ABSTRACT. This paper investigates relative constructions as in The gifted mathematician that you claim to be should be able to solve this equation, in which the head noun (gifted mathematician) is semantically dependent on an intensional operator in the relative clause (claim), even though it is not c-commanded by it. This is the kind of situation that has led, within models of linguistic description that assume a syntactic level of Logical Form, to analyses in which the head noun is interpreted within the CP-internal gap by reconstruction or interpretation of a lower element of a chain. We offer a solution that views surface representation as the input to semantics. The apparent inverted scope effects are traced back to the interpretation of the head nominal gifted mathematician as applying to individual concepts, and of the relative clause that Bill supposedly is as including an equational statement. According to this view, the complex DP in question refers to the individual concept that exists just in the worlds that are compatible with what is generally supposed to be the case, is a gifted mathematician in those worlds, and is identical to Bill in those worlds. Our solution is related to the non-reconstructionist analysis of binding of pronouns that do not stand in a c-command relationship to their binder, as in The woman that every man hugged was his mother in Jacobson (1994) and Sharvit (1996), and allows us to capture both similarities with and differences from the latter type of construction. We point out and offer explanations for a number of properties of such relative clauses – in particular their need for an internal intensional operator, their incompatibility with any determiner other than the definite article, and the fact that some of their properties are shared by demonstrably distinct kinds of relative clauses.

1. INTRODUCTION

The principal goal of this paper is to describe and analyze the semantic and pragmatic properties of a relative clause construction whose existence has only occasionally been mentioned in reference
works but, to the best of our knowledge, without full recognition of its special semantic properties\textsuperscript{2}. We will refer to it as EIR relatives, an acronym whose spelled-out form appears in the title of the paper and whose meaning will be revealed at the end of this section, after noting and illustrating some of its characterizing properties.

An illustration of EIR relatives is provided by the italicized phrase in the title, which we will not deal with directly due to its additional complications. Instead, we will mainly discuss the bracketed DP in (1), on the reading which says that Bill is supposed to be a gifted mathematician, and that in worlds in which this supposition is correct, he should be able to solve the problem in no time.

\begin{enumerate}
  \item [\textsuperscript{1}] [The gifted mathematician that Bill supposedly is _ ]
      should be able to solve this simple problem in no time.
\end{enumerate}

The full range of EIR relatives, and their similarities and differences to a number of other kinds of relative constructions, will become clearer in the course of the paper.\textsuperscript{3} But we would like to state from the outset that they need to be distinguished from two other constructions. First, imagine that Bill falsely claims to be a certain famous mathematician:

\begin{enumerate}
  \item [\textsuperscript{2}] [The famous mathematician that Bill claims to be _ ]
      is standing in front of him, and casting furious glances at him.
\end{enumerate}

This cannot mean that Bill claims to be a certain famous mathematician, and in all worlds in which these claims are correct, this mathematician is standing in front of Bill. In (2) we clearly talk about two distinct individuals, in contradistinction to (1).

The second construal is perhaps more difficult to distinguish from EIR relatives. It has been suggested that (1) does not make a claim about Bill \emph{per se}, but rather is a generic sentence about gifted mathematicians in general, and is as a matter of fact equivalent to (3).

\begin{enumerate}
  \item [\textsuperscript{3}] The kind of gifted mathematician that Bill supposedly is _ should be able to solve the problem in no time.
\end{enumerate}

\textsuperscript{2} We are aware of Huddleston & Pullum (2002), who mention cases like \textit{Her book displays the fine sceptical intelligence of the scholar she is} (cf. Chapter 12, section 3.3 (c), ex. 29) in the context of a taxonomy of relatives according to the syntactic position of their gap. This example is parallel to our (1), but is lumped together with other examples of relatives with postcopular gap like I \textit{don’t think it is the good investment they consider it to be} and \textit{Harry is basically a fat man searching for a thin man that he once used to be} which have different semantic properties, the first one being similar to (2), the second one representing Harry as two different persons.

\textsuperscript{3} EIR relatives are certainly not a particularly frequent phenomenon, and we work with constructed examples throughout the paper. But it should be stressed that EIR relatives occur in everyday discourse, as the following examples show (a number of related constructions will be discussed in section 3):

(a) Yet internally, within his own psyche, he hates the creature he has become (Saunders 1997:67)
(b) Immediately, we were trust into the atrocity that this software was (Paprocki 2004)
(c) those calling for it so vociferously must be charged with the war crime that this is and was (Kirwan 2006)
(d) Galen thanks his parents, and Yalu expresses pride in the ape he has become (http://rhandely0catch.com/POTA/timeline_08.html, retrieved Aug 16, 2007).
(e) The legend he has become will perhaps be fit enough to play cricket again (http://reverseswingmanifesto.blogspot.com/2007/05/hold-on-to-your-ego.html, retrieved Sep 30, 2007)
(f) How do you react when others treat you like the servant you claim to be? (www.authenticwalk.com/Lesson.phtml?Cat=417&P=3&M=A&L=, retrieved Sep 30, 2007)
(g) You however (the excellent tactician that you claim to be) should have ducked us instead of tacking beneath and getting lots of dirty wind and loosing even more height. (http://www.yachtsandyachting.com/forum/forum_posts.asp?TID=3189&PN=1&TPN=5, retrieved Sep 30, 2007)
We think that EIR relative clause DPs can receive a generic construal just as other definite DPs even without the phrase *kind of*, as illustrated in (4) with the kind predicate *rare*; witness the parallel cases with EIR relatives in (5), which appears to also have a generic construal without *kind of* for many speakers.

(4)  [The (kind of) beetle that Bill found _ ] is very rare nowadays.

(5)  [The (kind of) gifted mathematician that Bill supposedly is _ ] is very rare nowadays.

Furthermore, predicates like *should be able to* (...) can be interpreted as generic, as in (6), which explains why a generic construal is possible in (3) and perhaps also in our original example (1), even if this appears a bit forced.

(6)  The (kind of) beetle that Bill found should be able to survive even a hard winter.

However, observe that sentences like (7), with a predicate that cannot easily be interpreted as applying to a kind, show that EIR relatives cannot be reduced to kind interpretations of a definite DP.

(7)  [The (#kind of) brilliant mathematician that Bill unquestionably is _ ] has just solved a most difficult mathematical problem.

We should stress that EIR relatives are by no means restricted to English; similar examples in French, German, Romanian, Modern Hebrew and Russian can readily be constructed. These languages all exhibit externally headed relative clauses; it remains to be investigated whether EIR relatives also show up in languages with radically different patterns of relative clause construction, such as correlatives and internally-headed relative clauses.

Having taken care of these preliminaries, we now turn to the task of providing a working definition of EIR relatives. Observe that in (1) the CP-external NP *gifted mathematician* is interpreted as though internal to the relative clause, in particular, as though under the scope of the italicized intensional operator *supposedly* and *clearly*, respectively.

---

4 French:  [Le grand mathématicien que tu sembles être _ ]
*the great mathematician that you seem be*
e devrait avoir aucune difficulté à résoudre ce simple problème.
*Neg should have any difficulty at solve this simple problem*

German:  [Der fähige Mathematiker, der du _ angeblich bist]  
*the great mathematician who you supposedly are*
sollte dieses einfache Problem ohne Schwierigkeit lösen können.
*should this simple problem without difficulty solve be-able-to*

Romanian:  [Marele mathematician ce pare a fi _ Ion]
*great-the mathematician that seems to be Ion*
trebuie să rezolve această problemă în câteva minute.
*COND must SUBJ solve this problem in a few minutes*

Mod. Hebrew:  [Ha-matematikay ha-gadol she hu amur lihyot _ ]
*the-mathematician the-great that he said to-be*
yacliax le-lo safek liftor et ha-ba’aya ha-zot.
*will-succeed to-no doubt to-solve Acc the-problem the-this*

Russian:  Velikij mathematik, kotorym schitaetsja Petja,
*great mathematician who Instr is-considered Peter*
dolzhen byt’ v sostojanii reshit’ etu zadachu za pjad’ minut.
*ought be in state solve this problem in five minutes*
(8) Intuitive paraphrase of (1):
‘If Bill were a gifted mathematician, as he is supposed to be, he should be able to solve this problem in no time.’

On this view, EIR relatives form a sub-class of the broader class of ‘reconstruction relatives’, or R-relatives, which include functional relatives like (9) in which the pronoun his is construed as bound by the quantified DP every boy in a non-c-commanding configuration (for analyses see von Stechow 1990, Jacobson 1994, 2002a, Sharvit 1996, 1999).

(9) [The woman that every man, hugged _ ] was his, mother.

EIR relatives are R-relatives with reconstruction effects in the scope of an CP-internal intensional operator. This is, however, still too broad as a definition, as it also includes cases like the following:

(10) [The ideal wife that Bill has been vainly looking for _ all his life] may never be found.

We will see that that there are good reasons for distinguishing EIR relatives from other intensional ‘reconstruction’ relative constructions, or IR relatives, like (10), and to this end we note one further property that zeroes in on the desired class: the ‘gap’ of relativization is in the non-subject position of a copular structure, which is moreover necessarily construed as equational (this last point will be argued for in sections 2 and 3 below). In the examples so far, this equational structure was clearly marked by the copula is. The copula might be implicit, as in the small clause construction (11), or in implicit equational constructions as in (12):

(11) [The gifted mathematician that Bill is widely viewed as _] should be able to solve this problem.

(12) [The gifted mathematician that you claim to have hired _] should be able to solve this problem.
‘You have hired someone, and you claim that this person is a gifted mathematician, and this person (according to the claim) should be able to solve this problem.

We are now ready to provide a characterization of EIR relative constructions:

(13) Equational Intensional ‘Reconstruction’ Relatives (EIR relatives) are externally-headed relative clause constructions whose CP-external NP looks as though interpreted under the scope of an intensional operator inside the relative clause, the position of apparent reconstruction being the non-subject position of an equational copular structure or small clause.

The remainder of this paper is organized as follows. In the central section 2, we will develop the semantics of EIR relatives and compare them to the treatment proposed for functional relatives. In section 3, we focus on a feature that lies at the heart of our analysis, i.e., an equational interpretation of the copula, arguing that there is no natural way to achieve an adequate interpretation of EIR relatives under a predicative interpretation of the copula. Section 4 extends the scope of our investigation to EIR relatives with a greater range of copular subjects, section 5 specifies the nature of the definite article in EIR relatives and explains why we do not host other determiners, section 6 investigates the range of permissible intensional operators and discusses the pragmatic raison d’être of EIR relative clause constructions. Section 7 summarizes the results of the paper and notes the major conclusions that can be drawn from them.
2. THE SEMANTICS OF EIR RELATIVES

2.1 The Analysis of EIR Relatives: The Basics

The paraphrase of EIR relatives given in (8) was only meant to give an intuitive first idea of the overall meaning of this construction. We do not claim that EIR relatives are to be analyzed as anything that is structurally similar to this paraphrase. One reason is that (8) does not analyze EIR relatives as the referring expressions that they are – witness their syntactic form and their ability to license anaphoric pronouns:

(14) The gifted mathematician Bill claims to be should have solved this equation without problem. He should also have discovered the blunder in his attempt to prove Theorem 5.

We propose the following analysis of EIR relative constructions like (1), which will be developed in greater detail below. The subject DP the gifted mathematician that Bill supposedly is refers to an individual concept, a function from indices (worlds, times etc.) to individuals. Being a referring expression, it can be antecedent to a pronoun, which explains examples like (14). This individual concept is defined for all indices that stand in the accessibility relation of the epistemic modal operator supposedly, that is, for all indices that are compatible with what is supposed to be the case. For each index for which this individual concept is defined, the individual that is its value is a gifted mathematician, and is identical to Bill. Note that it need not be defined for the actual world of interpretation; example (1) in fact implies that it is not defined for the actual world, that is, that Bill actually is not a gifted mathematician. This meaning can be rendered formally as follows.

