 2001): Speaker undertakes commitments.

  -- Speaker takes on the liability to be blamed by the addressee if the proposition 'Mary petted a rabbit' is false.

  -- Speaker takes on the responsibility of making the addressee believe that that proposition is true.

# Speech acts and commitment states: Examples

We follow the normative view:

- Speech acts express commitments.
- By successfully performing a speech act, the speaker changes commitments of the speaker, the addressee, or the society.

(11) *Mary petted the rabbit.*
speaker guarantees that the proposition ‘Mary petted the rabbit’ is true.

(12) *Did Mary pet the rabbit?*
speaker puts addressee under obligation to state
whether the proposition ‘Mary petted the rabbit’ is true.

(13) *Mary, pet the rabbit!*
speaker obliges addressee to make ‘addressee pets the rabbit’ true.

(14) *I promise to pet the rabbit.*
speaker undergoes obligation to make ‘speaker pets a rabbit’ true.

(15) *I hereby name this rabbit Peter.*
speaker puts society under obligation to call this rabbit ‘Peter’

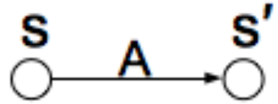
# Speech acts and commitment states: Implementation format

Commitment states: Sets of commitments, changed in discourse.
  cf. change of common ground in dynamic interpretation,
    as in Karttunen 1974, Stalnaker 1974, Heim 1983
  cf. Szabolcsi 1982 for speech acts as changes of world indices.

Speech acts: Illocutionary operators applied to a proposition
  Stenius 1967: “Sentence mood” and “sentence radical”;
  the illocutionary operator identifies type of commitment change,
  sentence radical specifies content of this change.

General format of commitment change, exemplified with assertion:

  s + ASSERT(speaker, addressee, Φ) = s′
  s: input commitment state
  s′: output commitment state
  s′ = s ∪ {speaker guarantees addressee that Φ is true}
  (reference to speaker and addressee will be omitted)



# Speech acts and commitment states: Conjunction and disjunction

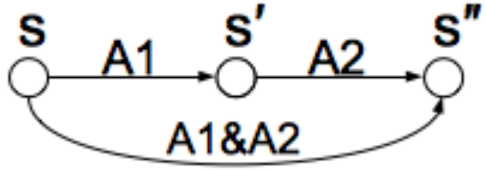
Speech acts can generally be **conjoined**:

(16) *Did Mary pet a rabbit, and did John poke at a chamaeleon?*

(17) *I promise to take you to the zoo, and I promise to take you to the movies.*

Speech act conjunction & is dynamic conjunction:

$s + [A_1 \& A_2] = [s + A_1] + A_2$




As states are sets of commitments:

$= [s + A_1] \cup [s + A_2]$

Speech acts cannot easily be **disjoined**:

(18) *Did Mary pet a rabbit, or did John poke a chamaeleon?*

Possible interpretation: replacement of first question, *or rather…*

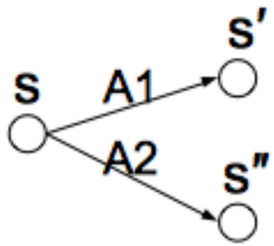
O.k. as rhetorical question, but then: assertive force.

(19) *I promise to take you to the zoo, or I promise to take you to the movies.*

Explanation: There is no dynamic disjunction.

Perhaps: $s + [A_1 \vee A_2] = \{s + A_1, s + A_2\}$,

but these are **alternative** commitment states!

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# Speech acts and commitment states: Quantification into speech acts

(20) *How many rabbits did every child pet?*
Pair-list answer: *Mary petted four, John two, and Bill three.*

Analysis by quantification into questions (cf. Karttunen 1977)

$\forall$x[**child**(x) → 'how many rabbits did x pet?']

But this is not a speech act, and the consequent is not a proposition!

