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RESEARCH ARTICLE

Superlative quantifiers and meta-speech acts

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Abstract Recent research has shown that the superlative quantifiers *at least* and *at most* do not have the same type of truth conditions as the comparative quantifiers *more than* (Geurts and Nouwen, Language 83:533–559, 2007) and *fewer than*. We propose that superlative quantifiers are interpreted at the level of speech acts. We relate them to denegations of speech acts, as in *I don't promise to come*, which we analyze as excluding the speech act of a promise to come. Calling such conversational acts that affect future permissible speech acts "meta-speech acts," we introduce the meta-speech act of a GRANT of a proposition as a denial to assert the negation of that proposition. Superlative quantifiers are analyzed as quantifiers over GRANTS. Thus, *John petted at least three rabbits* means that the minimal number n such that the speaker GRANTs the proposition that John petted n rabbits is $n = 3$. We formalize this interpretation in terms of commitment states and commitment spaces, and show how the truth conditions that are derived from it are partly entailed and partly conversationally implicated. We demonstrate how the theory accounts for a wide variety of distributional phenomena of superlative quantifiers, including the contexts in which they can be embedded.

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He is speaking that most superlative of tongues, O elocuted one.
Lestrade and the Dead Man's Hand
M. J. Trow

1 The meaning of superlative quantifiers

1.1 Commonly held intuitions

What do superlative quantifiers, like *at least* and *at most*, mean? Non-linguists have a clear intuition—for example, note the following discussion of *at least* from a book about computer databases:

> One important rule to remember is that there should be at least $(n-1)$ joins in an n-table query; thus, you need at least two joins for a three-table query, at least three joins for a query that involves four tables, and so on. The words "at least" are important: there could be more than $(n-1)$ joins…but if your multitable query has less than $(n-1)$ joins, the result will be [bad] (A. Kriegel and B. M. Trukhnov, *SQL Bible*, p. 319).

According to this intuition, *at least x* means x or more, but not less; *at most x* means x or less, but not more. In a context in which only integers are relevant, things are even simpler: *at least x* means more than $x-1$, and *at most x* means fewer than $x+1$.

Thus, since it is impossible to pet a non-integer number of rabbits, (1a) would mean (1b).

(1) a. John petted at least three rabbits.
b. John petted more than two rabbits.[1]

Similarly, (2a) would be equivalent to (2b).

(2) a. John petted at most three rabbits.
b. John petted fewer than four rabbits.

1.2 Keenan and Stavi (1986)

Until recently, such intuitions were widely shared by linguists as well, and have been formalized by Keenan and Stavi (1986). According to their theory, both superlative (*at least*, *at most*) and comparative (*more than*, *fewer than*) quantifiers are treated simply as generalized quantifiers, i.e. relations between sets. Thus, the meaning of (1a) is simply (3a), where R is the set of rabbits, and the meaning of (2a) is simply (3b).

(3) a. $|R \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| \geq 3$
b. $|R \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| \leq 3$

[1] There is another reading of (1a) (Horn 1972; Kadmon 1987), which can be made the preferred interpretation by focus, where John petted exactly three rabbits, and maybe other animals as well. The two readings behave differently, e.g., with respect to anaphora; but we defer discussion of this reading until Sect. 3.6.

Besides being intuitive, this definition has two important advantages. One advantage is that it gets the truth conditions right: if John petted two or fewer rabbits, (1a) is false, and if he petted four or more rabbits, (2a) is false.

The second advantage is that these truth conditions are extensional, which is as it should be. For example, suppose Mary likes rabbits but no other animal. Then, the extensions of the predicates *rabbit* and *animal that Mary likes* are the same. Note that the truth values of (1a) and (4) are the same, indicating that superlative quantifiers are extensional.

(4) John petted at least three animals that Mary likes.

However, Keenan and Stavi's theory, while getting the truth conditions right, fails to account for a number of phenomena. In particular, as Geurts and Nouwen (2007) point out, the meanings of superlative quantifiers differ from those of comparative quantifiers (*more/fewer than*) in subtle ways.

One of the observations of Geurts and Nouwen is that the distribution of comparative quantifiers is more restricted than that of superlative quantifiers. For example, only the latter, but not the former, can take sentential scope:

(5) a. John petted three rabbits $\left\{\begin{array}{l}\text{at most}\\ \text{*fewer than}\end{array}\right\}$.

b. $\left\{\begin{array}{l}\text{At least}\\ \text{*More than}\end{array}\right\}$, John petted three rabbits.

Additionally, superlative quantifiers, but not comparative ones, may combine with quantifiers, proper names, and specific indefinites:

(6) a. Mary petted $\left\{\begin{array}{l}\text{at least}\\ \text{*more than}\end{array}\right\}$ every young rabbit (and maybe some old ones too).

b. Mary petted $\left\{\begin{array}{l}\text{at most}\\ \text{*fewer than}\end{array}\right\}$ Bugs Bunny.

c. John petted $\left\{\begin{array}{l}\text{at least}\\ \text{*more than}\end{array}\right\}$ two rabbits, namely Bugs Bunny and Peter.

There are, however, cases where comparative quantifiers are acceptable, and it is superlative quantifiers that are odd. Suppose John petted exactly three rabbits, and we know this. Based on this fact, we would be justified in uttering (1b); however, it would be quite strange to utter (1a) or (2a).[2,3]

There are differences between superlative and comparative quantifiers not just in distribution, but also in interpretation: the former lack some readings that the latter have. For example, (7a) is ambiguous: it can mean either that it is permissible for you to have fewer than three martinis (say, because you don't like martinis), or that you

[2] A similar point is made by Nouwen (2010), with examples such as the following:

(i) #I know exactly how much memory my laptop has, and it is at most 2GB.

[3] Geurts and Nouwen (2007) point out additional cases where the distribution of superlative quantifiers is more restricted than that of comparative quantifiers; we will discuss such cases when we deal with embedded superlative quantifiers, in Sect. 5.

may not have more than two martinis. In contrast, (7b) is not ambiguous, and only receives the second reading.

(7) a. You may have fewer than three martinis.
b. You may have at most two martinis.

1.3 Geurts and Nouwen (2007)

In order to account for these phenomena, Geurts and Nouwen (2007) argue against the commonly held intuition, and propose that comparative and superlative quantifiers have different interpretations.

Following Krifka (1999), they propose that comparative quantifiers are focus sensitive NP modifiers. Roughly, (1b) means that there is a property that is higher on the relevant scale than the property of petting two rabbits, and this property applies to John.[4]

Formally, the meaning Geurts and Nouwen propose for *more than* α is:

(8) $\lambda x.\exists\beta(\beta > \alpha \wedge \beta(x))$

The relevant scale is affected by focus, which explains the difference between (9a) and (9b).

(9) a. John petted more than [two]$_F$ rabbits.
b. John petted more than [two rabbits]$_F$.

Sentence (9a), with focus on *two*, means that John petted a number of rabbits, and this number is greater than two; while (9b), with focus on *two rabbits*, is compatible with John having petted exactly two rabbits, provided that he petted additional animals.

Sentences (9a) and (9b) are evaluated with respect to different scales; and yet other scales account for examples such as the following:

(10) a. I will be more than happy to send you the necessary forms.
b. The telephone service here is less than satisfactory.

Sentence (10a) is presumably evaluated with respect to a scale involving properties such as being reluctant, indifferent, happy and ecstatic; (10b) presumably involves properties such as being terrible, bad, satisfactory, good, and excellent.

Importantly, Geurts and Nouwen restrict α and β in (8) to denote only first-order properties, i.e. expressions of type $\langle e, t\rangle$. The properties *happy* and *satisfactory* are clearly first-order. The property of being a group of two rabbits is also first-order in their system, since they treat groups as individuals. But propositions are not first-order properties, which is why comparative quantifiers cannot combine with them, and the unacceptability of the sentences in (5) is thereby explained. Similarly, quantifiers, names, and specific indefinites also do not denote first-order properties, which is why the sentences in (6) are bad.

[4] Incidentally, the meaning of (1b) is different from that of (i), since only the latter implicates that John petted no more than three rabbits.

(i) John petted three rabbits.

Regarding superlative quantifiers, Geurts and Nouwen propose that they are epistemic operators. Specifically, the meanings of (1a) and (2a) can be roughly paraphrased as (11a) and (11b), respectively.

(11) a. It is epistemically necessary that John petted three rabbits, and it is epistemically possible that he petted more.
b. It is epistemically possible that John petted three rabbits but it is epistemically impossible that he petted more.

Geurts and Nouwen demonstrate how their approach solves some of the problems with Keenan and Stavi's theory. In particular, they can explain why (2a) would be odd if it is known that John petted exactly three rabbits: according to Geurts and Nouwen's theory, (2a) entails that it is epistemically possible that John petted three rabbits. But in this case it is not only epistemically *possible*, but, in fact, epistemically *necessary* that John petted three rabbits, so the speaker makes a weaker statement than the one she can and ought to make. In other words, saying that it is epistemically possible that John petted three rabbits implicates that it is not epistemically necessary; but this implicature is not satisfied in a situation where it is known that John petted three rabbits.

According to Geurts and Nouwen (1a) is not merely odd, but actually false: this is because it entails that it is epistemically possible that John may have petted more than three rabbits; but if it is known that he petted exactly three, this entailment is plainly false.

Crucially, superlative quantifiers are not restricted to combine only with first-order properties: hence, their distribution is less restricted than of comparative quantifiers, and the facts exemplified by the sentences in (5) and (6) are thereby explained.