(15) \[ \text{[the gifted mathematician that Bill supposedly is]}_[i] = \iota x [\text{DOM}(x) = \text{SUPPOSED}(i) \land \forall i' \in \text{DOM}(x)[\text{GIFTED MATH}(i')(x(i')) \land x(i') = \text{BILL}(i')]] \]

Here x is a variable over partial individual concepts. DOM(x) is the domain of x, the set of indices for which x is defined. SUPPOSED(i) is the set of indices that are compatible with what is supposed to be the case at the index of evaluation, i. The domain of the individual concept x and this set are identical, that is, the individual concept is just defined at the indices that are compatible with what is supposed to be the case, and it need not be defined at the index of evaluation, i. At all indices i’ for which x is defined, it holds that the individual x(i’) is a gifted mathematician, and that x(i’) is identical to Bill. Note that this last equational statement, together with the restriction that the domain of x equals SUPPOSED(i), guarantees that the individual concept x is unique. This satisfies a necessary condition for the use of the iota operator in the formal language, and the use of the definite article in English. In fact, the definite article is the only felicitous option, since other determiners, like the indefinite article or the universal quantifier, would implicate that the concept under this description is not unique, a point to which we will return in section 4 below.

The VP should be able to solve this problem says of this individual concept that in all worlds that are compatible to what is expected, it is able to solve this problem. Predicating this property of an individual concept means that for all indices that are compatible with what is expected, the individual concept (that is, its value at the index) is able to solve this problem. More formally:

(16) \[ \text{[should be able to solve this problem]}_[i] = \lambda x \forall i'' \in \text{EXPECT}(i)[\text{ABLE TO SOLVE THIS PROBLEM}(i'')(x(i''))] \]

This is a predicate over individual concepts x. The intensional operator expressed by should is rendered by the accessibility relation EXPECT that provides, for the index of evaluation i, a set of
indices.\(^5\) It is claimed that for every index \(i''\) in this set, the individual \(x(i'')\) is able to solve this problem at \(i''\). For this claim to get a truth value, it must be the case that the individual concept \(x\) is defined for the index \(i''\); that is, it is presupposed that \(\text{EXPECT}(i) \subseteq \text{DOM}(x)\). This requirement is typically satisfied by accommodation, that is, the accessible indices are restricted to those for which the individual concept \(x\) is defined, \(\text{EXPECT}(i) \cap \text{DOM}(x)\).

Applying this VP meaning to the individual concept constructed in (15) we get the following interpretation:

\[
(17) \quad \lambda x \, \forall i'' \in \text{EXPECT}(i) \left[ \text{ABLE TO SOLVE THIS PROBLEM}(i'')(x(i'')) \right] \\
\quad = \lambda x \, \forall i'' \in \text{EXPECT}(i) \left[ \text{ABLE TO SOLVE THIS PROBLEM}(i'')(x(i'')) \right] \\
\quad = \forall i'' \in \text{EXPECT}(i) \left[ \text{ABLE TO SOLVE THIS PROBLEM}(i'') \right] \\
\quad = \forall i'' \in \text{EXPECT}(i) \left[ \text{ABLE TO SOLVE THIS PROBLEM}(i'') \right] \\
\quad = \forall i'' \in \text{EXPECT}(i) \left[ \text{ABLE TO SOLVE THIS PROBLEM}(i'') \right]
\]

What this says is: For all indices \(i''\) that are compatible with what is expected at \(i\), the individual that is the value of an individual concept \(x\) at \(i''\) is able to solve this problem, where \(x\) in turn is the individual concept that is defined at all indices \(i'\) that are compatible with what is supposed to be the case at \(i\), and for which it holds that it is a gifted mathematician at \(i'\) and identical to Bill at \(i'\).

The example illustrates that the indices that are accessible by way of the intensional operator \(\text{supposedly}\) and the indices that are accessible by way of the intensional operator \(\text{should}\) are in some sense compatible with each other. If these sets of indices were totally disjoint, no coherent predication could result.\(^6\) As we have seen in the discussion of (16), we have that \(\text{EXPECT}(i) \subseteq \text{DOM}(x)\) (perhaps after restriction of \(\text{EXPECT}(i)\) by accommodation), and \(\text{DOM}(x) = \text{SUPPOSED}(i)\); hence we have \(\text{EXPECT}(i) \subseteq \text{SUPPOSED}(i)\). It is this compatibility requirement that is responsible for the possibility of conditional paraphrases of EIR constructions as in (8), repeated here:

\[
(18) \quad \text{‘If Bill is a gifted mathematician, as he supposedly is, he should be able to solve this simple problem.’} \\
\quad \forall i'' \in \text{EXPECT}(i) \cap \text{SUPPOSED}(i) \left[ \text{ABLE TO SOLVE THIS PROBLEM}(i'')(\text{BILL}(i'')) \right]
\]

---

\(^5\) For simplicity of exposition, we will analyze intensional operators as simple quantifications over sets of indices and disregard contextual factors that concern the selection of a modal background, as well as any normality ordering between indices.

\(^6\) This property motivated the name that we used in previous versions of this paper, namely Modal Compatibility Relatives (see Grosu & Krifka 2004).
In the formal rendering of this paraphrase it is made explicit that the indices that are compatible with what is expected are restricted by those that are compatible with what is supposed to be the case, where it is presupposed that for all those indices, Bill is a gifted mathematician.\footnote{An anonymous reviewer pointed out that the parenthetical as he supposedly is belongs to the Conventional Implicature dimension, in the sense of Potts (2005). In the EIR relative example (1) the information that Bill is supposedly a gifted mathematician is not part of the regular assertion either, but is presupposed, as the descriptive content of definite descriptions usually is. We will not try to represent this presuppositional informational status here.}

The modal compatibility requirement is responsible for the oddness of (19), in which the VP is to be interpreted for the actual index i, which is not necessarily in the set \( \text{SUPPOSED}(i) \).

(19) #[The gifted mathematician that Bill supposedly is \_] solved this problem.

While the individual concept denoted by the EIR relative is only defined for the set of indices that are accessible to the modal operator within the EIR relative, we should add that this individual concept can be extended to other indices, including the index of evaluation of the whole sentence. In our example, it is assumed that Bill exists in the real world (though it is implicated that he is not a gifted mathematician). Pragmatically, EIR relatives are used to say something about an existing entity, but a particular \textbf{version} of this entity that in fact does not obtain. We will return to this \textit{raison d’être} of EIR constructions at the end, in section 6.

The main task will now be to analyze the semantics of EIR relative constructions. The biggest challenge is to derive the meaning of the subject DP, \textit{the gifted mathematician that Bill supposedly is}, in a compositional manner. To this we will turn in the next two subsections.

2.2 \textbf{“Reconstruction” Relatives Without Reconstruction}

It is a defining property of EIR relatives that their head noun appears to be under the scope of an intensional operator within the relative clause, which seems to require that it is reconstructed into this position. Hence this case appears to call for a head-raising analysis of relative clauses, for which a number of semantic arguments have been adduced in recent work (cf. Bhatt 2002, and Hulsey & Sauerland 2006 for an overview). The head-raising analysis, combined with the copy theory of movement (Chomsky 1993), yields the LF-representation (20), where the lower copy does not get pronounced, but is interpreted, and the higher copy does get pronounced, but is not interpreted.

(20) the [gifted mathematician [that [Bill [supposedly [is gifted mathematician]]]]]

should have solved this problem.

It is not obvious, though, how this structure could get interpreted in such a way that the reading paraphrased in (8) could be generated by the analysis in (20). In any case, it has not been done, Jacobson (2002a) has made it doubtful that it can be done without unmotivated assumptions, and has also shown that certain aspects of functional relative clauses like (9) raise serious if not intractable difficulties for syntactic reconstruction approaches. Furthermore, we would like to heed the general methodological advice of Jacobson (2002b), not to make use of syntactically unwarranted manipulations for the sake of semantic interpretation. Hence the scare quotes when we talk about ‘reconstruction’ relatives: This notion should not be taken literally; it just alludes to a
characterization of this type of relative clauses that is perhaps less bland and more meaningful than other possible terms, like ‘connectivity relative clauses’.

We have mentioned other kinds of apparent reconstruction relatives, such as (9), for which analyses have been developed that can serve as a model for EIR relatives. Based on the Skolem function (or choice function) analysis of questions such as (21) in Groenendijk & Stokhof (1983) and Engdahl (1986), an analysis of functional relative clauses like (22) was suggested in von Stechow (1990) in an unpublished paper, developed in detail by Jacobson (1994, see also 2002a), and extended by Sharvit (1996, 1999).

(21) [Which relative of his, does every boy, like _ ?

(22) [The relative of his, that every boy, likes _ ] is his, mother.

The meaning of (22), interpreted at an index i, can be characterized as in (23), which suggests a wide-scope analysis of the quantifier every boy by LF-movement as in (23.a), by which the head NP relative of his, gets into its scope, as indicated in (b).

(23) a. [every boy, [[the relative of his, that t, likes _ ] is his, mother]

b. \[\forall u [\text{BOY}(u) \rightarrow \exists v [\text{RELATIVE}(u)(v) \land \text{LIKE}(v)(u)] = \text{MOTHER}(u)(v)]\]

The cited authors discuss a number of problems of this analysis, in particular that it assumes movement out of a syntactic island, and predicts wrong truth conditions with quantifiers such as no boy. They suggest instead the following interpretation:

(24) \[tf [\forall u \in \text{DOM}(f) [\text{RELATIVE}(u)(f(u))] \land \forall v [\text{BOY}(v) \rightarrow \text{LIKE}(f(v))(v)] = \text{MOTHER}\]

This says that the function f such that for every u, f(u) is a relative of u, and for which it holds that every boy v likes f(v), is the mother-function (the function that maps every person to his or her mother). Note that this does not require that the universal quantifier over boys takes scope outside of the noun + relative clause construction. The analysis in (24) has to be qualified, as it turns out that we have to restrict the function f to “natural” ones, like the mother-function, or at least to functions that are given in ways beyond random pairings (cf. Jacobson 1994, Sharvit 1996), an issue which need not concern us here.

For EIR relative constructions we propose the following interpretation, which is a structural analogue of the analysis of the functional relative clause in (24).

(25) \[\text{The gifted mathematician Bill supposedly is}[[i] = \text{THE } x [\forall i' \in \text{DOM}(x) [\text{GIFTED MATH}(i')(x(i'))] \land \forall i'' \in \text{SUPPOSED}(i) [\text{BILL}(i'') = x(i'')]]\]

---

8 The most specific term, to our mind, would be ‘equational individual concept relatives’, but obviously this would presuppose the very analysis we would like to argue for.

9 The relation to this analysis of functional relatives, which the seasoned linguists we claim to be should have noticed ourselves when we developed our analysis of EIR relatives, had to be pointed out to us by an anonymous referee.

10 We skip dependency on indices in representations when they are unnecessary in formal phraphrases.
This identifies the individual concept \( x \) such that for every index \( i' \) for which \( x \) is defined, the individual is a gifted mathematician, and in addition for every index \( i'' \) that is compatible with what is supposed to be the case at the index of evaluation \( i \), Bill is identical to \( x \). Where we had functions from individuals to individuals in (24), we now have functions from indices to individuals in (25), and where we had quantification over individuals in (24), we now have quantification over indices in (25). Note that the intensional operator \textsc{supposed} gets into a position in which it apparently scopes over \textsc{gifted math}, as it is guaranteed that for all indices in \textsc{supposed}, \( x \), and therefore BILL, is a gifted mathematician.