Universal quantification into questions
as generalized speech act conjunction (Krifka 2001):

s + $\underset{\textbf{child}(x)}{\&}$ QUEST('how many rabbits x petted')

s + [QUEST('how many rabbits Mary petted') &
QUEST('how many rabbits John petted') &
QUEST('how many rabbits Bill petted')]

This explains why only universal quantifiers scope over speech acts:

(21) *How many rabbits did most children pet?*
No pair-list answer elicited.

# Speech acts and commitment states: Problem: Speech Act Denegation

(22) a. *I promise not to come.*
b*. I don't promise to come.*

Searle (1969):
a. PROMISE(¬ 'I come') -- o.k., expressing obligation to not come
b. ¬ PROMISE('I come') -- but what should this mean?

Hare (1970): In (b), the speaker is
"explicitly refraining from performing the speech act in question".

Are denegations (explicit) speech acts?
No. Linguistic evidence: impossibility of *hereby*.

(23) **I hereby don't promise to come.*

The utterance does not change a commitment state,
but delimits the way how commitment states may develop in the future.

We call such changes **meta speech acts**.

Meta speech acts require a more elaborate model of commitment change.

# Speech acts and commitment spaces: The notion of commitment space

Proposal: Sentences are interpreted w.r.t. **commitment spaces** which reflect the possible future development of commitments.

(24) S is a **commitment space** iff:

a. S is a set of commitment states,

b. the commitments in S are **consistent**.

c. S has a **least element**,
i.e. there is a commitment state s∈S such that for all s′∈S, s⊆s′.
We call this the **root** of S: √**S**.

The consistency requirement should exclude contradictory commitments in the same commitment state,
e.g. requirements from ASSERT(Φ), ASSERT(¬Φ).

The root of S is the current commitment state;
subsequent states express how the root can develop.

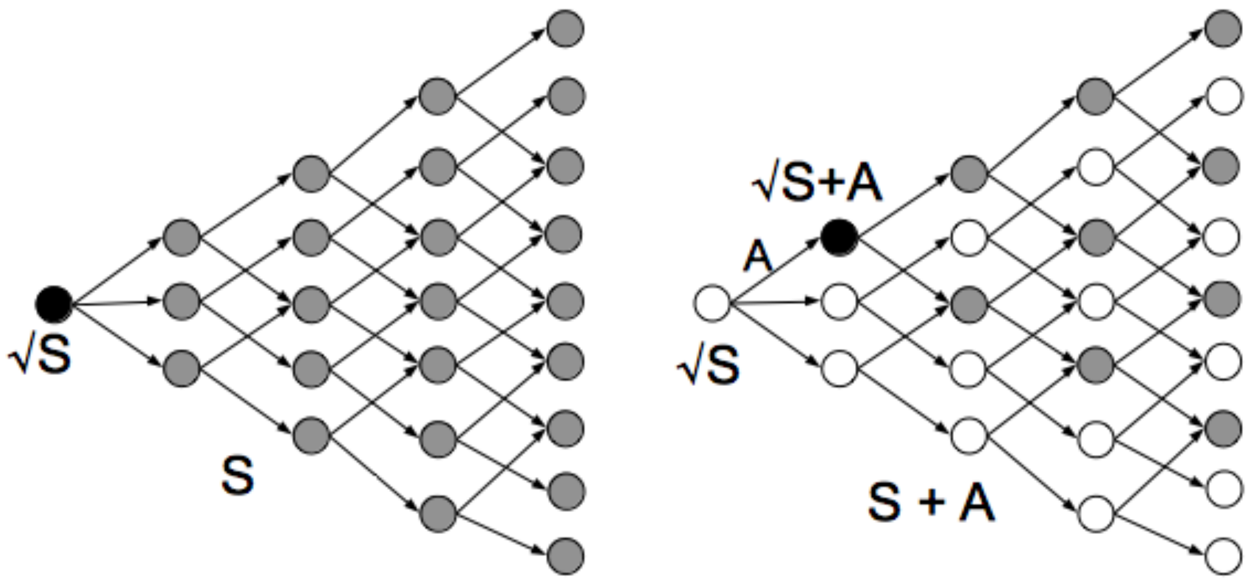
# Speech acts and commitment spaces: Simple speech acts

Update of commitment space S with speech act A:

(25) S + A = {s∈S | √S + A ⊆ s}

The output commitment space consist of all input commitment states s that contain the commitments of the root of the input space √S, updated by the speech act A.