The missing reading of (7b) is explained by the analysis of superlative quantifiers as epistemic modals: in general, deontic modals cannot take scope over epistemic ones.

Thus, Geurts and Nouwen's theory successfully accounts for a number of puzzling phenomena. However, they make these gains at a considerable cost.

One problem is that the superlative morphology of superlative quantifiers is ignored: it is not reflected in the analysis. There is nothing in the logical form proposed by Geurts and Nouwen for (1a) that indicates that three is the *least* number of rabbits that John petted.

Moreover, Geurts and Nouwen lose the two major advantages of Keenan and Stavi's theory: correct truth conditions, and extensionality.

Basing the truth conditions of superlative quantifiers on epistemic modality makes them subjective, dependent on epistemic states, which leads to incorrect predictions. Suppose John petted exactly four rabbits in the actual world; then (1a) would be true and (2a) would be false, regardless of any belief worlds.

One may try to save Geurts and Nouwen's theory by treating the epistemic modals as objective. Lyons (1977) draws a distinction between *subjective* and *objective* senses of epistemic modals. Thus, (12a) is subjective, dependent on the epistemic state of the speaker; but (12b) is objective, in the sense that it depends on the epistemic states of a large number of people, who are collectively the authority on the subject.

(12) a. Has anybody heard the news? I want to know who won the match. It might have been Mark.

b. There might have been life on Mars at some point in the past.

However, treating Geurts and Nouwen's epistemic modals as objective would not help. Suppose John committed exactly four traffic violations, but nobody knows this, not even the police (who are the authority on the subject), and not even John himself. Then, it would still be true that he committed at least three traffic violations, and false that he committed at most three traffic violations, and these truth values depend only on what actually happened, not on anybody's beliefs.

The analysis of superlative quantifiers as epistemic modals also leads to the incorrect prediction that they are intensional. However, as we have seen above, superlative quantifiers are extensional. Consider again the case where Mary likes rabbits, but no other animal. But this fact may not be known; hence, it may be epistemically necessary for John to pet three rabbits, without it being epistemically necessary for him to pet three of the animals that Mary likes. Hence, it is predicted that (1a) may have a different truth value from (4), repeated below:

(13) John petted at least three animals that Mary likes.

But this prediction is wrong.

Another problem is that *at most* licenses negative polarity items, whereas *at least* does not:

(14) At $\left\{\begin{array}{c} most \\ *least \end{array}\right\}$ three people have ever been in this cave (last century).[5]

However, nothing in Geurts and Nouwen's theory predicts this behavior.

Geurts et al. (2010) present the results of experiments that are claimed to support their theory; but some of the results are actually in conflict with it. Recall that, according to this theory, there is a fundamental difference between *at least* and *at most*: if John petted exactly three rabbits and the speaker knows this, (1a) would be false, whereas (2a) would be true but infelicitous. As a consequence, Geurts et al. predict that (1a) will be ruled out in this situation; however, citing Noveck (2001) and their own unpublished study, they conclude that true but infelicitous sentences such as (2a) ought to receive a mixed response, i.e. be acceptable about half the time.

Geurts et al.'s prediction, however, is not borne out. They asked subjects to judge whether (15b) and (15c) follow from (15a).

(15) a. Wilma had three beers.
b. Wilma had at least three beers.
c. Wilma had at most three beers.

The results were that *both* (15b) and (15c) are accepted about half the time.

Geurts et al. are aware of this difficulty, and they attempt to explain it by hypothesizing that people who accept (15b) do so because they interpret (15a) as saying that Wilma had three beers or more. As support for this claim, they demonstrate that when (15a) is replaced with (16), the acceptance of the inference to (15b) is reduced significantly.

[5] Note that this behavior obtains even when the superlative quantifier does not relate to a numeral:

(i) At $\left\{\begin{array}{c} most \\ *least \end{array}\right\}$ some bats and small rodents have ever been in this cave

Non-numerical scales will be discussed in Sect. 3.6.

(16) Wilma had exactly three beers.

However, this argument is not convincing, for two reasons. One is that the acceptance rate is about 20 %, still significantly higher than the predicted 0 %. The second reason is that there is a much simpler explanation for why *exactly* reduces the acceptance of the inference: plausibly, the word *exactly* itself acts, at least syntactically, as a superlative quantifier. Evidence for this possibility comes from the fact that *exactly* cannot co-occur with superlative quantifiers:

(17) *John petted $\left\{\begin{array}{l}\text{at least}\\ \text{at most}\end{array}\right\}$ exactly three rabbits.

Therefore, an utterance of (16) introduces a set of alternative sentences involving other superlative quantifiers—(15b) and (15c)—and implicates that these alternatives are false.[6]

Geurts and Nouwen point out that, in Dutch, the word *zeker* 'certainly' can sometimes be used to mean *at least*, as in:

(18)

een	restaurant	met	zeker	dertig	tafels
a	restaurant	with	certainly	thirty	tables

'a restaurant with at least thirty tables'

They treat *zeker* 'certainly' as a modal, hence claim that this phenomenon supports their analysis of superlative quantifiers as modal operators.

However, it is far from clear that adverbs (as opposed to adjectives) like *certainly* are really modal operators; it has been argued that, in fact, they are illocutionary operators, which modify the sincerity conditions of speech acts (Piñón 2006; Wolf and Cohen 2009). If this position is on the right track, the Dutch facts actually point in the direction of analyzing superlative quantifiers as illocutionary operators, rather than modals.[7]

This is the view we take in this paper: we propose that superlative quantifiers are illocutionary operators. The paper is structured as follows. We begin with a closer look at speech acts in Sect. 2. We discuss the nature of speech acts, and formalize them in a dynamic framework. We then introduce the notion of *meta-speech acts*, and incorporate them into our formalism. One particular meta-speech act, GRANT, is introduced: to GRANT ϕ is to refrain from asserting $\neg\phi$. This illocutionary operator turns out to be indispensable for the meaning of superlative quantifiers, as discussed in Sect. 3. We argue that an utterance of (1a) means that the speaker denies that John petted zero, one, or two rabbits. By implicature, the addressee concludes that the speaker GRANTs that John petted three rabbits, four rabbits, five rabbits, and so on. We provide a compositional derivation of this interpretation based on the superlative morphology of these quantifiers. We demonstrate how the truth conditions are

[6] We do not, however, make any claims about the meaning of *exactly*, except that it serves as an alternative to superlative quantifiers.

[7] Of course, the fact that a speech act operator is similar in meaning to superlative quantifiers does not *prove* that superlative quantifiers are illocutionary operators. In fact, there are quite a few operators with a similar meaning (see Nouwen 2010 for discussion), and in this paper we make no claim about whether any of the others is a speech act operator too.

derived from this meaning, and point out how the derivation of these truth conditions crucially relies on implicature. In Sect. 4 we proceed to account for the behavior of superlative quantifiers described in the current section; we account for the data that motivate Geurts and Nouwen's (2007) theory, as well as for data that are problematic for it. The data concerning embedded superlative quantifiers are particularly intricate, and they are discussed in Sect. 5. We find that superlative quantifiers are generally bad in downward entailing environments, and explain this by the fact that their interpretation relies crucially on scalar implicature, and that implicatures do not survive downward entailing contexts. We discuss cases where superlative quantifiers *are* fine in such contexts, and analyze them as demonstrating a distinct reading, the *evaluative* interpretation. Section 6 concludes the paper and discusses its implications for the semantics–pragmatic interface.

2 Modeling speech acts and meta-speech acts

2.1 Commitment states and commitment spaces

We understand speech acts as changing commitments of the interlocutors. For example, in asserting a proposition Φ, the speaker takes on a commitment to be responsible for the truth of Φ, and in promising to behave in a way described by a proposition Φ, the speaker takes on the commitment to behave in that way. In this normative approach to speech acts, we follow authors like Hamblin (1971), Gazdar (1981), Merin (1994) and Beyssade and Marandin (2006). The following implementation differs significantly from Cohen and Krifka (2011), and follows Krifka (to appear-b).

The basic notion of our implementation is one of a (basic) **commitment**. Commitments are expressed in some elementary representation language. For example, we should be able to express in this language that a speaker S_1 is responsible for the truth of a proposition Φ. This will not be developed further here.

A **commitment state** c is a set of commitments, the set of public commitments that the interlocutors have accumulated up to the current point in conversation. Commitment states play the role of common ground in other formal models of conversation. While common grounds typically are modeled by sets of propositions that are publicly accepted by all participants, commitment states are richer; for example, with assertive commitments, they also keep track of the participants that asserted a proposition, and hence are responsible for its truth.

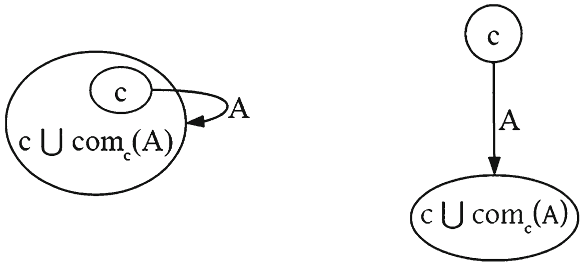
Commitment states are not arbitrary sets of commitments. The commitments have to be consistent with each other. For example, a commitment state that both contains the commitment of a speaker to a proposition Φ and to a proposition $\neg\Phi$ should be avoided by the speaker, as it would involve being responsible for conflicting propositions, and would invoke penalty in case the other participant presses on this issue.