The individual concept in (25) is construed from the following set of individual concepts by applying the meaning of the definite article:

\[
(26) \quad \lambda x [\forall i' \in \text{DOM}(x) (\textsc{gifted math}(i')(x(i'))) \land \forall i'' \in \textsc{supposed}(i) (\textsc{BILL}(i'') = x(i''))]
\]

This is the set of individual concepts \( x \) that are gifted mathematicians for all indices \( i' \) for which they are defined, and that are supposed to be identical to Bill in \( i \). How can we derive from this set the individual concept we are after? One option is to assume that \textsc{the} in (25) is the iota operator, which would require the set in (26) to be a singleton set. But is it a singleton set? Notice that \( x \) is unique for all the indices \( i'' \) in \textsc{supposed}(i), by virtue of the equation \( \textsc{BILL}(i'') = x(i'') \). That is, it is guaranteed that for all indices \( i'' \) that are compatible with what is supposed to be the case, \( x \) is identical to Bill, a unique individual. But \( x \) is not necessarily unique for indices \( i'' \) that are outside of \textsc{supposed}(i), as (25) restricts \( x \) only with respect to indices within \textsc{supposed}(i). For example, if \textsc{supposed}(i) = \{i_1, i_2, i_3\}, if \( \textsc{BILL}(i) = b \) for all indices \( i \), if \( b \) is a gifted mathematician in \( i_1, i_2 \) and \( i_3 \), and if \( j \) is a gifted mathematician in \( i_4 \) and \( m \) is a gifted mathematician in \( i_5 \), then (26) applies to at least the following individual concepts:

\[
\begin{align*}
(27) & \quad \{\{i_1, b\}, \{i_2, b\}, \{i_3, b\}\} \\
& \quad \{\{i_1, b\}, \{i_2, b\}, \{i_3, b\}, \{i_2, j\}\} \\
& \quad \{\{i_1, b\}, \{i_2, b\}, \{i_3, b\}, \{i_4, m\}\} \\
& \quad \{\{i_1, b\}, \{i_2, b\}, \{i_3, b\}, \{i_4, j\}, \{i_5, m\}\}
\end{align*}
\]

There are at least two options at this point. One is to discard functions like (27.b,c,d) because they are not constant. This would correspond the idea of using natural functions in functional relative clauses to ensure uniqueness. However, we sometimes might want to have non-constant functions, as the subject of the EIR relative might be truly index-dependent, as in \textsc{the gifted mathematician that the head of the mathematics department (whoever this might be) undoubtedly is} (cf. discussion of (56) below). Hence the better option seems to be to restrict the set of functions to the minimal functions, here (27.a), where minimality can be defined as follows:

\[
(28) \quad \text{Let } S \text{ be a set of functions,} \\
\text{then } \text{min}(S) = \{f \mid f \in S \land \forall g \in S [g \subseteq f \rightarrow g = f]\}
\]

Let us apply minimization to (26):

\[
(29) \quad \text{min}(\lambda x [\forall i' \in \text{DOM}(x) (\textsc{gifted math}(i')(x(i'))) \land \forall i'' \in \textsc{supposed}(i) (\textsc{BILL}(i'') = x(i''))])
\]

We can observe two important facts here. First, the domain of the individual concepts \( x \) in the set (29) is \textsc{supposed}(i). Proof: From the second conjunct it follows that \textsc{supposed}(i) \subseteq \text{DOM}(x), and by minimality it follows that \textsc{supposed}(i) = \text{DOM}(x). Second, there is maximally one such indi-
individual concept. Proof: Assume that there were two, and call them \( x, x^* \). We have for all \( i'' \in \text{SUPPOSED}(i) \), both \( x(i'') = \text{BILL}(i'') \) and \( x^*(i'') = \text{BILL}(i'') \), hence \( x(i'') = x^*(i'') \), and as \( \text{SUPPOSED}(i) = \text{DOM}(x) = \text{DOM}(x^*) \), we have \( x = x^* \), contrary to assumption. Note that it is crucial for this proof to have an equational interpretation of the copula, a point we will return to in section 3.

From the first fact it follows that the individual concept is defined only for the indices compatible with what is supposed to be the case, as stated in (15). From the second, it follows that we can indeed apply the definite article, semantically represented by the iota operator, as uniqueness is guaranteed, if the predicate applies at all. Using the more convenient syntax of \( \iota \) where it applies to a set, with \( \iota(\{x\}) = x \), we can write:

\[
(30) \quad [[\text{the [gifted mathematician] [that Bill supposedly is \_]]}](i) = \iota(\min \left( \lambda x \left[ \forall i' \in \text{DOM}(x)[\text{GIFTED MATH}(i')(x(i'))] \land \forall i'' \in \text{SUPPOSED}(i)[\text{BILL}(i'') = x(i'')] \right) )
\]

A natural question at this point is: What licenses minimization? We suggest that it is licensed pragmatically, by the Gricean maxim of quantity, according to which information is to be maximized. Note that an EIR relative is a definition of an individual concept, and we can assume that, as in all proper definitions, all the necessary information is given. In non-technical definitions, this is typically left implicit; in technical definitions, such as in the typical scheme of recursive definitions, this is often made explicit by saying that elements that can be generated by such-and-such rules are in a set, and nothing else is in the set. Similarly, the specification of the individual concepts by the literal meaning in (26) is strengthened in such a way that it is understood as a complete description, and by this it turns out that there is only one such individual concept.

This contrasts with other uses of relative clauses, such as the gifted mathematician that stood at the blackboard, which do not define an individual concept, but form a description that uniquely identifies an entity, given the context information. Hence we refer to the person that is a gifted mathematician and stood at the blackboard in a way that also encompasses this person when he or she does not stand at the blackboard, or was not a gifted mathematician, as when we continue with: was born under miserable conditions in a refugee camp. This is also why we need no minimality requirement in other cases of ‘reconstruction’ relatives, such as (22), as the MOTHER function is given independently, as a natural function.\(^\text{11}\)

The intensional operator in (30) is a strong one that expresses universal quantification over indices, and it may be doubted that uniqueness results also under intensional operators that express other types of quantification. And indeed, under weak intensional operators we do not get a unique individual concept:

\[
(31) \quad [[\text{[gifted mathematician] [that Bill might (turn out to) be \_]]}](i) = \lambda x \left[ \forall i' \in \text{DOM}(x)[\text{GIFTED MATH}(i')(x(i'))] \land \exists i'' \in \text{EPISTEMIC}(i)[\text{BILL}(i'') = x(i'')] \right)
\]

\(^\text{11}\) It is instructive to compare the use of minimization to achieve uniqueness with the way this is achieved by Sharvit (1996), who discusses functional relative clauses that are not based on natural functions, such as in [the woman that every man met \_ invited him]. Sharvit assumes a rather special interpretation of the quantifier every woman in which the unique minimal witness set of this quantifier is fed into an operator that stipulates that this set becomes the domain of the function to which the functional relative clause refers. We find the pragmatic approach outlined here more attractive, and would like to point out that it can also be applied to the kind of functional relative clause that Sharvit discusses.
The individual concepts \( x \) in this set are individual concepts for which it holds that they are gifted mathematicians for all their indices, as before, and for which there is some epistemically accessible index \( i'' \) for which they are identical to Bill. Now, when we apply the minimization operation to this set, we end up with many “small” individual concepts that are just defined for one index. For example, assume that \( \text{EPISTEMIC}(i) = \{i_1, i_2, i_3\} \), and that Bill is a gifted mathematician in \( i_1 \) and in \( i_2 \), then the individual concept \( \langle \{i_1, b\}, \{i_2, b\} \rangle \) is in the set, but also the individual concepts \( \langle i_1, b \rangle \) and \( \langle i_2, b \rangle \). When we apply minimalization as defined in (28) we end up with a set containing two functions, \( \{\langle i_1, b \rangle\}, \{\langle i_2, b \rangle\} \rangle \), to which the iota operator cannot apply.

Sharvit (1996) discusses equivalent data for functional relative clauses as a welcome result; her examples are the Modern Hebrew equivalents of constructions like The woman that {every man / #most men} invited arrived on time. While we leave aside the issue whether Sharvit’s treatment of this issue is on the right track, observe that EIR relative constructions with weak intensional operators such as (31) are well-formed, as in the following example:

(32) \[
\text{[the [gifted mathematician that Bill might turn out to be _ ]] would be a source of pride for his entire family.}
\]

We can achieve the intended uniqueness in this case if we apply set union on the minimalized set of individual concepts, as follows:

(33) \[
\begin{align*}
&\{[[\text{gifted mathematician}][\text{that Bill might (turn out to) be _ ]}]\}(i) \\
&= \cup \text{min}(\lambda x[\forall i' \in \text{DOM}(x) \text{[gifted mathematician]}(i')(x(i'))] \\
&\quad \wedge \exists i'' \in \text{EPISTEMIC}(i)[\text{Bill}(i'') = x(i'')])
\end{align*}
\]

In the model above, union would result in \( \cup \{\langle i_1, b \rangle\}, \{\langle i_2, b \rangle\} \} \} = \{\langle i_1, b \rangle, \langle i_2, b \rangle\} \} \), which is the intended individual concept (cf. section 5 for the treatment of the definite article). Notice that this is defined for a subset of the meaning of the epistemic operator (which was \( \{i_1, i_2, i_3\} \)), namely only for those for which Bill effectively is a gifted mathematician. This is as it should be, as the VP predication, like \text{would be a source of pride for his entire family}, applies to those worlds \( \langle i_1, i_2, \rangle \), not to \( i_3 \). In cases with strong modal operators such as (30) in which minimization yields a singleton set, first applying the union operation will have the same result. We will return to a motivation of the union operation and to the meaning of the definite article below, in section 5.

We now turn to the last part of our explanation of EIR relative construction, the verbal predicate. The full example (1) will receive the following interpretation:

(34) \[
\begin{align*}
&\{[[\text{the [gifted mathematician}][\text{that Bill supposedly is _ ]}] \\
&\text{[should have solved the problem]]\}(i) \\
&= \exists i'' \in \text{EXPECT}(i)[\text{ABLE TO SOLVE THIS PROBLEM}(i'')] \\
&\quad \cup \text{min}(\lambda x[\forall i' \in \text{DOM}(x) \text{[gifted mathematician]}(i')(x(i'))] \\
&\quad \wedge \exists i'' \in \text{SUPPOSED}(i)[\text{Bill}(i'') = x(i'')])
\end{align*}
\]

This states that for all indices \( i'' \) that are expectable with respect to the index of evaluation, \( i \), it holds that the value of the individual concept ‘the gifted mathematician that Bill supposedly is’ defined above is able to solve the problem. We have already seen in the discussion of (17) that this gives us the desired result, that the indices accessible via the \text{EXPECT} operator are linked to the ones accessible via the \text{SUPPOSED} operator through the domain of the individual concept \( x \). More precisely, it must be the case that \text{EXPECT}(i) is a subset of \text{SUPPOSED}(i), the set of indices \text{DOM}(x) for
which the individual concept exists. Assuming that the modal background of EXPECT can be accommodated, it will be accommodated in such a way as to satisfy the requirement $\text{EXPECT}(i) \subseteq \text{SUPPOSED}(i)$, that is, it will express an expectation on the basis of the indices of what is supposed to be the case.

2.3 Compositional Derivation of EIR Relatives

In the last subsection we developed the essentials of our analysis of EIR constructions. In this subsection we will show that a compositional derivation of the meanings involved is possible if one assumes certain type shifts, all of which are independently motivated. We will analyze our main example, (1), to a sufficiently detailed degree, showing that the meaning of each complex constituent can be described as the result of a combination of the meanings of its immediate syntactic parts.

It already became evident that the framework we are adopting makes heavy use of individual concepts. This is by no means novel; it was a crucial feature of Montague (1973), who assumed that common nouns and verbal predicates apply to individual concepts (cf. also Löbner 1979). Montague showed that in many cases predicates over individual concepts can be reduced to predicates over individuals via meaning postulates, whereas some predicates such as temperature or change are irreducible. We would like to suggest that we can entertain a more flexible system in which predicates typically apply to individuals, but can be type-lifted to predicates that apply to individual concepts. If $P$ is a predicate of individuals, then it can be type-lifted to a predicate that applies to an individual concept $x$ if for all indices $i$ for which $x(i)$ is defined, $P$ applies to $x(i)$ at $i$. That is, in addition to the regular meaning of gifted mathematician in (35.a), we also assume the type-shifted meaning in (b).

(35) $\text{[gifted mathematician]}(i)$
   a. $\lambda u [\text{GIFTED MATH}(i)(u)]$
   b. $\lambda x \forall i' \in \text{DOM}(x) [\text{GIFTED MATH}(i')(x(i'))]$

This corresponds exactly to the type-shift assumed for functional relative clauses like the woman that every man invited in Jacobson (1994) and Sharvit (1996) from the predicate over individuals $\lambda u [\text{WOMAN}(u)]$ to the predicate over functions to individuals $\lambda f \forall u \in \text{DOM}(f) [\text{WOMAN}(f(u))]$, which lends support to this kind of type-lift to functions as a general operation.