Notice: the output commitment space is a proper (rooted) set of commitment states.



# Speech acts and commitment spaces: Denegation

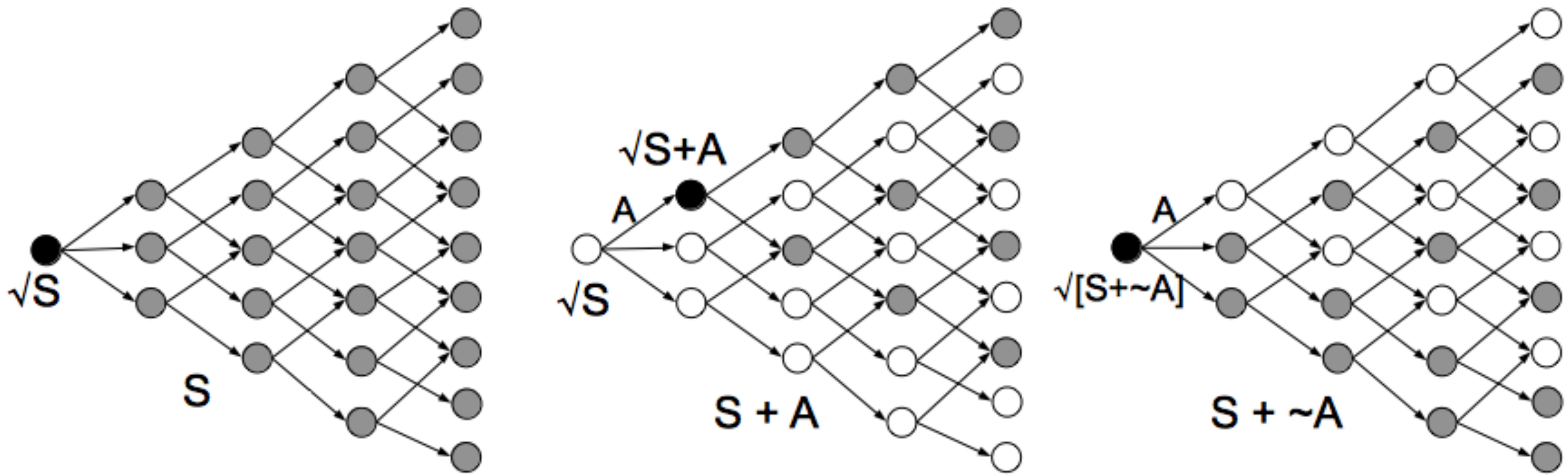
Denegation of speech acts ~A is interpreted as complement formation $\overline{\phantom{x}}$

(26) $S + {\sim}A = \overline{[S + A]}$

(27) $S + {\sim}\text{PROMISE}(\text{‘I come’}) = \overline{[S + \text{PROMISE}(\text{‘I come’})]}$

$= \overline{\{s \in S \mid \surd S + \text{PROMISE}(\text{‘I come’}) \subseteq s\}}$

Because a simple speech act A changes the root,
 its denegation ~A does **not** change the root,
 but still restricts the commitment space.