A (basic) **speech act** A is a function from an input commitment state to an output commitment state. The specific commitments that are conventionally associated with A are added to the current commitment state c, for which we write $c + A$, resulting in a commitment state $c \cup \mathrm{com}_c(A)$. The commitments of A might depend on the previous commitment state c, hence the index in $\mathrm{com}_c(A)$, the commitments generated by the speech act A in the context of the commitment state c.

(19) $c + A = c \cup \mathrm{com}_c(A)$.

Pictorial representations will turn out to be helpful. We will generally not use depictions like (a), which represent the set-theoretic relation between c and $c + A$, but rather representations like (b), which highlight the transitional nature of the update of commitment states by speech acts. But notice that in such diagrams, the commitments of the input commitment states are always understood as being contained in the commitments of the output commitment state.

(20) Update of commitment states by (basic) speech acts.



(a) by Venn diagram **(b)** by transition diagram

The execution of speech acts is governed by conversational rules. For example, performing a speech act A on a commitment state c that would lead to a contradictory commitment state would violate such rules, as it would impose conflicting commitments on the speaker. Furthermore, performing A on c should not be redundant, that is, the commitments expressed by A should not be already part of the commitment state c. The assumption that speakers adhere to such rules can generate conversational implicatures regarding the commitments. We will not go into these conditions for speech acts and their use in communication here, as they are tangential to our issues.

What is important for our purposes is that commitment states, and transitions between commitment states, are not sufficient to model speech acts in general. One case in point are denegations (cf. Searle 1969), as in the following cases:

(21) a. I don't promise to come.
b. I don't say that Bill is to be blamed.

Following Hare (1970), such denegations can be seen as refusals to perform a certain speech act. This cannot be expressed within a model in which speech acts have the effect of adding commitments to commitment states. Denegations rather restrict the admissible future development of commitment states. Hence we need a more complex notion: a commitment state c together with all the commitment states into which it can develop by basic speech acts, following the rules of conversation that obey consistency, non-redundancy, etc. We will call this a **commitment space**, and model it as a set of commitment states. In general, a commitment state c can develop into commitment state c' by a (series of) basic speech acts, which will add commitments to c'. Hence, if c can develop to c' by basic speech acts, we have $c \subseteq c'$. A commitment space then is a rooted set of commitment states, in the following sense:

(22) C is a commitment space iff:
- C is a set of commitment states;
- $\exists c \in C\ \forall c' \in C[c \subseteq c']$

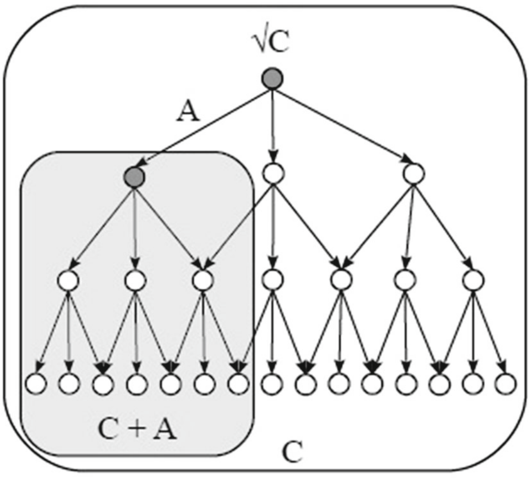
We call the unique commitment state c the **root** of the commitment space, and write $\surd C$. Notice that the root is the intersection $\bigcap C$; it is the set of commitments that all commitment states in C have in common.

We can now model the effect of basic speech acts with respect to commitment spaces. A commitment space C, updated by a speech act A, is the set of commitment states in C that are a superset of the root of C, updated with A.

(23) $C + A = \{c \in C | [\surd C + A] \subseteq c\}$

This is illustrated in (24). The commitment space C consists of the root commitment state $\surd C$, in grey, which is connected to other commitment states that contain increasingly more comprehensive commitments. One such transition, by the speech act A, is indicated. Performing A on C will yield the new commitment space $C + A$, indicated in light grey, with its own root.

(24) Execution of speech act A with respect to commitment space C:



2.2 Denegation, conjunction and disjunction of speech acts

We now can deal with the case of speech act denegation, cf. (21). This is not a basic speech act, but what we will call a meta-speech act. Formally, it cannot be expressed on the level of commitment states, then lifted to commitment spaces, as is the case with basic speech acts, cf. (23). Rather, it must be expressed on the level of commitment spaces directly, which is characteristic for meta-speech acts.

In the case of denegation of a speech act A, a speaker explicitly refrains from performing the speech act A. We can understand this in one of two ways: Either locally, i.e. the speaker refrains from performing A at the current point. Or globally, i.e. the speaker excludes A in all possible future developments. We assume here that denegation has a local character, because we find (25a) to be consistent, in contrast to (25b).

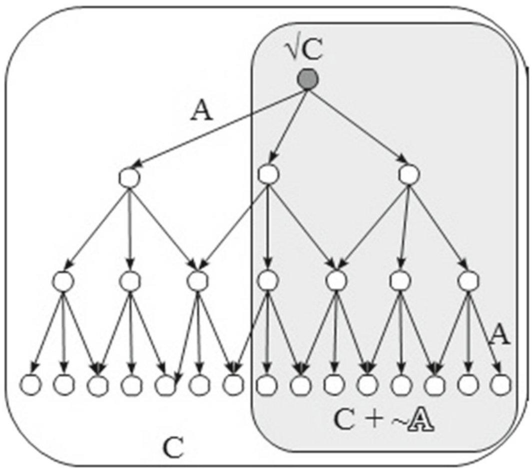
(25) a. I don't promise to come to your party, but I might do so later when I've had a look at my calendar.
b. #I promise to come to your party, but I might say that I won't come after I've had a look in my calendar.

The denegation of A, for which we will write $\sim A$, then can be stated as follows:

(26) $C + \sim A = C - [C + A]$

This is the complement of $C + A$ relative to C, which we can render as follows:

(27) Denegation of speech act A at commitment space C:



We see that with the denegation of a speech act, the root of the commitment state stays the same, and only the future developments are pruned. This is characteristic for the meta-speech acts that we will consider.

Notice that denegation, under this definition, has the expected property that two denegations cancel each other out. Hence, two meta-speech acts can amount to the effect of a basic speech act.

(28) $C + \sim\sim A = C - [C + \sim A] = C - [C - [C + A]] = C + A$

Speech acts can be conjoined, as has been argued for in Krifka (2001) for the case of questions.[8] We find conjunctions of basic speech acts, and conjunctions of meta-speech acts:

[8] See also Krifka (to appear-b) who proposes a theory of questions as meta-speech acts in which the speaker offers a restriction of the commitment space to those speech acts that are possible answers to the question.

(29) a. Put the coat on the hanger. And, put the shoes on the rack.
b. I promise to come to your party. And, I promise to bring a bottle of martini.

Arguably, the conjunction *and* in (29) is not Boolean, as it conjoins non-assertional conversational moves. One can also make the point that in cases like the following, two assertional speech acts, rather than two propositions, are conjoined.

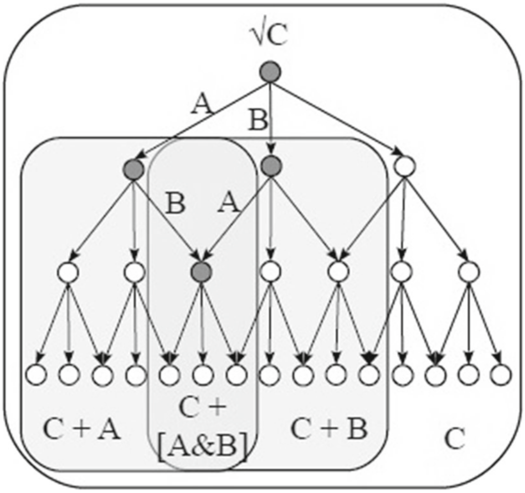
(30) John petted a rabbit. And Mary petted a goat.

Conjunction of speech acts can be modeled as usual as set intersection, on the level of commitment spaces. Using & for speech-act conjunction, we have:

(31) $C + [A\&B] = [C + A] \cap [C + B]$

If A and B are basic speech acts, this amounts to the concatenation of A and B, if this happens to be order-insensitive. This is illustrated in the following diagram.

(32) Conjunction of speech acts:



Notice that the resulting set is a commitment space, as it is rooted.

Can we also define a speech act disjunction? In Krifka (2001) it was argued that speech acts cannot be disjoined, as there is no natural operation like concatenation for conjunction. From this it was derived that non-universal quantifiers cannot scope over speech acts, as they require disjunction for their definition. In the current framework, we can easily define a disjunction for speech acts, based on set union:

(33) $C + [A \vee B] = [C + A] \cup [C + B]$

However, notice that the resulting set is not guaranteed to be rooted. For example, the set union of the two sets $C + A$ and $C + B$ in (32) has two minimal elements, not just one. Hence disjunction does not result in a proper commitment space.