12 More specifically, temperature is a functional noun that maps, for any index $i$, its individual argument $u$ to the temperature of $u$ at $i$. Lasersohn (2005) has argued that the need for a special treatment of temperature and price can be eliminated for the examples Montague had in mind, like The temperature is rising, and we concur. Another application has been proposed by Gupta (1980) to model criteria of identity originating from nouns. For example, Gupta argued that one and the same person can represent different passengers, thus allowing a representation of sentences like National Airlines flew 6 million passengers last year, where one and the same person may count as more than one passenger. We don’t think that this use of individual concepts is on the right track, as we have a similar reading of sentences like National Airlines flew 6 million persons last year. See Krifka (1990) for discussion.

13 For another use of this type shift, consider the basic predicative meaning of female: $\text{FEMALE} = \lambda u [u \text{ is female}]$. We also may want to form a predication over functional terms, as in Nieces are female. Using a shifted version of the predicate does the trick, as in $\forall f [\text{NIECE}(f) \rightarrow \text{FEMALE}(f)]$, where $\text{FEMALE'} = \lambda f \forall u \in \text{DOM}(f) [\text{FEMALE}(f(u))]$.

14 An anonymous reviewer argued that this type shift might also account for concealed questions, as in John guessed [the price of milk], in which the object DP denotes a function mapping indices $i$ to the price of milk at $i$. Why is it, then, that concealed questions cannot be formed with non-relational nouns like *John guessed [the gifted mathematician]? The reason is, it seems to us, that concealed questions in general are restricted to functions with measure terms as values, like price, temperature, height etc.; for example, John guessed Miss America does not have the reading ,John
We now turn to the formation of the relative clause \([that \; Bill \; supposedly \; is \; _]\). As we have seen, it is crucial that the copula is interpreted as identity of individual concepts. This copula can be reduced to individuals, as is the case for the copula in the following clause:

\[
(36) \quad \begin{align*}
[Cicero \; is \; Tullius](i) \\
= & \; IS(i)(TULLIUS)(CICERO) \\
= & \; \lambda x \lambda y[y(i) = x(i)](TULLIUS)(CICERO) \\
= & \; CICERO(i) = TULLIUS(i)
\end{align*}
\]

We follow standard assumptions about the interpretation of relative clauses: The gap of the relative clause is interpreted by a variable of the appropriate type (here, the type of individual concepts, see\(^{15}\)) that is lambda-abstracted to form a predicate.

\[
(37) \quad \begin{align*}
[that \; [Cicero \; is \; t_{se,1}]](i) \\
= & \; \lambda x \lambda y[I(i)(x, y)(CICERO)] \\
= & \; \lambda x \lambda y[x(i) = y(i)] \\
= & \; \lambda x \lambda y[y(i) = x(i)] \\
= & \; CICERO(i) = TULLIUS(i)
\end{align*}
\]

Modal operators like supposedly take a proposition as an argument, and the context typically provides them with some restriction of the modal base. The modal adverbial supposedly may appear as a sentence adverbial, or as a VP adverbial, a difference that we disregard here.

\[
(38) \quad \begin{align*}
[Cicero \; supposedly \; is \; Tullius](i) \\
= & \; \forall i \in \text{SUPPOSED}(i)[CICERO(i) = TULLIUS(i)]
\end{align*}
\]

We can combine (39) with the type.lifted meaning of the head noun gifted mathematician, as given in (35.b), according to the standard conjunctive rule of relative clauses:

\[
(40) \quad \begin{align*}
[[NP \; [NP \; gifted \; mathematician] \; [CP \; that, \; [Bill \; supposedly \; is \; t_{se,1}]]]](i) \\
= & \; \lambda x \left[ \forall i \in \text{DOM}(x) [\text{GIFTED MATH}(i(i)) \land \forall i' \in \text{SUPPOSED}(i)[\text{BILL}(i') = x(i)]] \right]
\end{align*}
\]

\(^{15}\) We assume e, t, s as basic types for entities, truth values and indices (worlds and times), respectively. The type of (possibly partial) functions from entities of type \(\sigma\) to entities of type \(\tau\) is given by \((\sigma)\tau\), and by \(\sigma\tau\) if \(\sigma\) is a basic type. This format (cf. Link 1979) allows for a more concise notation of complex types than the usual notation by an ordered pair, \((\sigma, \tau)\).
This concludes our sketch of a compositional interpretation of EIR relatives. It should be stated here that surprisingly little has to be assumed in addition to standard assumptions about the interpretation of relative clauses, and the little that has to be assumed – type raising of the head noun, the possibility of having traces or arguments of the type of individual concepts, the minimization and the union operator – appear to be well motivated.

3. THE EQUATIONAL CHARACTER OF THE COPULAR SENTENCE

In our analysis of EIR relatives we have assumed that the copula expresses an equation. It relates two individual concepts by identity, IS(i)(BILL)(x), which is true iff BILL(i) = x(i). This analysis may seem at odds with our intuitive understanding of EIR relative constructions insofar as they are felt to attribute a property to an individual, rather than to equate two potentially distinct individuals. For example, (1) is felt to attribute to Bill, in a modalized way, the property of being a gifted mathematician, rather than to assert Bill’s identity to some individual that is a gifted mathematician.

It is, however, hard to see how an appropriate meaning for EIR relatives could be construed on the basis of a predicative copular structure, at least under the standard assumption that the meaning of the relative is construed on the basis of its gap. In equational copular structures, the subject and the non-subject are assigned the same semantic type, while in predicational structures, the non-subject is of a higher type – if the subject is of type τ, the non-subject is of type τ′, the type of sets of entities of type τ, or rather of type σττ, the intension of such sets. In the case of an EIR relative, we would need to assign the gap to the type of predicates if we assume the copular structure to be predicative.

(42) [[CP that, [Bill is t_{sel,1}]]][i)](i), under predicational analysis: λP_i[P_i(i)(BILL(i))]

Let us call such structures property relative clauses. We cannot combine this meaning with the standard interpretation of the NP according to the regular rule for relative clauses. But a combination relying on the rule of functional application is possible:

(43) [[[NP gifted mathematician] [CP that, [Bill is t_{sel,1}]]][i]](i)
    = [[[CP that, [Bill is t_{sel,1}]]][i][[NP gifted mathematician]][i]](i)
    = λP_i[λu[P_i(i)(BILL(i))]|GIFTED MATH(i)(u)]
    = GIFTED MATH(i)(BILL(i))

However, this results in a meaning of the type of propositions, and not a meaning to which a determiner could apply. So we have to rule out this derivation for EIR relatives. But interestingly, strings like (43) do occur in English as an absolute construction with precisely this interpretation, as in (44).

(44) Gifted mathematician that Bill is, he solved the problem in no time.
    presupposed: (43), asserted: ‘Bill solved the problem in no time’

The presuppositional character of the italicized proposition might be related to the fact that it does not constitute a finite clause, hence cannot be the main communicative point of the whole utterance. There is also an implicature that the property of being a gifted mathematician was a cause for Bill’s
quick solution of the problem; this the sentence shares with other juxtapositions of sentences, as in 
*Bill will solve the problem, he is a gifted mathematician.*

If type-lifting the relative clause does not result in a possible interpretation for EIR relatives, then perhaps type-lifting the nominal predicate will do the trick. Lifting it to a predicate of individual concepts is of no help, but we might assume a type lift to a singleton set containing the original meaning of the nominal predicate, \( \lambda P[P = \text{GIFTED MATH}] \). In this case we arrive at the following meaning, according to the usual intersective rule of relative clause interpretation.

\[
\begin{align*}
\text{(45)} & \quad [\left[ [\text{NP gifted mathematician}] \left[ \text{CP that} \left[ \text{Bill is t}_{\text{set,1}} \right] \right] \right] ](i) \\
& \quad = \lambda P[P = \text{GIFTED MATH} \land P(i)(\text{BILL}(i))]
\end{align*}
\]

This is a second-order predicate that applies to exactly one property, namely to the property of being a gifted mathematician, provided that Bill is one, and to the empty predicate in case Bill is not, thus effectively creating a presupposition that Bill is a gifted mathematician. We know of no way to derive from that a meaning that refers to Bill, or to an individual concept related to Bill, and so this again appears to be a *cul de sac* for a possible derivation of the meaning of EIR relative clauses. But again, there is a relative clause construction that appears to be based on precisely this derivation, and which we may call **equational property relative clause** construction:

\[
\begin{align*}
\text{(46)} & \quad \text{Abdul is finally} \\
& \quad [\text{the naturalized American that his mother always wanted him to be}]
\end{align*}
\]

\[
\begin{align*}
\text{(47)} & \quad \text{Bill is [the gifted mathematician that his mother was} \\
& \quad \text{].}
\end{align*}
\]

Equational property relatives differ from EIR relatives; they occur within a copular sentence, and there is no requirement for a modal operator within the relative clause. We can derive the meaning of (47) as follows:

\[
\begin{align*}
\text{(48)} & \quad \text{a. } [\left[ \text{CP that} \left[ \text{his mother was t}_{\text{set,1}} \right] \right] ](i) = \lambda P_1[P_1(i)(\text{BILL’S MOTHER}(i))] \\
& \quad \text{b. } [\left[ \text{NP gifted mathematician} \right] ](i), \text{ after lifting: } \lambda P[P = \text{GIFTED MATH}] \\
& \quad \text{c. } [\left[ \text{NP [NP gifted mathematician] [CP that} \left[ \text{his mother was t}_{\text{set,1}} \right] \right] ](i) \\
& \quad = \lambda P[P = \text{GIFTED MATH} \land P(i)(\text{BILL’S MOTHER}(i))] \\
& \quad \text{d. } [\left[ \text{DP the [NP [NP gifted mathematician] [CP that} \left[ \text{his mother was t}_{\text{set,1}} \right] \right] ](i) \\
& \quad = \text{GIFTED MATH,} \\
& \quad \text{under the presupposition that GIFTED MATH}(i)(\text{BILL’S MOTHER}(i))}
\end{align*}
\]

---

16 This interpretation is quite similar to the interpretation of topicalization constructions, as in *John, Mary likes _,* in which the meaning of *Mary likes*, \( \lambda x[\text{LIKES}(x)(\text{MARY})] \), is applied to *JOHN*. Also, notice that the presuppositional nature and the conventional implicature is also present in other absolute constructions, such as *Being a gifted mathematician, Bill solved the problem in no time*, or *A gifted mathematician, Bill solved the problem in no time*.

17 This type of relative clause construction appears to occur far more frequently than EIR relatives, and sometimes it is not easy to distinguish from them, as in the following examples:

(a) If you were the tt “expert” you claim to be you would have known about this system
(http://skepdic.com/comments/ttcom.html, retrieved Sep 30, 2007)

(b) And he isn’t the straight talker he claims to be
(www.blog-city.com/community/tagshare/?mccain, retrieved Sep 30, 2007)
This predicate is applied to the subject, BILL, in the regular fashion. Note that the main clause is predicational by nature. Being a predicate, the complex DP in (46) or (47) could not end up in a referential argument position, in sharp contrast to EIR relatives.

Summarizing what has been achieved so far in this section, we see that relative clauses with predicational gaps exist, but that they are distinct from EIR relatives. As our attempts to explain EIR relatives as predicational structures have lead nowhere, we think we have made a strong case for the equational analysis of EIR relatives.

In the remainder of this section we will point out three other types of relative clauses that show superficial similarities to EIR relatives. We start with the property relative clauses that are arguments of non-copular predicates, which is possible provided that they are non-referential:

(49) Bill wants to marry the blonde, blue-eyed woman that his mother was.

(50) Bill wants to marry the blonde, blue-eyed woman that his father married.