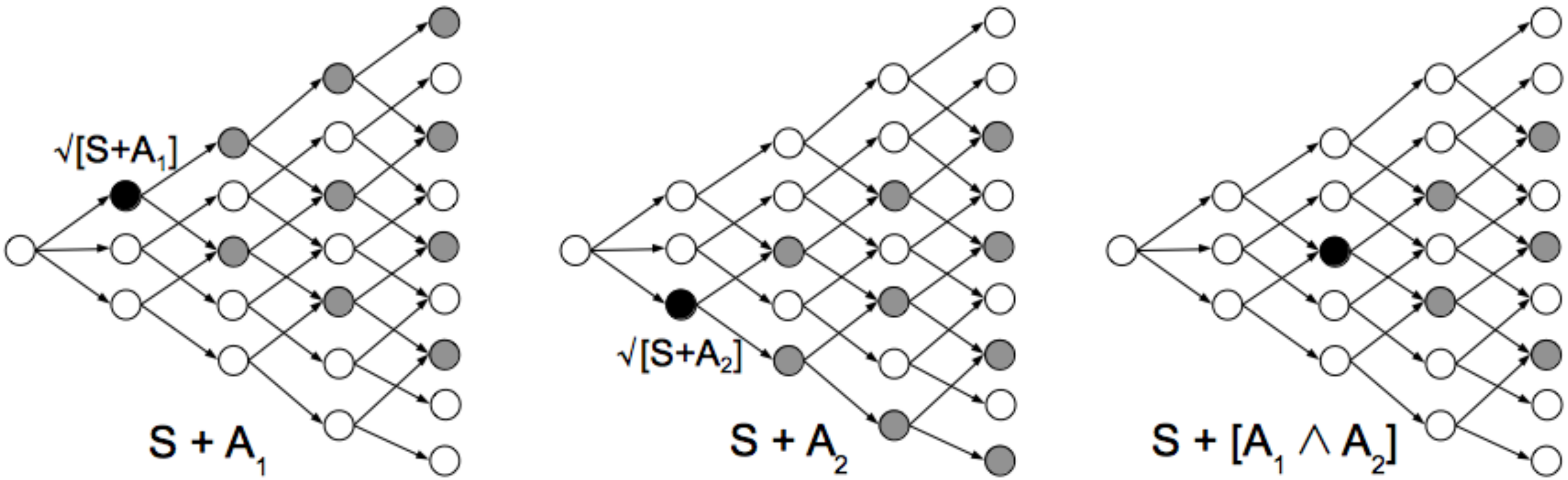

Observe: ~~A = A, as S + ~~A = S + A

# Speech acts and commitment spaces: Conjunction

Conjunction of speech acts can now be modelled by set intersection:

(28) $S + [A_1 \wedge A_2] = [S + A_1] \cap [S + A_2]$

Notice that conjunction results in a rooted set of commitment states:




Relation to dynamic conjunction: $\surd S + [A_1 \wedge A_2]) = \surd S{+}A_1{+}A_2 = \surd S{+}A_2{+}A_1$

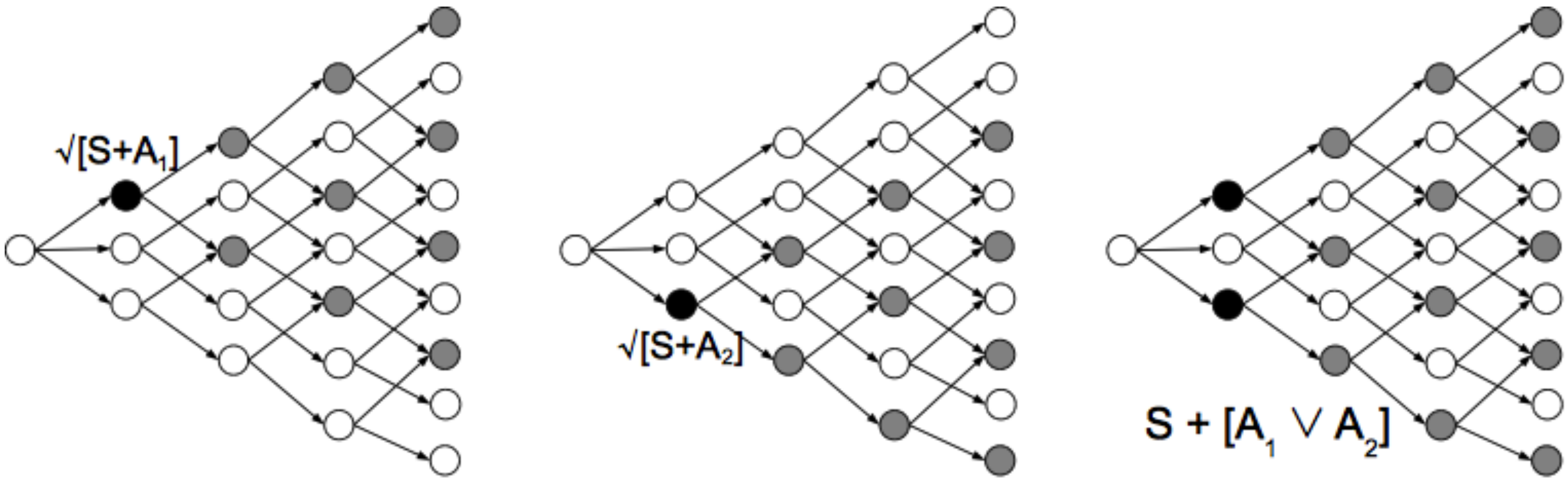
As universal quantifiers are generalized conjunctions,
this explains universal quantification into speech acts.