Even if disjunction is not generally defined, it is also not the case that it is generally undefined. If $C + A$ and $C + A'$ have the same root, then their union also has a root, namely the same commitment state. This is the case, for example, for $C+\sim A$ and $C+\sim B$ in (32), as both have the same root, $\surd C$. Hence the disjunction of two denegations is well-defined. In the case of (32), $C+[\sim A\vee\sim B]=[C+\sim A]\cup[C+\sim B]$ is the set of commitment states outside the two shaded areas, which of course also can be gotten by $C+\sim[A\&B]$. Obviously, this is one of de Morgan's laws, now for denegations:

(34) $[\sim A\vee\sim B]=\sim[A\&B]$

Notice that the following holds as well:

(35) $\sim[A\vee B]=[\sim A\&\sim B]$

It is true that $[A\vee B]$ fails to be a proper commitment space, as it is not rooted; but denegation creates a commitment space again, and so the whole operation should be defined. For the empirical motivation of de Morgan's rule with denegation, cf. the following example:

(36) a. I don't promise to marry you or swear to stay with you.
b. $\Leftrightarrow$I don't promise to marry you and I don't swear to stay with you.[9]

2.3 Assertions and GRANTs

The main use of denegation in this paper is to define the meta-speech act of a GRANT.[10] A GRANT indicates a willingness to go along with a possible assertion of a proposition by the other interlocutor. Let us assume the following representation of assertion:

(37) $\mathrm{ASSERT}_{\mathrm{Sp,Ad}}(\Phi)$

This expresses that the speaker asserts the proposition Φ to the addressee. Performed with respect to a commitment state c, this will add the responsibility of Sp for the truth of Φ with respect to Ad. This effect on commitment states can be lifted to commitment spaces, following (23).

A GRANT of a proposition Φ then is a denegation to assert the negation of Φ:

(38) $C+\mathrm{GRANT}_{\mathrm{Sp,Ad}}(\Phi)=C+\sim\mathrm{ASSERT}_{\mathrm{Sp,Ad}}(\neg\Phi)$

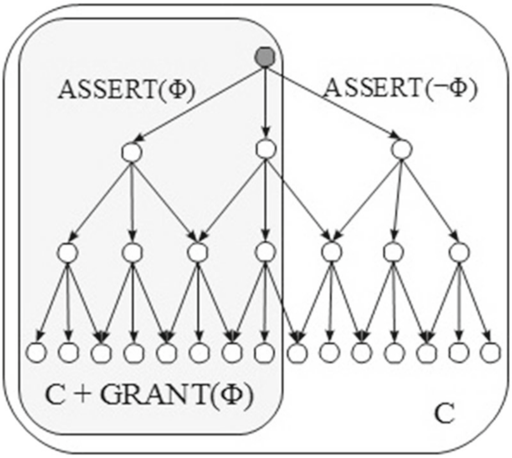
As the speaker and addressee do not change, and we in general do not consider interaction phenomena in this paper, we will drop the Sp and Ad indices henceforward.

9 We thank Barbara Partee for a modification of our original example.

10 The name we chose for this speech act is of course meant to be suggestive, but we are definitely not claiming that the English verb *grant* denotes the speech act GRANT. This is just as with the English verb *assert*, which does not denote the speech act ASSERT, but rather a special case of a strong, formal assertion (cf. Vanderveken 1990–1991).

The result of GRANTing a proposition Φ is illustrated in the following diagram. Notice that GRANTs, as denegations, do not change the root. The GRANT includes, but does not enforce, the assertion of Φ.

(39) GRANTing a proposition

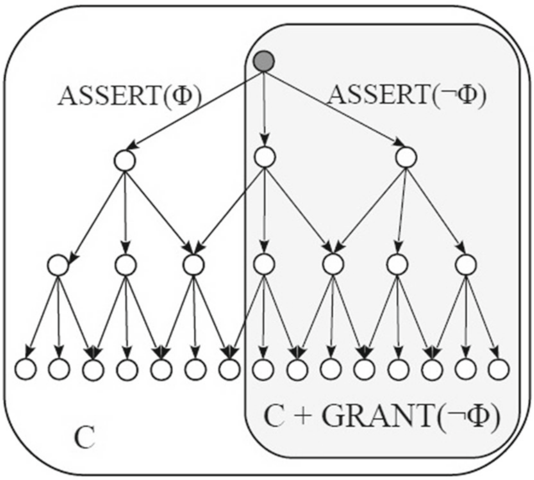


We have the following equivalence, familiar from the modal logic equivalence $\Box\Phi \equiv \neg\Diamond\neg\Phi$:

(40) For all commitment spaces C, $C + \text{ASSERT}(\Phi) = C+ \sim \text{GRANT}(\neg\Phi)$.

The following diagram illustrates GRANT($\neg\Phi$); note that its complement is ASSERT(Φ), illustrating the rule (40).

(41) $C + \text{GRANT}(\neg\Phi)$



This concludes the introduction to the pragmatic framework for speech acts that we assume for this paper. The crucial step consists in the notion of commitment spaces, which allow for speech acts like denegation that affect future possible conversational moves, and that also allow for a limited disjunction on the level of speech acts. For the purpose of the current article, we will need commitment spaces, as they allow to define the speech act notion of GRANT.[11]

Modeling denials, and in particular GRANTs, as changes of commitment spaces is structurally similar to using modal notions. Going from commitment states to commitment spaces corresponds to evaluating propositions at a particular possible world, vs. evaluating them with respect to a set of modally accessible possible worlds. Hence, ASSERTing Φ appears similar to $\Box\Phi$, and GRANTing Φ similar to $\Diamond\Phi$. However, it is important to notice that in the present modeling, it is not the factual content of Φ that is at stake, but rather the conversational commitments that arise with Φ, e.g. the commitments to the truth of Φ in case with assertions. With this in mind, we now turn to a conversational theory of superlative quantifiers.

3 The pragmatics and semantics of superlative quantifiers

3.1 Modelling superlative operators

We are now in a position to model *at least* and *at most* as speech-act related operators, or rather, as operations that help to express meta-speech acts. Take the following example:

(42) Mary petted at most three rabbits.

Intuitively, this says that the maximal number n such that the speaker GRANTs that Mary petted n rabbits is n = 3.

(43) $max\ n:\ \mathrm{GRANT}(|\boldsymbol{rabbit}\ \cap\ \lambda x.\boldsymbol{pet}(\boldsymbol{m}, x)| = n)\ =\ 3$

In our current setup, (43) cannot be interpreted directly. But notice that we can interpret it as saying that for all numbers n with n > 3, the speaker does not GRANT that Mary petted n rabbits, which is a conjunction of denegations of GRANTs:

(44) $C\ +\ \underset{n>3}{\&} \sim \mathrm{GRANT}(|\boldsymbol{rabbit}\ \cap\ \lambda x.\boldsymbol{pet}(\boldsymbol{m}, x)|\ =\ n)$

This generalized conjunction is tantamount to:

(45) $C\ +\ \begin{bmatrix} \sim \mathrm{GRANT}(|\boldsymbol{rabbit}\ \cap\ \lambda x.\boldsymbol{pet}(\boldsymbol{m}, x)|\ =\ 4)\ \& \\ \sim \mathrm{GRANT}(|\boldsymbol{rabbit}\ \cap\ \lambda x.\boldsymbol{pet}(\boldsymbol{m}, x)|\ =\ 5)\ \& \\ \ldots \end{bmatrix}$

With *at most*, the speaker does not make an assertion but rather excludes assertions—here, the assertions that Mary petted 4 or more rabbits. Notice that with (40), (44) is equivalent to the following assertion:

(46) $C\ +\ \underset{n>3}{\&}\mathrm{ASSERT}(|\boldsymbol{rabbit}\ \cap\ \lambda x.\boldsymbol{pet}(\boldsymbol{m}, x)|\ \neq\ n)$

[11] In Krifka (to appear-b) the proposed model is slightly more complex; it involves sequences of commitment spaces, which reflects the dynamics of communication, as in question-answer sequences.

The speaker excludes that Mary petted four or more rabbits, but leaves it open whether she petted three, two, one, or no rabbit at all.

We now consider the case of *at least,* with the following example:

(47) Mary petted at least three rabbits.

This says that the *minimal* number n such that the speaker GRANTs that Mary petted n rabbits is n = 3:

(48) $min\ n:\ \text{GRANT}(|\textbf{\textit{rabbit}} \cap \lambda x.\textbf{\textit{pet}}(\textbf{\textit{m}}, x)| = n) = 3$

This translates into the denegation of GRANTs that Mary petted n rabbits for n smaller than 3:

(49) $C + \underset{n<3}{\&} \sim \text{GRANT}(|\textbf{\textit{rabbit}} \cap \lambda x.\textbf{\textit{pet}}(\textbf{\textit{m}}, x)| = n)$

Which is tantamount to:

(50) $$C + \begin{bmatrix} \sim \text{GRANT}(|\textbf{\textit{rabbit}} \cap \lambda x.\textbf{\textit{pet}}(\textbf{\textit{m}}, x)| = 2)\ \& \\ \sim \text{GRANT}(|\textbf{\textit{rabbit}} \cap \lambda x.\textbf{\textit{pet}}(\textbf{\textit{m}}, x)| = 1) \wedge \\ \sim \text{GRANT}(|\textbf{\textit{rabbit}} \cap \lambda x.\textbf{\textit{pet}}(\textbf{\textit{m}}, x)| = 0) \end{bmatrix}$$

With *at least*, the speaker excludes assertions that Mary petted 2, 1, or 0 rabbits. This is equivalent to the following assertion:

(51) $C + \underset{n<3}{\&} \text{ASSERT}(|\textbf{\textit{rabbit}} \cap \lambda x.\textbf{\textit{pet}}(\textbf{\textit{m}}, x)| \neq n)$

The speaker excludes that the number of rabbits Mary petted is smaller than 3.

3.2 Truth conditions

Representing superlative quantifiers as modifiers of meta-speech acts raises an immediate question. Recall that we hailed as one of the advantages of Keenan and Stavi (1986) the fact that they get the truth conditions right, and one of our complaints against Geurts and Nouwen (2007) was the fact that they don't. But if, as we claim, the meaning of (1a) is not a proposition, how can it get *any* truth conditions, let alone the correct ones?