Both examples have non-incestuous readings; (49) does not mean that Bill wants to marry his own mother, and (50) does not mean that he wants to marry the wife of his father; rather, Bill wants to marry a blonde, blue-eyed woman like his mother or his father’s wife, respectively. To construct the meaning of the object DP, we have to assume that the meaning of marry can be type-lifted so that it can take a property as its argument (cf. van Geenhoven 1998, McNally & van Geenhoven 2005). In the following, we give a partial derivation of (50), where both occurrences of marry are interpreted as applying to property arguments.

\[
\begin{align*}
(51) \quad &a. \quad [\text{married}]^{(i)}(i) \\
&= \lambda P_2 \lambda x \exists y [P_2(y) \land \text{MARRIED}(i)(y)(x)]
\end{align*}
\]

\[
\begin{align*}
b. \quad &[[\text{CP} \ (\text{that}_1) \ \text{his father married} \ t_{set,1}]]^{(i)}(i) \\
&= \lambda P_3 \exists y [P_3(y) \land \text{MARRIED}(i)(y)(\text{BILL’S FATHER})]
\end{align*}
\]

\[
\begin{align*}
c. \quad &[[\text{NP blonde, blue-eyed woman}] \ [\text{NP blonde, blue-eyed woman}] \ [\text{CP} \ (\text{that}_1) \ \text{his father married} \ t_{set,1}]]^{(i)}(i) \\
&= \lambda P[P = \lambda i \lambda x [\text{BLOND}(i)(x) \land \text{BLUE-IFIED}(i)(x) \land \text{WOMAN}(i)(x)]
\end{align*}
\]

\[
\begin{align*}
d. \quad &[[\text{NP blonde, blue-eyed woman}] \ [\text{CP} \ (\text{that}_1) \ \text{his father married} \ t_{set,1}]]^{(i)}(i) \\
&= \lambda P[\lambda P = \lambda i \lambda x [\text{BLOND}(i)(x) \land \text{BLUE-IFIED}(i)(x) \land \text{WOMAN}(i)(x)]
\end{align*}
\]

\[
\begin{align*}
&= \lambda P[P = \text{BBW}], \text{for short}
\end{align*}
\]

\[
\begin{align*}
&= \{\text{BBW}\}, \text{under the presupposition that Bill’s father married a BBW}
\end{align*}
\]

---

18 We wish to point out that one objection that might be raised against the equational analysis does not go through. Thus, one might try and make the equative construal of copular clauses explicit, as in Cicero is identical to Tullius, or Cicero is the same person as Tullius, and one might expect to be able to use this locution in EIR relatives, contrary to fact, cf. The gifted mathematician [that Bill supposedly is {identical to / the same person as} ... ] should be able to solve this problem. This sentence can only be understood in such a way that the possibility is entertained that there are two persons, a gifted mathematician, and Bill, and that they may be one and the same person. This is different from the EIR relative in (1), which does not presuppose that there might be another person that Bill might be identical with. It appears that the locution is identical to can only be used if the two individual concepts it relates are given independently, which is precisely not the case with EIR relatives.
d. \[ \left[ \left[ DP \right] \right] \left[ \left[ NP \right] \left[ \left[ CP \right] \right] \right] \left[ \left[ \left[ \left[ CP \right] \left( that \right) \right] \left( his \ right) \left( father \ right) \left( married \right) \right] \right] \left[ t_{set,1} \right] \right] \right] \left[ \right] \left( i \right) \]
\[ = \left( \lambda P \left[ P = BBW \land \exists y \left[ P(i)(y) \land MARRIED(i)(y)(BIL\acute{\text{L}}'S \ FATHER) \right] \right] \right) \]
\[ = BBW, \ under \ the \ presupposition \ that \ Bill's \ father \ married \ a \ BBW. \]

e. \[ \left[ marry \right] \left[ \right] \left( i \right) \]
\[ = \lambda P \lambda x \exists y \left[ P(i)(y) \land MARRIED(i)(y)(x) \right] \]
\[ \text{(again, property argument reading)} \]

f. \[ \left[ marry \left[ \right] \left[ \left[ \left[ \left[ CP \right] \right] \right] \right] \left[ \left[ \left[ \left[ \left[ CP \right] \left( that \right) \right] \left( his \ right) \left( father \ right) \left( married \right) \right] \right] \right] \left[ \right] \right] \right] \left[ \right] \left( i \right) \]
\[ = \lambda x \exists y \left[ BBW(i) \land MARRIED(i)(y)(x) \right], \]
\[ \text{under the presupposition that Bill's father married a BBW} \]

The second type of relative constructions that is easily confused with EIR relatives was mentioned at the beginning of this paper, cf. (3). This construction consists of generic DPs that refer to a subkind, as in Bill is the same mathematician his mother was. The copula construction that this type of sentence is based on is the one we find in sentences like His mother was the kind of mathematician that was good at geometry but bad at arithmetics.\textsuperscript{19} We assume that this construction presupposes a taxonomic hierarchy and refers to either a kind or a property that stands in a taxonomic relation to the kind or property named by the head noun. If we assume a property analysis of subkinds, we can assume a relation \textsc{subkind} that maps a property (like the mathematician property) to a subproperty of this property, one that applies to entities that share an additional feature.

\[ \text{(52)} \]

a. \[ \left[ \left[ CP \left( that \right) \left( his \ right) \left( mother \ right) \left( was \right) \right] \right] \left[ t_{set,1} \right] \right] \left( i \right) \]
\[ = \lambda P \left[ P(i)(BIL\acute{\text{L}}'S \ MOTHER(i)) \right] \]
b. \[ \left[ \left[ NP \left( kind \ of \right) \left( mathematician \right) \right] \right] \left( i \right) \]
\[ = \lambda P \left[ \textsc{subkind}(i)(MATH)(P) \right] \]
c. \[ \left[ \left[ NP \left[ NP \left( kind \ of \right) \left( mathematician \right) \right] \right] \left[ CP \left( that \right) \left( his \ right) \left( mother \ right) \left( was \right) \right] \right] \left[ t_{set,1} \right] \right] \left( i \right) \]
\[ = \lambda P \left[ \left[ \left[ \left[ \left[ \left[ \left[ \textsc{subkind}(i)(MATH)(P) \right] \right] \land P(i)(BIL\acute{\text{L}}'S \ MOTHER(i)) \right] \right] \right] \right] \right] \]

With the standard intersective semantics for relative clauses, we arrive at a set of predicates P that define subkinds of mathematicians (which entails that P \subseteq \textsc{Mathematician}) such that they also apply to Bill’s mother. The definite article either presupposes that there is exactly one such subkind, or identifies the most general such subkind. In any case, this appears to lead to an adequate semantics for sentences like (3), which we may call \textit{subkind property relative clauses}.

The third type concerns relative clause constructions like Bill is (twice) the mathematician his mother was\textsuperscript{20}. In such cases we talk about degrees of mathematician-hood that can be measured. We

\textsuperscript{19} Real-life examples of this type of construction are easy to find, and again some of them might be mixed up with EIR relatives:
(a) If Dole were truly the leader he claims to be, he would be seeking to bring the G.O.P. back to his brand of pragmatism.
(b) If he is the kind of Christian he claims to be, he should let everyone have a go.
(www.b3ta.com/board/5913327, retrieved Sep 30, 2007)

\textsuperscript{20} Examples of this type:
(a) If he was twice the man he is, he would still be only half as intelligent as a politician and a tenth as popular as a lawyer. (rafman.deviantart.com/, retrieved Sep 30, 2007)
can capture this in a similar way as with the SUBKIND-relation assumed above, namely by a DEGREE relation; in a sense, SUBKIND gives us a qualitative scale, and DEGREE a quantitative scale. In the following, DEGREE(i)(MATH) applies to properties P that are equally good in their mathematical abilities.

\[(53)\]

a. \[\llbracket \text{CP} (\text{that}_t \text{ his mother was } t_{\text{set},1}) \rrbracket (i) = \lambda P_1[P_1(i)(\text{BILL's mother}(i))]\]

b. \[\llbracket \text{NP} [\text{mathematician}_t] \rrbracket (i) = \lambda P[\text{DEGREE}(i)(\text{MATH})(P)]\]

c. \[\llbracket \text{NP} [\text{mathematician}_t] \llbracket \text{CP} (\text{that}_t \text{ his mother was } t_{\text{set},1}) \rrbracket \rrbracket (i) = \lambda P[\text{DEGREE}(i)(\text{MATH})(P) \land P(i)(\text{BILL's mother}(i))]\]

d. \[\llbracket \text{DP} \llbracket \text{the} \llbracket \text{NP} [\text{mathematician}_t] \llbracket \text{CP} (\text{that}_t \text{ his mother was } t_{\text{set},1}) \rrbracket \rrbracket \rrbracket (i) = \lambda P[\text{DEGREE}(i)(\text{MATH})(P) \land P(i)(\text{BILL's mother}(i))]\]

e. \[\llbracket \text{DP} \llbracket \text{twice} \llbracket \text{DP} \llbracket \text{the} \llbracket \text{NP} [\text{mathematician}_t] \llbracket \text{CP} (\text{that}_t \text{ his mother was } t_{\text{set},1}) \rrbracket \rrbracket \rrbracket \rrbracket (i) = 2 \lambda P[\text{DEGREE}(i)(\text{MATH})(P) \land P(i)(\text{BILL's mother}(i))]\]

We assume for simplicity that degrees are unique, that is, only one degree P of a given dimension, like mathematician-hood, can apply to an individual; then the definite article can be expressed by the iota operator (otherwise, we would have to assume a maximality operator). Furthermore, we assume that for degrees certain operations like addition and multiplications are defined. An appropriate name for such cases, then, is degree property relative clause.

This concludes our discussion of related types of relative clauses and brings us back to the central concern of this article.  

4. THE COPULAR SUBJECT WITHIN EIR RELATIVES

So far we have confined our discussion to EIR relatives whose subject is a singular proper name. Here we will consider cases in which the copular subject is something other than a name; of particular interest will be quantifiers and their scopal properties. We begin with a case in which the subject denotes a sum individual:

\[(54)\]  
[The gifted mathematicians that \{Bill and Mary, the Johnsons\} clearly are _ ]  
have undoubtedly won many distinctions in the course of their careers.

(b) Flintoff would not be half the player he is today (...)  
(www.cricket365.com/story/0,18305,6673_2754596,00.html, retrieved Sep 30, 2007)

21 For completeness, we note that EIR relatives share with the other relative clauses discussed here the property that they do not allow the full range of wh-pronouns as complementizers, but generally prefer that or the empty complementizer (cf. *the gifted mathematican who Bill claims to be and Bill wants to marry the blonde woman who his father married, which only has an incestual reading). We see these restrictions as the result of a restriction of who to (human) individuals of type e. The relative pronoun which also appears to be possible with property relatives, a fact that does not come as a surprise as it is compatible with property antecedents in general, as in John is intelligent, which his brother is not.
Treatment of such examples is straightforward once we allow for sum individuals and individual concepts that map indices to sum individuals. We take sum individuals to be of the semantic type of individuals, e, hence the trace of their individual concepts is of type se:

$$(55) \quad [[[\text{gifted mathematicians} \ [\text{that, Bill and Mary clearly are t}_{w,1}]]](i)$$

$$= \lambda x[\forall i' \in \text{DOM}(x)[\text{GIFTED MATH}(i')(x(i'))]$$

$$\quad \land \forall i'' \in \text{CLEARLY}(i)[\text{BILL}(i'') \oplus \text{MARY}(i'') = x(i'')]]$$

After minimization, cf. (28), this is a singleton set containing the individual concept x that maps each index in its domain to a sum individual that falls under the predicate GIFTED MATHS at that index (the $-$S in GIFTED MATHS should indicate that this is a predicate over sum individuals).

Furthermore, as expressed by the second conjunct, for all indices that are compatible with what is clearly the case, x is identical to the sum individual of Bill and Mary, which means that x consists of two atomic parts, one being identical to Bill, and one to Mary. To illustrate, if CLEARLY(i) = \{i_1, i_2, i_3\}, and Bill and Mary (b and m) are gifted mathematicians in these worlds, then this is the singleton set containing the function \{(i_1, b@m), (i_2, b@m), (i_3, b@m)\}.

Our next example contains a **definite description** whose extension varies with the chosen index:

$$(56) \quad [[[\text{The gifted mathematician that the head of the department certainly is } _ ]$$

will be able to solve this simple problem.