# Speech acts and commitment spaces: Disjunction

We can define disjunction of speech acts by set union:

(29) $S + [A_1 \vee A_2] = [S + A_1] \cup [S + A_2]$

But this does not generally result in a rooted set of commitment states:




Hence, disjunction is not generally well-defined.

This explains why non-universal quantifiers do not scope over speech acts.

# Speech acts and commitment spaces: De Morgan

Disjunction is not generally well-defined for commitment spaces, but De Morgan holds:

(30) $S + {\sim}[A_1 \vee A_2]$

$= \overline{[[S + A_1] \cup [S + A_2]]}$

$= [\overline{[S + A_1]} \cap \overline{[S + A_2]}\,]$

$= S + [{\sim}A_1 \wedge {\sim}A_2]$

Example with explicit performatives:

(31) *I don't promise to marry you or threaten to leave you.*

⇔ *I don't promise to marry you and I don't threaten to leave you.*

# Speech acts and commitment spaces: The speech act of granting

In conversation about facts, we often do not just assert propositions, we also **grant** propositions to the opponent (cf. Merin 1994).

Grants are expressions of willingness to go along with assertions by the opponent – hence denegations of asserting the contrary.

(32) S + GRANT(Φ) = S + ~ASSERT(¬Φ)

Example:

(33) *The evidence is inconclusive, but I grant that you are innocent. I still have my doubts, though.*

Equivalences (similar to modal logic):

(34) GRANT(Φ) = ~ASSERT(¬Φ)
ASSERT(Φ) = ~GRANT(¬Φ)

Grants are meta speech acts:
They do not change the root of a commitment space,
but restrict the future progress of commitments.

# SQs as quantifications over grants: *at most*

(35) *Mary petted at most three rabbits.*

A meta speech act restricting commitment spaces.

Intuitive formulations:

- The maximal number n such that the speaker grants that Mary petted n rabbits is n = 3.
- $\max\{n \mid \mathrm{GRANT}(|\mathbf{rabbit} \cap \lambda x[\mathbf{pet}(\mathbf{m},x)]| = n)\} = 3$
- $\forall n[\mathrm{GRANT}(|\mathbf{rabbit} \cap \lambda x[\mathbf{pet}(\mathbf{m},x)]| = n \rightarrow n \leq 3]$
- $\forall n[n > 3 \rightarrow {\sim}\mathrm{GRANT}(|\mathbf{rabbit} \cap \lambda x[\mathbf{pet}(\mathbf{m},x)]| = n]$

Formalization as changes of commitment spaces:

- $S + \bigwedge_{n>3} {\sim}\mathrm{GRANT}(|\mathbf{rabbit} \cap \lambda x[\mathbf{pet}(\mathbf{m},x)]| = n)$
- $S + [\,{\sim}\mathrm{GRANT}(|\mathbf{rabbit} \cap \lambda x[\mathbf{pet}(\mathbf{m},x)]| = 4) \wedge$
  ${\sim}\mathrm{GRANT}(|\mathbf{rabbit} \cap \lambda x[\mathbf{pet}(\mathbf{m},x)]| = 5) \wedge \ldots]$

Speaker does not make an assertion, but excludes certain future assertions.