The heart of our proposal is that superlative quantifiers are modifiers of speech acts, not propositions. What this means is that there is no proposition Φ such that a speaker who utters, say, (1a), can be said to assert Φ—there is no Tarski biconditional "(1a) is true iff Φ." Therefore, (1a), strictly speaking, does not really have truth conditions. However, we do intuitively judge (1a) to be true or false in various circumstances. How so?

The answer is that what we perceive to be truth conditions are derived. We take the natural view that if one makes a series of assertions, then the truth conditions of what the speaker did will be the conjunction of the propositions asserted.

We have argued that (1a) means that the minimal n s.t. the speaker GRANTs that John petted exactly n rabbits is three. As we have seen in (51), from this it follows that the speaker makes the following three assertions:

(52) a. John did not pet exactly two rabbits.
b. John did not pet exactly one rabbit.
c. John did not pet exactly zero rabbits.

This interpretation of (1a) accounts straightforwardly for cases when it is false. Suppose, for example, that John petted exactly two rabbits. Then the content of the assertion (52a) is false, which accounts for the falsity of (1a).

Things are more interesting when the sentence is true. Suppose John petted exactly four rabbits—then (1a) ought to be true. Indeed, the content of all the assertions in (52) would be true. Is this sufficient to account for the truth of (1a)?

Not quite. It *would* be sufficient, if we could establish that the meaning of (1a) is really the three assertions in (52). But if the minimal n s.t. the speaker GRANTs that John petted exactly n rabbits is three, it follows that the speaker makes the assertions in (52), but it does not follow that these are *all* the assertions that she makes. It is quite compatible with our definition of the meaning of superlative quantifiers that the speaker could, for example, also assert (53).

(53) John did not pet exactly four rabbits.

In this case, it would still be the case that the minimal n s.t. the speaker GRANTs that John petted exactly n rabbits is three, yet (1a) would be false, rather than true.

We are, therefore, faced with the following situation. Since it does follow semantically, from our definition, that the speaker who uttered (1a) made the three assertions in (52), it follows that if the state of affairs contradicts one of these assertions, what the speaker said is false. So, if John petted 0, 1, or 2 rabbits, this contradicts one of the assertions we know the speaker made, and therefore we know that what the speaker said was false.

However, if John, in fact, petted exactly 4 rabbits, it doesn't follow semantically that (1a) is true, since it is not excluded by our definition that the speaker also asserted (53). In order to account for the truth of (1a), then, we need to rule out assertions like (53). We cannot do so on logical grounds, since an assertion of (53) would be perfectly consistent with the interpretation of (1a) we are proposing. However, we *can* rule it out on pragmatic grounds, specifically by conversational implicature.

By using a superlative quantifier, the speaker took the trouble to indicate that she accepts the commitments of all the assertions in (52); if she also wanted to commit to the claim that John did not pet exactly n rabbits for other values of n, the maxim of Quantity dictates that she should have indicated that as well. From the fact that she didn't, we can conclude, by a straightforward implicature, that she is not committed to such an assertion. Therefore, the only relevant assertions that the speaker is committed to are those in (52), and since all of them are true, it follows that (1a) is true if John petted exactly four rabbits, which is the result we want.

Note that, according to our view, what (1a) says about values of $n < 3$ is an entailment, while what it says about $n \geq 3$ is an implicature. This asymmetry between the falsity of the sentence (which follows semantically) and its truth (which follows pragmatically) captures the intuition (which also underlies Geurts and Nouwen 2007) that when one says (1a) one doesn't know what the number of rabbits that John petted is; but one does know what that number is *not*.

Let us put this another way: when we deal with propositions, if we know when the propositions is false, we know when it is true. For example, $\Phi \wedge \Psi$ is false iff Φ is false or Ψ is false (or both); therefore, it is true iff this is not the case.

However, things are different when we deal with speech acts. Suppose the speaker talks at some length, and we are told that she made two assertions:

(54) $\mathrm{ASSERT}_{\mathrm{Sp,Ad}}(\Phi)\&\mathrm{ASSERT}_{\mathrm{Sp,Ad}}(\Psi)$

But she might have made additional assertions, which we don't know about. The question is: is what the speaker said true or false?

If either Φ or Ψ is false, we know that what the speaker said was false, because at least one of her speech acts was an assertion of a false proposition. But what if Φ and Ψ are both true? In this case, the two assertions we know about were assertions of true propositions, but it still does not follow that what the speaker said was the truth: because there might have been other relevant assertions, which we don't know about, and which were false. Only if we can rule out, by way of implicature or some other means, that the speaker asserted anything besides Φ and Ψ, can we conclude that what she said was true.

Since superlative quantifiers are illocutionary operators, this is precisely the situation we face. The superlative quantifier tells us what propositions the speaker is committed to: if one of those is false, the sentence is false. But if all are true, it still doesn't follow that the sentence is true, because the superlative quantifier doesn't tell us that the speaker is not committed to additional relevant propositions, which might turn out to be false; we can only conclude this by implicature, and only in this way can we conclude that the sentence is true.

3.3 The role of implicature

The proposal that the truth of sentences with superlative quantifiers comes from implicature has received some experimental support. Hacohen et al. (2011a, b) have presented subjects with pictures and sentences with superlative quantifiers, and asked them to indicate whether the sentences accurately describe the pictures. Reaction times for (correct) true judgments turned out to be significantly longer than reaction times for (correct) false judgments. In contrast, no significant difference was found between true and false judgments with comparative quantifiers. On the assumption that the computation of implicature takes additional time (Bott and Noveck 2004), our theory provides a natural explanation of these results, which would be mysterious under Geurts and Nouwen's account.

Additional experimental support comes from investigations of cancelability. We argue that it follows by implicature that the speaker who utters (1a) GRANTs that John petted exactly three rabbits; the inference, therefore, ought to be cancelable.

Geurts et al. (2010) and Cummins and Katsos (2010) have shown that speakers do not judge (55b) to follow from (55a).

(55) a. Brian has at most two children.
b. Brian has at most three children.

Geurts et al. (2010) follow Geurts and Nouwen's theory: according to them, (55a) entails that it is epistemically impossible that Brian has exactly three children, while (55b) entails that it is epistemically possible; and this is a logical contradiction.

The theory proposed here offers a different explanation: while (55a) *entails* that the speaker denies that Brian has exactly three children, (55b) merely *implicates* that the speaker GRANTs this.

The two theories lead to different predictions. If, as we argue, GRANTing that Brian has three children is an implicature, it ought to be cancelable; and in context in which it is actually canceled, the inference from (55a) to (55b) should go through. In contrast, Geurts and Nouwen's theory predicts that this inference will not be licensed under any context.

Cummins and Katsos (2010) set out to test this issue experimentally.[12] They have found that, in contrast with (55), the inference from (56a) to (56b) *is* accepted as valid.

(56) a. Anne has three children but Brian has at most two children.
b. Anne and Brian each have at most three children.

Cummins and Katsos note that this judgment requires acceptance of the inference from (55a) to (55b). If it were an entailment of (55b) that the speaker GRANTs that Brian has exactly three children, it would not be cancelable, and the inference in (56) would stand no chance of being valid. However, in the context of (56), the implicature is canceled: (56b) merely implicates that the speaker GRANTs for one of Anne and Brian that she or he has three children; since (56a) explicitly says that Anne has three children, the implicature regarding Brian is canceled, and the inference goes through.

The proposal that the truth of superlative quantifiers follows pragmatically can explain yet another way in which they differ from comparative quantifiers. Suppose John petted over a thousand rabbits. Then an utterance of (1b), reproduced below, would clearly be rather odd.

(57) John petted more than two rabbits.

However, we believe it is unquestionable that (57) is true in the situation described. It is usually odd to talk about more than n individuals satisfying a certain property, when the number of individuals actually satisfying the property is substantially higher than n. However, in the right context, such a statement may be acceptable. In fact, we have been able to find on the Web freely occurring examples, with $n = 1$:

(58) a. If one person throws one piece of rubbish on the ground per day then that person throws 365 pieces of rubbish in one year, but Koh Tao has more than one person, in fact 320,000 people.[13]
b. In some instances, your e-mail message may be seen by more than one person…in fact, it may be forwarded to the entire mailing list.[14]

12 Cummins and Katsos's own theory makes crucial use of implicature, but in a way fundamentally different from our theory. We will describe and discuss their proposal in Sect. 5.2.2.

13 Based on http://www.savekohtao.com/index.php?option=com_content&view=section&layout=blog&id=3&Itemid=4&lang=en.

14 http://www.angelfire.com/mi/FAST/inputform.html.

c. I was experiencing this strange feeling with more than one person; in fact, with lots of people of both sexes.[15]

Not that all the examples in (58) the would become much worse if we replace the comparative *more than one person* with *at least two people*.

This phenomenon is easily explained by our proposal that the truth of a sentence containing a superlative quantifier comes from an implicature. A speaker uttering (1a) is taken to assert that John did not pet exactly 2, 1, or 0 rabbits. As the speaker leaves it open whether John did not pet exactly 3, 4, 5…rabbits, the implicature arises that the speaker considers it possible that John did pet exactly 3, 4, 5…rabbits. But this implicature may get weaker the higher the numbers get, because there might be additional reasons why the speaker might consider it impossible that John petted, say, 1,000 rabbits—and then, in fact, would be ready to assert that John did not pet 1,000 rabbits.