There is a de-re reading referring to the head of the department that is independently given by the context, and a de-dicto reading referring to whoever is the head of the department at a given index. In the de-re reading, the content of the description is evaluated with respect to the index of evaluation. This results in an interpretation that is essentially like the case of EIR relatives with proper names. In the de-dicto reading, the content of the description is evaluated with respect to the local index of the modal operator that c-commands the description.

$$(57) \quad [[[\text{gifted mathematician}] \ [\text{that, the head (of the department) certainly is } t_{w,1}]]](i)$$

\hspace{1cm} de re: $\quad \lambda x[\forall i' \in \text{DOM}(x)[\text{GIFTED MATH}(i')(x(i'))]$$

$$\quad \land \forall i'' \in \text{CLEARLY}(i)[\text{HEAD}(i''(u)) = x(i'')]]$$

\hspace{1cm} de dicto: $\quad \lambda x[\forall i' \in \text{DOM}(x)[\text{GIFTED MATH}(i')(x(i'))]$$

$$\quad \land \forall i'' \in \text{CLEARLY}(i)[\text{HEAD}(i''(u)) = x(i'')]]$$

In contrast to the cases considered so far, in the de-dicto reading the individual concepts x are not constant functions, but identify whoever is head of the department for the indices in the set CLEARLY(i). Everything else is equal; minimalization will give us a singleton set to which the iota operator can apply. To illustrate, let us assume that a, b, c, d, e are the heads of the mathematics department in the worlds $i_1, i_2, i_3, i_4, i_5$, respectively, that CLEARLY(i) = \{i_1, i_2, i_3\}, and that a, b, c are gifted mathematicians in $i_1, i_2, i_3$, respectively, then the description **gifted mathematician that the head of the mathematics department certainly is** denotes the set containing the four individual concepts in (58), and minimalization will identify the singleton set containing (58.a).

$$(58) \quad \begin{align*}
\text{a. } & \{(i_1, a), (i_2, b), (i_3, c)\} \\
\text{b. } & \{(i_1, a), (i_2, b), (i_3, c), (i_4, d)\} \\
\text{c. } & \{(i_1, a), (i_2, b), (i_3, c), (i_5, e)\} \\
\text{d. } & \{(i_1, a), (i_2, b), (i_3, c), (i_4, d), (i_5, e)\}
\end{align*}$$
We now turn to cases in which the copular subject is a **quantifier**, and we begin with one that features a plural quantifier:

(59)  [The heroic fighters that most soldiers in this unit unquestionably are _] will certainly receive a medal of honor.

We analyze the quantifier *most* as an existential quantifier that introduces a sum individual that is a part of the sum individual of all soldiers, and larger than its complement. Hence MOST(SOLDIERS(i))(u) is true iff u is a sum individual for which it holds that u ≤ ⊕SOLDIERS(i), and #(u) > #(⊕SOLDIERS(i) − u).

\[
\begin{align*}
&[[\text{heroic fighters}] \ [\text{that, most soldiers (in this unit) unquestionably are } \text{t}_{\text{xei}}]](i) \\
&= \lambda x[\forall i' \in \text{DOM}(x)[\text{HEROIC FIGHTERS}(i')(x(i')) \\
& \quad \land \exists u[\text{MOST}(\text{SOLDIERS}(i))(u) \land \forall i'' \in \text{UNQUESTIONABLE}(i)[u = x(i'')])]
\end{align*}
\]

The first conjunct restricts the individual concepts x to those that map each index in their domain to a certain number of heroic fighters, and that for all i” that are unquestionably the case, u is identical to x(i”).

Let us look at this more carefully, considering a model in which there are five soldiers in i, namely the individuals a, b, c, d, e, four of which are unquestionably heroic fighters, namely a, b, c, d. Assume that UNQUESTIONABLE(i) = {i_1, i_2}. We denote sum individuals like a⊕b by ab, for short. Note that five individuals exist that qualify for MOST(i)(SOLDIERS) and for HEROIC FIGHTERS, namely abc, abd, acd, bcd and abcd. Minimization of (60) will give us the set that contains the following twenty-five functions:

\[
\begin{align*}
&\{(i_1, abc), (i_2, abd), \ldots, (i_1, abc), (i_2, abd)\}, \\
&\{(i_1, abd), (i_2, abc), \ldots, (i_1, abd), (i_2, abc)\}, \\
&\{(i_1, acd), (i_2, abc), \ldots, (i_1, acd), (i_2, abc)\}, \\
&\{(i_1, bcd), (i_2, abc), \ldots, (i_1, bcd), (i_2, abc)\}, \\
&\{(i_1, abd), (i_2, abc), \ldots, (i_1, abd), (i_2, abc)\}
\end{align*}
\]

What we want is the very last one of these functions, \{(i_1, abc), (i_2, abd)\}, which is the biggest: it represents the individual concept that identifies the sum of all soldiers that are unquestionably heroic fighters, and (59) says that all of the heroic fighters among the soldiers in this unit will certainly receive a medal of honor.

We can achieve this with the help of a **sum operation** over the set of individual concepts in (61), which is defined as follows:

(62)  If x and y are individual concepts with the same domain, \\
then \(x \oplus y = \{(i, u) \mid u = x(i) \oplus y(i)\}\)

We write ⊕S for the sum of all individual concepts in S, provided that S is a set of individual concepts with the same domain. More precisely, ⊕ can be defined as follows:

\[
\text{\textbf{82} This analysis is to be preferred over the standard Generalized Quantifier analysis of most. It allows us to explain cases in which most is combined with a mass noun, as in most (of the) water is poisoned, and cases in which most allows for collective predications, as in most students gathered in the hallway. Furthermore, it explains why anaphoric relations are possible with most+NP antecedents, as in Most students were late. They apologized.}
\]
(63) a. If \( x, y \) are two individual concepts, then \( x \leq y \) iff \( \forall i[x(i) \leq y(i)] \)

b. \( \oplus S = x \) iff \( \forall y \in S[y \leq x \land \forall z[y \leq z \rightarrow z \leq x] \)

(64) \[ \left[ \left[ \text{the [heroic fighters]} \right] \left[ \text{that, most soldiers (in this unit) unquestionably are } t_{se,1} \right] \right] \right] (i) \\
= \oplus \min(\lambda x[\forall i' \in \text{DOM}(x)[\text{HEROIC FIGHTERS}(i')(x(i'))] \\
\land \exists u[\text{MOST(SOLDIERS}(i))(u) \land \forall i'' \in \text{UNQUESTIONABLE}(i)[u = x(i'')]]) \]

In the model given above, this will single out the largest individual concept, \( \{i_1, \text{abcd}\}, \{i_2, \text{abcd}\} \).

Notice that in cases with a referring subject within the EIR relative clause, such as gifted mathematician that Bill supposedly is, the operation will get us the right result as well, as the set of individual concepts is a singleton set in this case. We also get a singleton set for the nominal constructions in cases like the following in which the head noun is modified by a number word (where \( |u| \) specifies the number of atoms that a sum individual consists of).

(65) \[ \left[ \left[ \text{the [two gifted mathematicians]} \right] \left[ \text{that Bill and Mary supposedly are } \_ \right] \right] \right] (i) \\
= \oplus \min(\lambda x[\forall i' \in \text{DOM}(x)[\text{GIFTED MATH}(i')(x) \land |x(i')| = 2] \\
\land \forall i'' \in \text{SUPPOSED}(i)[\text{BILL}(i'') \oplus \text{MARY}(i'') = x(i'')]) \]

Let us now turn to scope preferences of quantificational subjects. In (60) we have assumed that most soldiers scopes over the intensional operator, unquestionably. There is a strong tendency for wide-scope interpretation of quantifiers in EIR relative clauses, which can be appreciated by comparing (59) with the less felicitous (66), where the quantifier is preferably interpreted with narrow scope:

(66) \#[The heroic fighter that it seems that someone or other is _ ] \\
will certainly receive a medal of honor.

The reason for this, we suggest, concerns the raison d’être of EIR relatives revealed in full below in (101). EIR relatives state something about an individual concept under a particular aspect that may not hold at the index under which the sentence is evaluated. For example, (1) states something about Bill under the additional assumption that he is a gifted mathematician, which may not hold at the index of evaluation. While our analysis of the meaning of the EIR relative clause in (15) or (30) yields an individual concept that is strictly defined only for indices in the domain of the modal operator, this is supposed to be a restricted or intentionally limited version of an individual concept that is also defined for the index of evaluation. With names, as in the gifted mathematician Bill supposedly is, this is straightforward when we assume that Bill denotes a constant function. The same holds for wide-scope quantifiers and for definite descriptions. The situation is different, however, for narrow-scope quantifiers. There is no guarantee that the individual concept defined for the indices accessible for the modal operator can be suitably extended to the index of evaluation, or to any index other than those introduced by the modal operator. For example, a sentence like It seems that someone or other is a heroic fighter does not guarantee that there is an individual outside worlds of appearance such that its version in worlds of appearance is a heroic fighter.

We turn to an example with a singular quantifier:

(67) [The heroic fighter that every (single) soldier in this unit certainly is _ ] \\
will hopefully do his utmost to defend the fatherland.

The technique used for plural quantifiers that relies on sum individuals cannot be used in this case. But examples like this one clearly are functional relative clauses of the type discussed by Jacobson (1994) and Sharvit (1996, 1999), and so they should be treated as a combination of classical functional relative clauses and EIR relatives. In the example at hand, we have a function that maps every
soldier $u$ to an individual concept $x$ that in turn maps indices $i$ to $u$. That is, the type of the trace is $\text{ese}$, and we get the following meaning of the NP modified by the relative clause:

$\lambda f [\forall u \in \text{DOM}(f) \ \forall i' \in \text{DOM}(f(u)) [\text{HEROIC FIGHTER}(i')(f(u)(i'))] \ \land \\
\forall u \in \text{SOLDIER}(i) \ \forall i'' \in \text{CERTAIN}(i) [u = f(u)(i'')]]$)

The functions $f$ in this set map individuals to individual concepts, which in turn map indices to entities. This embodies the combined functional dependency, on individuals and indices. The first conjunct states that for all entities $u$ in the domain of $f$, $f(u)$ is an individual concept such that for all indices $i'$ in the domain of $f(u)$, the value of the individual concept $f(u)$ in $i'$ is a heroic fighter in $i'$. The second conjunct determines that for all soldiers in this unit $u$ it holds that for all indices $i''$ that are compatible with what is certain, $u$ is identical to $f(u)(i'')$. In order for this condition to be satisfied, the domain of $f$ must contain the soldiers at $i$, $\text{SOLDIER}(i) \subseteq \text{DOM}(f)$, and for each $u$ in the domain of $f$, the indices that are compatible with what is certain at $i$ must be in the domain of $f(u)$, $\text{CERTAIN}(i) \subseteq \text{DOM}(f(u))$. As before, this set of functions is minimized. This means that only those functions $f$ survive for which $\text{DOM}(f) = \text{SOLDIER}(i)$, and for which for all $u \in \text{DOM}(f)$ it holds that $\text{DOM}(f(u)) = \text{CERTAIN}(i)$. This in turn is one, and only one function:

$\lambda u \in \text{SOLDIER}(i) \ \lambda i'' \in \text{CERTAIN}(i) \ [\forall u \in \text{DOM}(f) \ \forall i' \in \text{DOM}(f(u)) [\text{HEROIC FIGHTER}(i')(f(u)(i'))] \ \land \\
u = f(u)(i'')]$)

Note that the treatment of this example, though quite complex, is a straightforward extension of the interpretation of functional relative clauses and of EIR relatives we have developed so far.

The two quantifier examples, with a plural quantifier like most soldiers, and a singular quantifier like every soldier, illustrate how cases with quantified DPs in the position of the subject of the EIR relative can be dealt with. We do not intend here to consider the full range of quantifiers that are possible in this position, e.g. existential quantifiers as in the experienced defenders that at least some of you seem to be, or universally interpreted any-clauses, such as the lucky winner that any of you can in principle become. But we need to say something about cases with downward entailing quantifiers or a negative copular structure, which are not automatically accounted for under our approach, as Fred Landman (p.c.) pointed out to us. The challenge is that some way needs to be provided in order to avoid making predications about non-existent objects, as in the following infelicitous examples.