By this, speaker allows that Mary petted 3, 2, 1, or 0 rabbits.

# SQs as quantifications over grants: *at least*

(36) *Mary petted at least three rabbits.*

Intuitive formulations:

- The minimal number n such that the speaker grants that Mary petted n rabbits is n = 3.
- min{n | [GRANT(|**rabbit** ∩ λx[**pet**(**m**,x)]| = n)} = 3
- ∀n[GRANT(|**rabbit** ∩ λx[**pet**(**m**,x)]| = n → n ≥ 3]
- ∀n[n < 3 → ~ GRANT(|**rabbit** ∩ λx[**pet**(**m**,x)]| = n]

Formalization as changes of commitment spaces:

- S + $\bigwedge_{n<3}$ ~GRANT(|**rabbit** ∩ λx[**pet**(**m**,x)]| = n)
- S + [ ~GRANT(|**rabbit** ∩ λx[**pet**(**m**,x)]| = 2) ∧
  ~GRANT(|**rabbit** ∩ λx[**pet**(**m**,x)]| = 1) ∧
  ~GRANT(|**rabbit** ∩ λx[**pet**(**m**,x)]| = 0) ]

Speaker does not make an assertion, but excludes certain future assertions.

By this, speaker allows that Mary petted 3, 4, 5, … rabbits.

# SQs as quantifications over grants: Problem #1: Superlative morphology

The superlative morphology is clearly represented in the meaning of SQs:

(37) *Mary petted at least three rabbits.*

$\textbf{min}\{n \mid \text{GRANT}(|\textbf{rabbit} \cap \lambda x.\textbf{pet}(\textbf{m},x)| = n)\} = 3$

(38) *Mary petted at most three rabbits*.

$\textbf{max}\{n \mid \text{GRANT}(|\textbf{rabbit} \cap \lambda x.\textbf{pet}(\textbf{m},x)| = n)\} = 3$

# SQs as quantifications over grants: Problem #2: Truth conditions

Problem of Geurts & Nouwen: No objective truth conditions.

Here: SQs are meta speech acts, how can they have truth conditions?

(1) *Mary petted at least three rabbits.*

- Suppose Mary petted exactly two rabbits.

  Uttering (1) includes: ~GRANT(|**rabbit** ∩ λx[**pet**(**m**,x)]| = 2)
  which is equivalent to: ASSERT(|**rabbit** ∩ λx[**pet**(**m**,x)]| ≠ 2)

  But this cannot be truthfully uttered in the scenario.

- Suppose Mary petted exactly four rabbits.

  Uttering (1) means: ~GRANT(..0..) ∧ ~GRANT(..1..) ∧ ~GRANT(..2..),

  by **implicature**: ~~GRANT(..3..), ~~GRANT(..4..), …

  equivalent to: GRANT(..3..) ∧ GRANT(..4..) ∧ GRANT(..5..)…

  This can be truthfully uttered in the scenario,
  as one of the options granted can be truthfully asserted.

# SQs as quantification over grants: Problem #2: Certainty

Notice:

**Falsity is determined semantically,**
**Truth is determined pragmatically, via implicature.**

This captures the intuition that
when one says (1), one doesn't know how many rabbits Mary petted;
but one does know how many rabbits he did *not* pet.