The same sort of asymmetry can be seen, perhaps even more clearly, when we consider other speech acts besides assertion. Take requests, for example:

(59) Give me at least three cookies.

The conditions under which this request is *not* satisfied are clear: if the hearer gives the speaker fewer than three cookies—two, one, or no cookies at all. But the conditions under which the request *is* satisfied are less clear: not any number $n \geq 1$ of cookies will equally satisfy the speaker. Four cookies may be better than three, but one thousand may be too many.

3.4 The superlative morphology

Recall that one of the arguments against Geurts and Nouwen (2007) was that they ignore the superlative morphology of superlative quantifiers. In contrast, our theory can account for it, since we take (1a) and (2a) to mean (60a) and (60b) respectively.

(60) a. The minimal ("least large") number n s.t. the speaker GRANTs that John petted exactly n rabbits is 3.

b. The maximal ("most large") number n s.t. the speaker GRANTs that John petted exactly n rabbits is 3.[16]

In order to demonstrate how this superlative reading is derived, we need a brief discussion of the meaning of superlative morphology in general.

The superlative morpheme *-est* usually applies to gradable adjectives. These are usually treated as measure functions from entities to degrees or extents on a scale (Bartsch and Vennemann 1972; Kennedy 1999), so that **high**(x) is the extent to which x is high—the height of x.

[15] http://www.circleofa.org/articles/FallingInLoveWithEveryone.php.

[16] Nouwen (2010) proposes a somewhat different epistemic theory of superlative quantifiers (and other constructions), and Penka (2010) argues that this theory can get a superlative semantics; but Schwarz et al. (2012) argue against this theory, mainly on the grounds of its inadequate handling of NPIs. We will not get into this account further here (but see Sect. 5.4), except to point out that, as an epistemic theory, it suffers from the other problems we note with Geurts and Nouwen (2007).

According to the (more or less) standard view of *-est* (Heim 1999; Kiss and Farkas 2000), its meaning is:

(61) $\lambda A.\lambda R.\lambda x.(R(x) \wedge \forall y((R(y) \wedge y \neq x) \rightarrow A(x) > A(y)))$[17]

Thus, the meaning of *highest mountain* is derived compositionally, by applying (61) twice, to the predicates **high** and **mountain**:

(62) $\lambda x(\mathbf{mountain}(x) \wedge \forall y((\mathbf{mountain}(y) \wedge y \neq x) \rightarrow \mathbf{high}(x) > \mathbf{high}(y)))$

If we apply (62) to the Everest, denoted by the constant **e**, we get:

(63) $\mathbf{mountain}(\mathbf{e}) \wedge \forall y((\mathbf{mountain}(y) \wedge y \neq \mathbf{e}) \rightarrow \mathbf{high}(\mathbf{e}) > \mathbf{high}(y))$

This says that the Everest is a mountain, and that every other mountain is lower than it.

Note that (63) entails that the Everest is a mountain. But is this inference really an entailment? According to Heim's (1999) logical form it is; but nowhere does she argue for this. In our view, *the highest mountain* is presupposed to be an element on a relevant scale of highness—in this case, it is presupposed to be an elevation of the ground. But not all elevations of the ground are mountains—some are hills; hence, *the highest mountain* it is not entailed to be a mountain. Rather, this inference is an implicature, and is therefore cancelable.

Indeed, Heim herself seems to imply that the inference is cancelable. Consider her example:

(64) John climbed the highest mountain.

Heim acknowledges that (64) "may be true on the grounds of John's climbing some puny hill" (p. 1). But a hill (usually defined to be an elevation of the ground less than 2,000 feet high) is not a mountain (2,000 feet and up).

It should be noted that Heim refers here to the comparative reading, rather than the absolute reading; but since Heim's logical forms for both the absolute and comparative readings entail that John climbed a mountain, her willingness to cancel this implication for the comparative reading indicates perhaps that she might have been willing to do so for the absolute reading too.

Be that as it may, it is possible to find attested examples where the absolute reading of the superlative is clearly used, and which show the cancelability of the inference that the highest mountain is a mountain:

(65) a. Our [The Netherlands's] highest mountain is really just a big hill as it's only 322 m high.[18]
b. did I mention the highest mountain in Florida is a fire ant hill…?[19]
c. Nothing like Denmark, where the highest mountain is a mere hill and everything is flat like a pancake[20]

[17] This is the *absolute* reading of *-est*; we are ignoring the *comparative* reading here.

[18] http://www.rnw.nl/english/article/weird-and-wonderful-stories-lowlands.

[19] http://www.datehookup.com/Thread-262702.htm.

[20] www.myspace.com/176016690.

The inference from *x is the P-est Q* to *x is a Q* is cancelable not only when *Q* is the property *mountain*; this fact holds with other properties as well:

(66) a. Does Michigan really want…a president who hails from a Southern state where the biggest lake is a puddle?[21]
b. Science has proved that Pluto (the smallest planet) is not a planet[22]
c. The best defense is a good offense
d. The worst crime is faking it (Kurt Cobain)

We therefore propose revising the theory of superlatives somewhat, so that the meaning of the superlative morpheme is:

(67) $\lambda A.\lambda R.\lambda x.\forall y((R(y) \wedge y \neq x) \rightarrow A(x) > A(y))$

Note that unlike (61), (67) no longer entails $R(x)$; this inference is now derived only by implicature.

The meaning of *highest mountain* is now (68a), and when we apply this to the Everest we get (68b), which does not entail that the Everest is a mountain.

(68) a. $\lambda x.\forall y((\mathbf{mountain}(y) \wedge y \neq x) \rightarrow \mathbf{high}(x) > \mathbf{high}(y))$
b. $\forall y((\mathbf{mountain}(y) \wedge y \neq \mathbf{e}) \rightarrow \mathbf{high}(\mathbf{e}) > \mathbf{high}(y))$

Everest is indeed the highest mountain on earth, and its height is 29,035 feet. It therefore follows logically from this fact combined with (68b) that anything that is higher than 29,035 feet is not a mountain—for then Everest would not be the highest mountain. We can see this even more clearly if we replace the property *highest mountain* with *highest hill*: technically, anything higher than 2,000 feet is a mountain, not a hill. Given that Cavanal Hill, Oklahoma, is the highest hill on earth (at 1,999 feet), it follows that any elevation higher than it is not a hill anymore.

We have seen that the Everest itself is implicated to be a mountain; what is the status of individuals that are lower than the Everest, i.e. lower than 29,035 feet? Of course, we are only talking of individuals in the relevant universe of discourse, i.e. elevations of the earth. Take, for example, K2 in the Himalayas, whose height is 28,251 feet. Can we conclude that it is a mountain? Not logically, but there is a clear implicature that it is. An elevation whose height is 28,000, or 27,000, or 26,000 feet would also be implicated to be a mountain, although this implicature seems to grow weaker as the height goes down, and is ultimately cancelable: indeed, elevations that are under 2,000 feet will no longer be considered mountains.

3.5 Lexical entries for superlative quantifiers

The lexical entry for *at most* is based on the meaning of the *-est* morpheme, as in (67), with one important difference: the inclusion of the illocutionary operator GRANT.

Where does GRANT come from? Geurts and Nouwen face a similar problem, although they use an epistemic operator rather than an illocutionary one; their solution

[21] http://invasivespecieseast.blogspot.co.il/2008/01/week-of-january-6-2008.html.

[22] http://anoushkakohli.blogspot.co.il/.

is to stipulate that the operator is provided lexically.[23] We follow suit, assuming that GRANT is specified lexically; but we go further in speculating that the introduction of GRANT is motivated by the preposition *at*.

While we cannot really substantiate this speculation, we note that illocutionary operators introduced by prepositions are not unheard of. For example, Rivero (1994) notes that in Spanish, if certain prepositions precede a verb in the infinitive, the result is a direct command:

(69) a. ¡A correr!
to run+INF
'Run!'

b. ¡Sin empujar!
without push+INF
'Do not push!'

Assuming that *at* contributes the GRANT operator, and *most* is the superlative morpheme, the lexical entry for *at most* is the following:

(70) $\lambda A.\lambda R.\lambda x.\forall n((\text{GRANT}(R(n)) \wedge n \neq x) \rightarrow A(x) > A(n))$

To illustrate the derivation of the meaning of superlative quantifiers, take (42), repeated below:

(71) Mary petted at most three rabbits.

The meaning of this sentence is built out of the application of *at most*, whose meaning is (70), to its three arguments.

The complement of *most*, *A* in (70), is empty, so we take it to be the identity function. Note that since the value of the identity function is equal to its argument, it trivially gets the properties of its argument. Specifically, if the identity function applies to an item on a scale, such as a number, then its value will again be an item on a scale.

In dealing with the second argument, we adapt Hackl's (2001) account of comparative quantifiers (see also Nouwen 2010). Hackl, following earlier work by Ross and Bresnan, treats *more* as *many+er*; correspondingly, we will treat *most* (as it occurs in *at most*) as *many+est*.

Many is interpreted as a gradable adjective, similarly to *tall*: just like *tall* is the set of individuals that are tall to degree *d*, *many* is the set of (plural) individuals that are numerous to degree *d*, or *d*-many. If the degree is numerical, the lexical meaning of *many* is simply (72).