(70) The responsible father {that none of you was _ / that Bill wasn’t _ } prevented his son from making a fatal mistake.

The deviance of (70) is predicted because minimization yields the null set, for which the iota operator is not defined. There are, however, comparable cases in which infelicity is circumvented by pragmatic accommodation made possible by certain features of the matrix predication. We illustrate and briefly comment on two such situations. A first form of accommodation takes place in data with at most:

(71) [The able-bodied fighters that at most ten individuals in this entire battalion still were _] nonetheless managed to stop the enemy’s advance.

If we analyze at most ten following standard Generalized Quantifier theory as ‘having a cardinality between zero and ten’, minimization and union yield the empty set, predicting the same kind of infelicity as in (70). Since the example is felicitous, we must assume that the matrix predicate rules out the zero option, forcing the copular subject to define a plurality with cardinality greater than one. This is the same kind of accommodation that allows for anaphoric expressions, as in At most ten soldiers broke through the enemy’s lines. They never returned.
A second type of accommodation is found in data with EIR relatives comparable to those in (70), but which crucially differ from the latter in having an epistemic modal in the matrix predicate, as, e.g., in (72).

(72) The responsible father \{that none of you was \_ / that Bill wasn’t \_\} could have prevented his son from making a fatal mistake.

We suggest that the irrealis modal in the main clause implicates an unrealized state of affairs in which the property denoted by the CP-external NP of the EIR relative is in fact possessed by an individual or individuals. As a result of this implicature, data like (72) acquire the essential import of (73.a,b).

(73) The responsible father \{that (at least) one of you might have been \_ / that Bill might have been \_\} prevented his son from making a fatal mistake.

In sum, pragmatic accommodation can in principle overcome the problem created by downward entailing copular subjects and negative copular structures.

5. THE DEFINITE ARTICLE AND THE DEFINITENESS RESTRICTION

In this section we will turn to the issue how the definite article that we find with EIR relative clauses is to be interpreted, and we will also explain why the definite article is the only determiner for EIR relatives (at least in languages that have definite articles – cf. footnote 4 which includes a Russian example that predictably lacks any article).

So far, we have not given a clear idea how the definite article should be interpreted in this position. We have interpreted it by the iota operator applying to a singleton set of minimized individual concepts in (30), we have assumed an additional union operation for cases with weak modal quantifiers in (33) and (34), and we have assumed sum formation for cases with non-universal copular subjects in (64). As for minimization, we have argued that this is a pragmatically motivated operation due to the definitional character of EIR relatives (cf. discussion of (30)). We would now like to suggest that union and sum formation is achieved by the definite article when applied to a set of individual concepts.

It is well-known that the definite article in a domain for mass nouns and plurals can be represented by the sum operation (cf. Sharvy 1980, Link 1983). More specifically, a phrase like the (three) apples refers to the sum of all entities that fall under the predicate (three) apples, with the presupposition that this sum individual itself falls under it (where the presupposition makes sure that three apples is undefined if there are more than three or less than three apples in the domain):

(74) If \( \alpha \) denotes a set of entities,
then \([[(the \ \alpha)] = \Theta[\alpha]],\) provided that \(\Theta[\alpha] \in [\alpha]\),
where \(\Theta[\alpha]\) is the sum individual that consists of all entities in \([\alpha]\).

How can this definition be generalized to individual concepts? We have defined the notion of a sum of a set of individual concepts in (63). This definition is restricted, as it presupposes that all individual concepts in the set have the same domain. In cases with weak modal quantifiers this is not the case; we have used set union there to form a large individual concept from a set of smaller ones. We can combine these two operations to a sum operation of a set of individual concepts (and the concept of a sum of a set of functions in general) \(\Theta\), which is defined as follows:
If \( x, y \) are individual concepts, then \( x \otimes y = \{ (i, u) \mid x(i) * y(i) \} \), where:
- \( x(i) * y(i) = x(i) \oplus y(i) \) if \( x(i) \) and \( y(i) \) are defined,
- \( x(i) \) if \( x(i) \) is defined and \( y(i) \) is undefined,
- \( y(i) \) if \( y(i) \) is defined and \( x(i) \) is undefined,
- undefined if \( x(i) \) and \( y(i) \) are undefined.

For example, we have \( \{ (i_1, a), (i_2, b) \} \otimes \{ (i_2, c), (i_3, d) \} = \{ (i_1, a), (i_2, b \oplus c), (i_3, d) \} \). The operation \( \otimes \) defines a part relation \( \preceq \), and can be generalized to a general sum operation for sets of individual concepts in the same way as done for \( \oplus \) in (63). We then can give the following meaning rule for the definite article when combined with an NP that applies to set of individual concepts:

If \( \alpha \) denotes a set of individual concepts, then \( \llbracket \text{the } \alpha \rrbracket = \otimes \min \llbracket \alpha \rrbracket \), provided that \( \otimes \min \llbracket \alpha \rrbracket \in \llbracket \alpha \rrbracket \).

Again, the minimization operation is pragmatically motivated, but is integrated here into the semantics for reasons of perspicuity, and it could well be argued that it is a pragmatic element that became part of semantics. The “provided”-clause does not actually give any restriction here and could be left out. Observe that the sum operation \( \otimes \) combines the functions of \( \cup \) and \( \oplus \), which were necessary for deriving the proper meaning of EIR relatives under all circumstances, and can be seen as the version of the general meaning of the definite article suitable for the type of individual concepts. It would give us the right result also in more trivial cases like \( \text{the pope} \), which would refer under the given theory for each index \( i \) to the pope at \( i \) (and for the time of schism to the sum of the individuals that were rightly called pope).

We now turn to the question why the definite article is the only option for EIR relatives. Indeed, other determiners, even universal quantifiers and the definite article combined with \( \text{only} \), are infelicitous, as can be appreciated by considering the following examples:

\[
\begin{align*}
\text{a. } \{ \text{The / #the only / #a certain / #some / #every / #that} \text{ gifted mathematician} & \text{ that Bill supposedly is } _{1} \text{ } \} & \text{ should be able to solve this problem.} \\
\text{b. } \{ \text{The / #all (the) / #some / #most / #those} \text{ gifted mathematicians that} & \text{ Bill and Mary / all of you / most of you / at least some of you} \text{ supposedly are } _{1} \text{ } \} & \text{ should be able to solve this problem.}
\end{align*}
\]

We have mentioned the option of specifying number by numerals in plural constructions as in \( \text{the two gifted mathematicians that Bill and Mary claim to be} \), which is compatible with our theory as shown in (65). Notice the contrast with the quantifier \( \text{both} \), which is not possible as a determiner of EIR relatives. But numerals are properly analyzed as part of the NP, not as determiners, so this does not constitute an exception. Moreover, in languages that distinguish between definite that address entities which are unique due to shared world knowledge and entities that are unique for other reasons, we find that EIR relatives occur with the latter type of article. For example, German allows for the combination of prepositions and definite article (e.g. \( \text{von dem} > \text{vom} \)) in case the uniqueness is due to world knowledge, cf. (78), which is not possible for EIR relatives, cf. (79). This can be explained by the definitional character of EIR relatives; they do not refer to an entity that is already given, but construct a new concept.

\[
\text{Vom Papst erwartet man, dass er keine vorschnellen Entscheidungen trifft.}
\]

‘One expects of the pope that he doesn’t make hasty decisions.’
Von dem begabten Mathematiker, der Bill zu sein behauptet, erwartet man, dass er dieses Problem lösen kann.

‘One expects of the gifted mathematician that Bill claims to be that he can solve this problem.’

In order to explain why the definite determiner remains as the sole option, we would have to show for every determiner why they cannot be used. In many cases, minimization necessarily yields a singleton set, for which the definite article is the only option. In such cases, the use of other quantifiers is odd (cf. John’s only mother, every mother of John, that mother of John). In case the uniqueness is guaranteed by the way an entity is referred to, quantifiers other than the definite article lead to ungrammaticality. This can be seen with superlatives:

a. Mary is the / the only / every / some / that best mathematician of her class.

b. Bill and Mary are the / the two / all the / some / those best mathematicians of their class.

Now, we have seen that there are cases that do not result in a singleton set as the NP meaning, and here the question arises why other quantifiers cannot be used. Let us discuss one example:

#Every gifted mathematician that Bill might possibly be should at least be able to solve basic quadratic equations without problems.

Recall that with weak modal operators, we get after minimization a set of small individual concepts, like {{i1, b}, {i2, b}, ...}, where the indices i1, i2, etc. together form the epistemically accessible set of indices, and for each index Bill is a gifted mathematician. This appears like an appropriate set to quantify over. But notice that the indices represent every possibility compatible with the present knowledge, where two indices i, i′ might differ only due to arcane circumstances that have nothing to do with the issues at hand. Such possibilities cannot easily be distinguished, they cannot be counted, and hence they cannot be quantified over. In cases in which we do count or quantify over possibilities, as in two possibilities or every possibility, we group indices together into discernable subclasses, as in there are two possibilities by which she could have climbed the mountain.

The restriction to definite articles is remarkable because it is not found with all the relative constructions with restricted determiner choice, in particular, with what Dayal (1995) called “definite relatives” and Grosu and Landman (1998) called “strange relatives of the third kind”, as can be seen in (82) with English amount relatives (Carlson 1977) and in (83) with Hindi correlatives, respectively (see Grosu 2002 for detailed discussion and additional illustration).

a. Bill is {the / the only / a / every} gifted mathematician that his mother was.
b. These guys are {the / #the only / #some / #all /# all the} gifted mathematicians that their parents were.

Even though EIR relatives and equational property relatives are distinct, as we demonstrated in section 3, they both contain an equation in their definition that guarantees that they apply to singleton sets, and hence they are constructionally unique.

The other two types of relative clauses mentioned in section 3, subkind and degree property relative clauses, show a similar restrictions in their determiner choice:

(85) a. John is {the / #the only / #a / #every} (kind of) mathematician his mother was.

b. John is twice {the / #the only / #a / #every} mathematician his mother was.

Again, we can argue for constructional uniqueness under the natural assumptions that the SUBKIND relation identifies mutually non-overlapping sets, and that DEGREE maps entities to a unique degree. However, the SUBKIND and DEGREE relation are dependent on an index of evaluation, which can vary in modal contexts. This means that in such contexts we may also find other determiners, as in *John is a mathematician that his mother never quite managed to be* (cf. Grosu 2000).

6. THE RANGE OF ADMISSIBLE INTENSIONAL OPERATORS, AND THE RAISON D’ÊTRE OF EIR RELATIVES

All the data with EIR relatives that we have examined so far contained intensional operators in the relative clause. In this section, we will take a closer look at the range of operators that are admissible in EIR relatives, and will also show that the presence of *some* kind of operator is necessary for EIR relative constructions to be felicitous.

We find a wide range of intensional operators in EIR relatives. Examples with epistemic operators we have seen already; the following cases exemplify deontic, buletic and temporal operators in the relative clause.

(86) [The reliable friend Mary should have been _ in this difficult situation] would have stopped John from making a fool of himself.

(87) [The brave and selfless fighter for justice that Rose wished to be _ ] would certainly have improved the lives of the villagers.

(88) [The idealist you once were _ ] would have jumped into action on hearing about this violation of basic human rights.

Of particular interest are cases of *judgmental* operators as in (89). They appear to be deontic in character, yet do not specify how people should act, but rather how events should be valued.

(89) [The abominable atrocity that the killing of the hostages was _ ] must not go unpunished.

This example can be paraphrased as follows: At all indices that are compatible with some ethical norm, the killing of the hostages is evaluated as an abominable atrocity. The main clause then states that for all these indices, indices at which the killing does not go unpunished are closest to the ethical ideal.
EIR relative constructions can also be formed with **predicates of personal taste** that are dependent on an index that represents the personal source of a judgment:

(90) [The delicacy that this Schwarzwald cherry cake was _ ]
    could not have been topped by anything else.

In section 2.1 we have argued that the indices of the individual concept defined by the EIR relative and the ones of the intensional operator of the main clause have to be compatible to each other. This can perhaps best be shown with temporal operators, as both operators must restrict the same set of times:

(91) [The Maoist that Bill once was _ ]
    sincerely {believed, #believes} in the principles of perpetual revolution.