# SQs as quantifications over grants
# Problem #3: Extensionality

Geurts & Nouwen: Modal theory predicts non-replacability of expression with the same extension salva veritate, which does not hold:

(1) *John petted at least three rabbits.*

(3) *John petted at least three animals that Mary liked.*

In the current theory, there is no modal context;
replacement of expressions with the same extension should work.

# SQs as quantifications over grants: Problem #4: Licensing of NPIs

Recall:

*At most* but not *at least* licenses negative polarity items:

(8) *At most three people have **ever** been in this cave*.

(9) **At least three people have **ever** been in this cave*.

Distribution of NPIs: Kadmon & Landman 1993, Krifka 1995:
NPIs denote the least specific meanings among their alternatives,
but lead to the most specific (strongest) overall meaning.

Assume:
*ever* denotes unrestricted time,
alternatives denote specific times (e.g., 'last year').

To be shown:

- *At most 3 people have been in this cave at some time or other*
  is **stronger** than *At most 3 people have been in this cave last year.*
- *At least 3 people have been in this cave at some time or other*
  is **not stronger** than *At least 3 people have been in this cave last year.*

Here: Only the first point will be shown.

# SQs as quantifications over grants: Problem #4: Licensing of NPIs

Def. of strength for (meta) speech acts:

$A_1$ is as strong as or stronger than $A_2$ iff $S + A_1 \subseteq S + A_2$

Clear and obvious logical entailment:

¬ ‘n people in cave, some time or other’ |= ¬ ‘n people in cave, last year’

Consistency requirement for commitment states:

if s contains the commitment of ASSERT(¬ ‘n people, some time or other’)
then s contains the commitment of ASSERT(¬ ‘n people, last year’)

Hence:

$$S + \bigwedge_{n>3} \text{ASSERT}(\neg\text{‘n people in cave, some time or other’})$$

$$\subseteq S + \bigwedge_{n>3} \text{ASSERT}(\neg\text{‘n people in cave, last year’})$$

# Some additional topics: Embedded superlative quantifiers

It is often assumed that speech acts cannot be embedded.

But this is wrong:

We have seen that speech acts can be embedded,
subject to constraints of interpretability
(cf. quantification into question acts).

Example: Propositional attitudes.

(39) *Mary thinks that John had at least three martinis.*

- Reading 1, no embedding:
  Mary thinks that John had *n* martinis,
  and the speaker says that *n* is at least three.
- Reading 2, embedding:
  Mary thinks: “John had at least three martinis.”
  Speech act is used to characterize Mary’s thought from her perspective.

# Some additional topics: Embedded superlative quantifiers

Keenan and Stavi would predict that SQs are freely embeddable

Geurts and Nouwen: SQs are embeddable in exactly the same environments that epistemic modals are.

But they only have two examples, both of downward entailing contexts:

(40) a. **None of the guests danced with at least/most three of waitresses.*

b. *?Betty didn't have at least/most three martinis.*

Geurts and Nouwen: (40a) is bad because *none* is a weak quantifier.

But other weak quantifiers do embed SQs:

(41) *Some/two of the guests danced with at least/most three of the waitresses.*

Other downward-entailing contexts do not embed SQs (Nilssen 2007):

(42) a. *John hardly ate ??at least/more than three apples.*

b. ??*Policemen rarely carry at least two guns.*

c. ??*This won't take at least 50 minutes.*

# Some additional topics: SQs in downward-entailing contexts

Recall:

- the falsity of a SQ follows semantically,
- the truth requires a scalar implicature.

Hence, the interpretability of a SQ requires scalar implicature.

But implicatures generally do not survive downward entailing contexts (Chierchia 2004)

(43) *If you drink or smoke, you will become ill.*

# Some additional topics: The good, the bad, and the puzzling…

Superlative quantifiers are acceptable in the antecedent of conditionals and quantifiers if the consequent is "good" (Nilsen 2007)

(44) a. *If you click the Finalize your order button at least twice you will get a discount.*

b. ??*If you click the Finalize your order button at least twice you will be charged double.*

# Some additional topics: The good, the bad, and the puzzling…

Kay (1992): Evaluative reading of *at least*

(45) *At least this hotel is centrally located.*

- Presupposition: being centrally located is good.
- Entailment: being centrally located is the minimal requirement.
- Implicature: this is less than the maximally good property.