(72) $\lambda n.\lambda y.|y| = n$

The second argument, *R* in (70), is the property of being a number *n* s.t. the set of rabbits that Mary petted is *n*-many:

(73) $\lambda n.\mathbf{many}(n, \lambda z(\mathbf{pet}(\mathbf{m}, z) \wedge \mathbf{rabbit}(z)))$[24]

[23] Nouwen (2010) also makes use of an epistemic operator; he assumes that it is phonologically null, but admits that he has no independent evidence for it.

[24] See Hackl (2001) for the details of how a correspondingly similar argument is identified on the basis of the surface syntax.

Plugging in the meaning of **many** from (72), we get (74) as the second argument of *at most*.

(74) $\lambda n.|\lambda z(\mathbf{pet}(\mathbf{m}, z) \wedge \mathbf{rabbit}(z))| = n$

The last argument, x in (70), is just the number 3.
Therefore, the formulation of (71) is (75).

(75) $\forall n((\text{GRANT}(|\lambda z(\mathbf{pet}(\mathbf{m}, z) \wedge \mathbf{rabbit}(z))| = n) \wedge n \neq 3) \rightarrow 3 > n)$

This is, of course, equivalent to (76).

(76) $\forall n(\text{GRANT}(|\lambda z(\mathbf{pet}(\mathbf{m}, z) \wedge \mathbf{rabbit}(z))| = n) \rightarrow n \leq 3)$

Note that (76) entails that the speaker does not GRANT that Mary petted n rabbits for values of n greater than 3; by implicature, it follows that she does GRANT this for values less than or equal to 3. This is exactly the interpretation of superlative quantifiers we have proposed, now derived in a principled way. As we have seen, (76) can be given a precise semantics in our framework for representing speech acts.

The meaning of *at least* is derived in a similar fashion.

3.6 Non-numerical scales

Superlative quantifiers do not always apply to numerical scales. Consider the following attested examples, where the *if not* continuation makes it clear what the relevant scale is:

(77) a. Rehabilitation without invasive procedures is at least comparable if not superior to more invasive and costly procedures. [25]
b. This is at least misleading, if not wrong.[26]
c. The agent who bills such expenses is at least unethical, if not criminal.[27]
d. [That] prices continued to rise after the announcement…is at least possible if not certain in at least some parts of Russia.[28]
e. The "Gab Drag" is at least plausible, if not necessary, from a dramatic perspective.[29]
f. This is at least confusing, if not conflicting.[30]

Non-numerical interpretation is possible even when a numeral is involved. As Horn (1972) and Kadmon (1987) have shown, (1a) and (2a) have readings that can be roughly paraphrased as, respectively:

(78) a. John petted exactly three rabbits, and maybe he petted more animals.
b. Maybe John petted exactly three rabbits, but he petted no other animal.

[25] http://blog.myphysicaltherapyspace.com/2010/08/is-acl-surgery-the-spinal-fusion-of-the-lower-extremity.html.

[26] http://stackoverflow.com/questions/3815411/stored-procedure-and-permissions-is-execute-enough.

[27] http://ezinearticles.com/?Avoiding-Publishing-and-Agent-Scams&id=197784.

[28] http://www.kyivpost.com/news/opinion/op_ed/detail/77654/print/.

[29] http://www.whoosh.org/issue28/schultz1.html.

[30] http://www.psg.com/lists/radiusext/2007/msg00147.html.

In these examples, the superlative quantifier applies to entailment (Horn) scales: being superior entails being comparable, being wrong entails being misleading, being criminal entails being unethical, etc.

We formalize this phenomenon by generalizing '$\geq$' beyond numerical comparisons: if P and Q are propositions, and P is higher on the relevant Horn scale than Q, we write $P \geq Q$.

This phenomenon is a generalization of our previous treatment of superlative quantifiers over numerals. To see this, note that one of the equivalent formulations of (1a) is (79).

(79) $\forall n(\mathrm{GRANT}(|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| = n) \rightarrow n \geq 3)$

Now, any proposition of the form $|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| = n$, for $n \geq 3$, entails $|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| \geq 3$. Hence, we can rewrite (79) as (80), where it is assumed that context restricts P to range only over statements about the number of rabbits that John petted.[31]

(80) $\forall P(\mathrm{GRANT}(P) \rightarrow P \geq {}^\wedge|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| \geq 3)$

Similarly, (2a) can be formulated as (81a), which is equivalent to (81b).

(81) a. $\forall n(\mathrm{GRANT}(|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| = n) \rightarrow n \leq 3)$
b. $\forall P(\mathrm{GRANT}(P) \rightarrow P \geq {}^\wedge|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| \leq 3)$

However, we will continue to use the conceptually simple numerical comparison, whenever the scale is numerical.

4 Accounting for the data

Our proposed account can explain the facts motivating Geurts and Nouwen's approach, and also the facts that constitute challenges to their theory.

For the purposes of this paper, we accept Geurts and Nouwen's account of comparative quantifiers: in particular, that they are restricted to combine with first-order properties only. We therefore accept their account of the distributive facts concerning such quantifiers.

Regarding superlative quantifiers, we have seen that our theory provides truth conditions that are intuitively correct, something that Geurts and Nouwen fail to do. We have also seen that our approach, unlike theirs, does justice to the superlative morphology of superlative quantifiers.

Moreover, we have argued that the truth conditions proposed by Geurts and Nouwen are intensional, where, in fact, they ought to be extensional. It might seem that our theory, because it takes superlative quantifiers to be illocutionary operators, does not provide an extensional semantics, but this is wrong.

Sentence (1a) expresses quantification over meta-speech acts, as formalized in our framework, but these are *not* its truth conditions. The truth conditions are *derived* from this representation (partly with the help of implicature). The derived truth conditions are, in effect, the same as Keenan and Stavi (1986), and paraphrased as in (82).

[31] ${}^\wedge\phi$ indicates the intension of ϕ.

(82) The number of rabbits that John petted is greater then or equal to 3

Now, suppose rabbits are Mary's favorite animals. Then (82) is, of course, equivalent to (83).

(83) The number of Mary's favorite animals that John petted is greater than or equal to 3.

Of course, a speaker may wish to assent to one and deny the other, but such a speaker would simply be mistaken. Thus, the truth conditions of superlative quantifiers are extensional, although they are derived from a representation that is not.

Recall that Geurts and Nouwen point out that (1a) and (2a) would be odd if we knew that John petted exactly three rabbits, but for different reasons: the former is false, whereas the latter is literally true but odd. Under the proposed approach, the reason for both is the same: if the speaker knows that John petted exactly three rabbits, then she ought to assert this, and not GRANT that John petted any other number of rabbits. However, both (1a) and (2a) implicate that the speaker GRANTs, but does not assert, that John petted exactly three rabbits, and that the speaker GRANTs this statement for other values of n ($n > 3$ for (1a), $n < 3$ for (2a)). Since this is an implicature, and not an entailment, both (1a) and (2a) are infelicitous rather than false, and the experimental results reported by Geurts et al. (2010) are explained.[32]

We can also explain why *at most*, but not *at least*, licenses NPIs. Following Kadmon and Landman (1993) and Krifka (1991, 1995), an NPI introduces alternatives, and implicates that all the stronger alternatives than the one asserted are false. For example, *anything* introduces alternative properties, but denotes the most general property: **thing**. Usually, all the other properties are stronger, hence are implicated to be false. But this is a contradiction: how can something be a **thing**, without having any more specific property? Therefore, the use of *any* is unacceptable in most environments. However, in a downward entailing context, the other alternatives are weaker: for example, under negation, not being a **thing** is stronger than not having any more specific property. Therefore, the alternatives are not implicated to be false, and *any* is licensed.

Now consider (14), repeated below:

(84) a. *At least three people have ever been in this cave (last century).
b. At most three people have ever been in this cave (last century).

The NPI *ever* introduces alternative sets of times within the reference time—last century—and denotes the most general of them—the entire century. The clause inside the scope of the superlative quantifier implicates that all the stronger alternatives are false.

In order to explain why the NPI is licensed in (84b), we need to show that (85a) is stronger than (85b)

(85) a. At most 3 people have been in this cave at some time or other.
b. At most 3 people have been in this cave last year.

[32] cf. Brasoveanu (2009) who proposes that the infelicity of superlative quantifiers in such cases is pragmatic, rather than semantic. For him, however, this does not follow from the meaning of these quantifiers, but is a stipulated felicity condition.

Since we propose that superlative quantifiers are illocutionary operators, we need to define the notion of relative strength for speech acts. A natural definition is the following:

(86) A_1 is as strong as or stronger than A_2 iff for all commitment spaces C, $C + A_1 \subseteq C + A_2$

In words, A_1 is stronger than A_2 iff every commitment that A_1 creates is also a commitment that A_2 creates, but not vice versa.

In order to show that (85a) is stronger than (85b), one more piece of the puzzle is needed. A speaker who asserts ϕ is committed to the truth of ϕ; but what if ϕ entails ψ? Is the speaker also committed to the truth of ψ? It is arguable whether, in general, this is a reasonable requirement, since often it is very hard to figure out the entailments of a proposition. However, we certainly *can* require this if the entailment from ϕ to ψ is, in some sense that we will not try to make precise, clear and obvious. In this case, indeed, the consistency requirement on commitment spaces requires that a speaker who asserts ϕ is committed to the truth of ψ.

The update with (85a) is (87a), and the update with (85b) is (87b).