This **compatibility requirement** is the reason for the oddness of sentences like the following, as there is no plausible connection between the worlds of Bill’s claim in which he is a gifted mathematician, and those at which he would be able to lift 200 kilos.

(92) #[The gifted mathematician that Bill claims to be _ ]
    is certainly able to lift 250 kilos.

So far we have considered EIR constructions in which the main clause contained an intensional operator that was quantificational by nature. But it could well be another intensional construction:

(93) I admire very much [the idealist you once were _ ],
    not [the cynic you have become _ ]

Predicates like **admire** are intensional, as their object argument need not exist in the world of evaluation; they share this property with verbs like **seek**, **worship** or **write about**. The class of predicates that can be understood this way is wider than one might think, considering cases like the following:

(94) I am {addressing this appeal, talking} to [the idealist you once were _ ],
    not to [the opportunist you have become _ ].

The argument position of **addressing** and **talking to** is arguably intensional here, as the person addressed or talked to is not simply the addressee, but the addressee thought of as having a certain property that he or she does, in fact, not have anymore. By doing so, the speaker tries to revive this property so that the individual concept identified by **the idealist you once were** starts to exist again. What we need here is some notion of an extension of the individual concept beyond the domain for which it was initially defined. This is also illustrated by the following examples with modal and temporal indices, respectively.

(95) [The happy couple that Bill and Sue seemed to be _ ]
    is in fact a reality.

(96) After a long period of opportunistic behavior,
    [the sincere idealist that Max once was _ ] exists again.

We now turn to the issue of where the index of evaluation, i.e. the actual world and time as proposed by the speaker, can be located within the domains of the intensional operator of the EIR relative clause. We have seen a number of cases in which the indices of the operator in the relative
clause fails to include the index of evaluation; for example, (1) implicates that Bill is not a gifted mathematician. This can also be expressed directly:

(97)  [The competent mathematician that Bill falsely claims to be _ ]
      should have solved this simple problem with greater ease.

At the same time, it is not necessary for the indices of the operator to exclude those evaluations, as shown by (98), and also by (99), where the intensional indices are temporal.

(98)  [The competent mathematician that Bill clearly is _ ]
      has just solved a highly complex problem.

(99)  [The celebrity you have become _ ]
      seems to have forgotten its humble beginnings.

In both examples, the indices of the operator are included in those of the speaker’s beliefs at the moment of speech. In these cases the predication of the main clause must also include the actual index. However, what crucially distinguishes such data from others with ‘ordinary’ relative clause constructions is that other indices are taken into account. In (98), the speaker’s use of clearly considers all the possibilities for which there is clear evidence. In (99), become indicates that the denotatum of you is viewed by the speaker as not having always been a celebrity.

There are cases in which the predicate of the EIR construction cannot strictly apply to the individual concept defined by the EIR relative, but to a continuation of it:

(100)  The rich man that you may become some day will hopefully show more generosity towards his brothers and sisters.

Notice that the predicate does not state something about you for the moment or time of becoming rich, but for the following time of being rich. Hence (100) states something about the temporal continuation of the individual concept that is you and is becoming rich.

Having discussed the semantics of EIR relatives as well as the various restrictions it imposes, in particular the conditions on the intensional operators discussed in the last section, it is time to consider the larger picture of the pragmatic conditions under which EIR constructions are used. More specifically, what could be the reason for expressing a claim about an arcane individual concept that just exists for the indices given by some intensional operator?

In section 2.1 we noted that even though the individual concepts of EIR relatives are restricted by their intensional operator, it is generally supposed that the restricted individual concept can be naturally extended to other indices (cf. also section 4 on admissible quantifiers within EIR relatives). Intuitively, our example (1) is about Bill in the world of evaluation, not about some arcane partial individual concept that exists only in certain possible worlds, just as the paraphrase in (8) suggests: ‘if Bill were a gifted mathematician, then he would be able to solve this problem’.

EIR relatives allow us to talk about certain versions of individuals that potentially differ from the real individual by having certain properties. We can formulate what we consider to be the raison d’être of EIR relatives as follows:

(101)  EIR relative clauses denote, at a set of indices properly included in the full set of indices that are contextually taken into account, different ‘versions’ of the individuals that fall under the denotation of the copular subject, these versions being distinguished from other contextually relevant ones by the possession of the property denoted by the CP-external NP.
Examples like the following fail to satisfy this condition:

(102) #[The secretary that Mary is _] works for Bill.

The denotation of the subject would be identical to the one of the simpler expression, Mary. The property secretary mentioned in the relative clause is no different from any other property possessed by that concept, and singling it out for special mention is without pragmatic justification. Under these circumstances, the denotatum of the EIR relative becomes indistinguishable from that of the copular subject, and the use of the more complex expression is unmotivated.23

The choice of property that helps to define a possibly counterfactual individual concept is guided by the speaker’s intention to specify a relation between the indices at which the individual concept is defined, and the indices at which the predication is made.

(103) [Assume that Bill claims to be a professional weight lifter, and he also claims to be a gifted mathematician.]  
#The gifted mathematician Bill claims to be should be able to lift 200 kilos.

This example is odd, even though at the indices compatible with Bill’s claim he is both a weight lifter and a mathematician. In the semantic theory developed here, the two expressions the gifted mathematician Bill claims to be and the professional weight lifter Bill claims to be actually have the same denotatum in the given context. In fact, they have the same impact as the according-phrase in the following example:

(104) According to what Bill claims to be,  
he should be able to {solve this equation, lift 200 kilos}.

The choice of the predicate (gifted mathematician, professional weight lifter) is governed by pragmatic considerations that appeals to certain expectations of normality, e.g. that being a gifted mathematician helps to solve mathematical problems, but not to lift weights. We find similar principles at work in the choice of absolute constructions like the following:

(105) (Being) a gifted mathematician,  
Bill should be able {to solve this problem, #lift 200 kilos}

Let us return to the point illustrated with examples like (102), which was deemed infelicitous because it expresses the same as Mary works for Bill. It is instructive to compare the oddity of unnecessarily using a more complex expression that makes use of equation instead of a simpler one with the related case of transparent free relatives (cf. Grosu 2003):

(106) John is eating his soup with [what _ may seem to be a fork],  
(but _ is in fact a fancy spoon).

Like EIR relatives, transparent free relatives rely on an equational structure within the relative, the post-copular term is construed in the scope of a CP-internal intensional operator, and the copular subject has wider scope than the intensional operator. The crucial difference is that the CP-internal gap is in the position of the copular subject, with the result that the denotation of the relative clause

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23 One could see a possible motivation for the secretary that Mary is as an expression that refers to Mary and introduces the presupposition that she is a secretary. One reason why English does not allow for this type of construction might be that it is preempted by the simpler appositive construction Mary, a secretary, or by the absolute construction Being a secretary, Mary…
is defined at matrix indices, and is subject to none of the compatibility restrictions that have been noted in relation to EIR relatives. Now, in the absence of an intensional operator, transparent free relatives become indistinguishable from the post-copular term, with resulting infelicity:

(107)  a. #John is eating his soup with [what _ is a fork].

Just like EIR relatives, transparent free relatives become infelicitous when they do not live up to their raison d’être, which we take to be to denote something that is in fact something else at a proper subset of the total set of indices that are contextually taken into account.

Having established the need for an intensional operator within EIR relatives, we note that the operator in question does not need to be overtly expressed, so long as it can be inferred in some way or other with sufficient ease. The ability to construe an implicit operator depends on the wording, on contextual assumptions, and also on the ability and/or readiness of individual language users to perform this construal. We have thus found a certain amount of inter-personal variation in relation to the acceptability of EIR relatives without overt intensional operators, which ranged from strong qualms to (almost) complete acceptance.

One factor that seems to favor the recovery of an implicit operator is the use of a CP-external NP whose denotation can be viewed as being a personal evaluation, typically, of the speaker, rather as something taken for granted by everyone in the universe of discourse. Thus, the full versions of (108) and of (109) tend to be preferred to the reduced version.

(108) The hospital is more than ready to hire [the (fabulous) nurse that Mary is _ ].

(109) [The (brilliant) mathematician that Bill is _ ] will hopefully solve this problem.

This is not to say that the CP-external NP must necessarily contain an (overt) adjective. (110) is in general readily accepted, presumably because the matrix VP implies that the speaker views himself as a conscientious doctor.

(110) [The doctor that I am _ ] cannot refuse to see a patient who is unable to pay.

We feel that (110) has a reading that is distinct from the taxonomic reading which can be spelled out by the kind of doctor that I am and is compatible with generic predicates, such as is rare nowadays. It can be paraphrased as ‘Being a doctor, I cannot refuse to see patients who are unable to pay.’

7. CONCLUSION

In this paper, we have dealt with a rare and hitherto virtually unstudied type of relative clause construction, which raises prima facie problems for surface structure interpretation, and which we proposed to call “Equational Intensional ‘Reconstruction’ relative”, in virtue of two properties that we view as necessary components of its analysis – a CP-internal equational statement, and a CP-internal intensional operator – and a third property that hints at an analysis it may seem to invite – namely, syntactic reconstruction of the head NP in the position of the CP-internal gap. We suggested that EIR relatives denote individual concepts that are defined only within the domain of the intensional operator that they contain. This allowed us to analyze them as a variety of functional relative clauses, and to interpret them taking surface structure to be the input to semantics, following proposals that were developed for relative clauses that rely on functions from individuals to individuals. We also amended those early proposals by attributing reconstruction effects in both
types of relatives to a minimality operator that applies to the output of intersection of NP and CP. At the same time, our analysis keeps the two types of functional relatives distinct by assuming internal equation for EIR relatives only, something that enabled us to account for the fact that EIR relatives, but not relatives based on functions from individuals to individuals, allow only definite determination. We accounted for these definiteness effects in a way that enabled us to capture partial similarities with superficially similar predicative DPs, as well as differences from other relatives which, in addition to definite articles, also allow other types of quantifiers.

We would like to conclude this paper by noting a few more general contributions it has made, with implications for future research. A first point concerns the role of individual concepts in natural language semantics. While their importance was already recognized in Montague (1973) – they are the key for his solution to sentences like the temperature is ninety and rising – and they have been used in a number of additional studies, e.g., Gupta (1980), we have shown that this notion can be fruitfully exploited to solve a number of other problems in natural language semantics and the syntax/semantics interface.

A second point concerns the head-raising analysis of relative clauses. To the extent that such an analysis involves a more complex syntax than one which relies on surface structure interpretation, it is important to see whether analyses that rely on surface structure interpretation can be devised. This paper has shown that this is feasible for one kind of relative clause construction, EIR relatives. The solution we propose is similar to the analysis of another well-known type of relative clauses that has invited a head-raising analysis, in particular Jacobson’s and Sharvit’s analysis of functional relative clauses. Our approach, combined with the approach adopted by these two authors, provides a unified approach to these two types of ‘reconstruction’ relatives, capturing in a natural way both similarities and differences between them, such as the presence or absence of uniqueness effects. A comparison of the detailed analyses provided by Jacobson and Sharvit and by ourselves with the head-raising analysis is not possible at this stage, since, to our knowledge, there are no analyses of the latter type at the moment.

The third point is that we hope to have maid a contribution to a better understanding of the semantic typology of relative clause constructions. While Grosu & Landman (1998) and Grosu (2002) proposed that a theory of the syntax-semantics interface must recognize at least one semantic type of relative in addition to the traditionally recognized restrictive and appositive types, in particular, relatives that disallow existential quantification, we have tried to show that even more restricted types exists, in particular, types that allows only definite determination. Beyond EIR relatives, we have discussed equational property relatives, subkind property relatives, and degree property relatives. This points to the possibility that there may exist additional semantic types of relatives, a point that is worth keeping in mind when undertaking subsequent research.

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Alexander Grosu  
Dept. of Linguistics  
Tel Aviv University, Tel Aviv, Israel  
grosua@post.tau.ac.il

Manfred Krifka  
Humboldt Universität zu Berlin  
& Zentrum für Allgemeine Sprachwissenschaft, Berlin  
krifka@rz.hu-berlin.de