Proposal:

The antecedent of (44a) and (44b) is reinterpreted with the evaluative interpretation of *at least*.

- Presupposition: Clicking (exactly) twice is good
  — satisfied in (44a) (getting a discount)
  — fails in (44b) (charged double)
- Implicature: You click less than some maximally good limit
  — canceled.
- Entailment: You click minimally twice

(44a) = If you click minimally twice, you will get a discount

# Some additional topics: Requirements

(46) *John needs at least three martinis.*

(a) The minimal n s.t. the speaker is willing to assert that John needs exactly n martins is n = 3.

(b) The minimal number n of martinis that satisfies John's needs is n = 3

Geurts and Nouwen account for reading (a) straightforwardly.

For (b), they are forced to propose that the epistemic modal of *at least* becomes deontic as a consequence of the explicit requirement indicated by *need*.

Our explanation: Scope ambiguity.

(a) $\forall n[\text{GRANT}_{\text{speaker}}(\text{John requires: John has n martinis}) \rightarrow n \geq 3]$

(b) John requires: $\forall n[\text{GRANT}_{\text{John}}\ (\text{John has n martinis}) \rightarrow n \geq 3]$

Note:

-- For (b): *require* must be able to subcategorize for speech acts.

-- The subject of (b) must be able to grant:

(47) *The project needs at least three years to complete.* – only reading (a)

# Conclusions

General points:

- Interlocutors often express meta speech acts: they are not moves in the conversation, but indicate which moves are possible.
- Speech acts (including meta speech acts) can be modelled as changes of commitment spaces.
- Thus they become semantic objects, and hence part of semantic recursion. Therefore they can be embedded, provided their embedding is interpretable.

Particular points:

- SQs are quantifiers over meta speech acts.
  -- At least 3 Φ: the minimal number n s.t. the speaker grants n Φ is 3.
  -- At most 3 Φ: the maximal number n s.t. the speaker grants n Φ is 3.
- The falsity of a SQ is determined semantically,
  the truth of a SQ is determined pragmatically, via scalar implicature.
- Hence, SQs can be embedded in environments where scalar implicature survives.

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Comments:

-- How general is denegation? E.g., not for exclamatives. (M. Gerner)
For exclamatives: Condition that momentaneous emotions are expressed, can only modify the root.

-- Introduction of discourse referents (N. Kadmon)
*Mary petted at least two rabbits. They are sitting in the garden.*
Explanation: Introduction of discourse referent, separate
ASSERT(Mary petted x); (introduces discourse referent x; maximal DR)
$\forall n[\text{GRANT}(n \text{ rabbits}(x)) \rightarrow n \geq 2]$
ASSERT(x is sitting in the garden)
Maximalization cf. Evans:
*John has some rabbits. Mary petted them.* (= all the rabbits John has).

Perhaps there are other ways to handle this too:
GRANT(Mary petted 2 rabbits) ∧ GRANT(Mary petted 3 rabbits) ∧ …
this garantees the existence of rabbits, E-type pronoun possible.

Problem with evaluative reading (different syntax) – B. Partee, #

Proposal:

$\forall n$[GRANT(If you click n times then you get a price) → n≥2]

Speaker grants addressee something; grants are generally in the interest of the addressee. Therefore bad with ‘you will get fined’

Multiple quantifiers:

At least three children petted at least seven rabbits.

At most three children petted at most seven rabbits.

# At least three children petted at most seven rabbits.

Hence: Global interpretation.

$\forall n$[GRANT(n children petted n′ rabbits) → n ≥3, n′ ≥7]

cumulative reading!

Assume that at least/at most introduces alternatives that are only compatible with GRANT as a speech act operator.