(87) a. $C + \underset{n>3}{\&}$ ASSERT($\neg$ 'n people have been in this cave at some time or other')

b. $C + \underset{n>3}{\&}$ ASSERT($\neg$ 'n people have been in this cave last year')

Now, for any n, it is clear and obvious that $\neg$ 'n people have been in this cave at some time or other' entails $\neg$ 'n people have been in this cave last year'; therefore, if a commitment state contains the commitments of the former, it ought, by consistency, also to contain the commitments of the latter. We can therefore conclude that the commitment space in (87a) is a subset of the commitment space in (87b), i.e. the former is stronger than the latter, which explains the licensing of NPIs with *at most*.

In order to explain why (84a) is bad, we need only point out that (88a) is not stronger than (88b).

(88) a. At least 3 people have been in this cave at some time or other.

b. At least 3 people have been in this cave last year.

Intuitively, if the speaker minimally GRANTs that 3 people have been in this case last year, then by a simple logical inference, the speaker must be prepared to minimally GRANT that 3 people have been in this cave at some time or other.

In order to account for the missing readings of superlative quantifiers, as exemplified by (7b), we need to discuss more thoroughly the phenomenon of embedded superlative quantifiers.

5 Embedding

5.1 Constraints on embedding

Because Keenan and Stavi (1986) claim that superlative quantifiers are just generalized quantifiers, their theory predicts that they should be freely embeddable. This prediction is not borne out, as pointed out by Geurts and Nouwen. They present the following examples:

(89) a. $\left\{\begin{array}{l}\text{Each}\\ \text{Most}\\ \text{?About five}\\ \text{*None}\end{array}\right\}$ of the guests danced with at least/most three of the waitresses.

b. ?Betty didn't have at least/most three martinis.

When superlative quantifiers are replaced in these examples by epistemic modals, they claim, the judgments are the same. Geurts and Nouwen do not explain why epistemic modals behave in this way, but consider this fact evidence for their theory, according to which superlative quantifiers *are* epistemic modals.

We would be hesitant to draw such a conclusion based on only two examples. But even if it is true that superlative quantifiers are embedded in the same environments in which epistemic modals are (which is an empirical question), all that such a fact would demonstrate is that there is something similar in the meanings of superlative quantifiers and epistemic modals. In fact, it is often claimed that at least some epistemic modals are speech act modifiers themselves (see Cohen 2010a for a recent argument to that effect). If, as we believe, superlative quantifiers are also speech act modifiers, this would explain the similarity between them.

What is the prediction of our theory regarding embeddings of superlative quantifiers? At first sight, it might appear that we predict that embedding is impossible, since superlative quantifiers are speech act modifiers. But this is not quite right: as we have seen in Sect. 2 (see also Krifka to appear-a), although there are contexts where speech acts operators cannot be embedded, there are also contexts where they *can*.

We therefore ought to investigate the contexts under which superlative quantifiers can be embedded, and those where they can't, and see whether this distribution can be accounted for under the assumption that they are illocutionary operators. Let us look at a few examples.

5.2 Downward entailing environments

5.2.1 Quantifiers

On the basis of (89a), Geurts and Nouwen claim that superlative quantifiers are good in the scope of strong quantifiers, but bad in the scope of weak ones. But the use of the partitive *Q of the guests* actually prefers the strong reading of the quantifiers, so this conclusion is dubious.

Note that the only quantifier that is really bad in (89a) is *none*, whose scope is a downward entailing environment; in order to explain the unacceptability of the sentence, we need to look more closely at the behavior of superlative quantifiers in such environments.

5.2.2 Superlative quantifiers are often bad…

As example (89b) indicates, superlative quantifiers are not good in the scope of negation. Nilsen (2007) observes that superlative quantifiers are generally bad in downward entailing contexts:

(90) a. ??John hardly ate at least three apples.
b. ??Policemen rarely carry at least two guns.
c. ??This won't take at least 50 minutes.

Compare this with the acceptability of comparative quantifiers in the same environments:

(91) a. John hardly ate more than three apples.
b. Policemen rarely carry more than two guns.
c. This won't take more than 45 minutes.

As we have seen, Geurts and Nouwen's explanation, namely that superlative quantifiers behave like epistemic modals, is not satisfactory.

Alternative approaches are taken by Büring (2007) and Cummins and Katsos (2010), who can be seen as proposing more sophisticated versions of Keenan and Stavi's (1986) ideas. While there are important differences between the two theories, both propose, in essence, that (1a) is interpreted as (92).

(92) John petted three or more rabbits.

Büring and Cummins and Katsos demonstrate how the use of the disjunction can explain many of the puzzling facts about superlative quantifiers discussed above.

However, these theories, just like Keenan and Stavi (1986), predict that superlative quantifiers can be freely embeddable; in particular, they predict that superlative quantifiers can occur in the scope of downward entailing contexts. Indeed, although *at least three* is bad in downward entailing contexts, *three or more* is perfectly good, as can be seen by the following attested examples:

(93) a. [A]gainst Pittsburgh, five players had two receptions, and nobody had three or more.[33]
b. Nobody wants three or more inboxes and calendars in the government[34]
c. Nobody wants to spend three days or more in hospital if they could be safely back home within 24 hours[35]
d. Five of you managed to guess two numbers correctly, but alas nobody got three or more right[36]

None of the previous theories, then, successfully accounts for the inability of superlative quantifiers to be under the scope of downward entailing operators. But what, then, is the explanation of this fact?

To answer this question, let us consider the meaning of the affirmative counterpart to (89b):

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33 http://www.baltimorebeatdown.com/2010/10/8/1738491/ravens-pass-defense-vs-broncos-pass-offense.

34 http://www.bluespace.com/blog/type1.

35 www.cdhb.govt.nz/communications/healthline/issue61.pdf.

36 http://diamondgeezer.blogspot.com/2004/04/youre-predictable-noon-deadline-has.html.

(94) Betty had at least three martinis.

According to the theory proposed here, (94) means that the speaker asserts, for all values of $n < 3$, that it is false that Betty had exactly n martinis.

Therefore, if Betty had fewer than three martinis, (94) is false. If Betty had n martinis, for some $n \geq 3$, the sentence is true, but this truth follows pragmatically, by way of conversational implicature, rather than semantically. The speaker could have also denied that Betty had n martinis, and this would have been a stronger statement than the one she actually chose to make. But the fact that the speaker chose not to make that statement, implicates that the speaker actually GRANTs, for all $n \geq 3$,[37] that Betty had n martinis. Since we assume that the speaker is committed to the disjunction of all her contextually relevant GRANTs, it follows that she is committed to the disjunction, one of whose elements is n, and (94) is therefore true.

Thus, the falsity of (94) follows semantically, but its truth requires a scalar implicature. So (94) does not have standard truth conditions: to get a proposition from it, we need a scalar implicature. The implicature gets us a strengthened reading, which does have truth conditions.

It is well established that scalar implicatures do not survive when triggered in downward entailing contexts. For example, implicature usually causes disjunction to receive an exclusive interpretation. But in the antecedent of a conditional, disjunction is usually interpreted inclusively, because the implicature doesn't apply:

(95) If you drink or smoke, you will become ill

Sentence (95) cannot mean that one who drinks *and* smokes will escape illness!

In fact, Chierchia (2004) observes that scalar implicature cannot be embedded in any context that licenses *any*, and explains this fact as follows. Normally, the reading derived by scalar implicature is stronger than the literal meaning. For example, exclusive disjunction is stronger than inclusive disjunction. However, in contexts that license *any*, the statement derived by the implicature is actually weaker than the original statement, and this is not allowed, by what he calls "the Strength Condition". Thus, (96), in which disjunction is exclusive, is actually weaker than (95), and this is why this interpretation is not generated.

(96) If you drink or smoke but not both, you will become ill.

What about superlative quantifiers? Since their complete truth conditions are generated by scalar implicature, it follows that it should not be possible to embed them in a downward entailing context.

Let us, for concreteness, see how this comes about in the case of negation. The formalization of the affirmative (94) is:

(97) $$\begin{aligned} C + &\sim \mathrm{GRANT}(|\mathbf{martinis} \cap \lambda x.\mathbf{have}(\mathbf{b}, x)| = 2)\& \\ &\sim \mathrm{GRANT}(|\mathbf{martinis} \cap \lambda x.\mathbf{have}(\mathbf{b}, x)| = 1)\& \\ &\sim \mathrm{GRANT}(|\mathbf{martinis} \cap \lambda x.\mathbf{have}(\mathbf{b}, x)| = 0) \end{aligned}$$

If we treat (89b) as the denegation of (94), and apply de Morgan and the complement rule, its formalization would be:

[37] Though not, perhaps, for values of n that are *much* greater than 3.

(98) $C+\text{GRANT}(|\textbf{martinis} \cap \lambda x.\textbf{have}(\textbf{b}, x)| = 2)\vee$
$\text{GRANT}(|\textbf{martinis} \cap \lambda x.\textbf{have}(\textbf{b}, x)| = 1)\vee$
$\text{GRANT}(|\textbf{martinis} \cap \lambda x.\textbf{have}(\textbf{b}, x)| = 0)$

What are the derived truth conditions? As before, they ought to be determined by what the speaker GRANTs. But we have, in fact, very little information about this. We know that the speaker GRANTs at least one statement with $n \leq 2$, but we don't know which. And the speaker may GRANT other statements, with various values of n—this is not precluded by (98). Thus, we can say very little about the truth conditions of (89b).

Suppose Betty had exactly two martinis; if the speaker GRANTs the corresponding statement, her utterance would be true; but we don't know this. Now suppose Betty had exactly four martinis. Then we know that one of the statements the speaker GRANTs is false; but perhaps she also GRANTs that Betty had exactly four martinis, which would be true. In short, unlike the case of non-negated superlative quantifiers, truth conditions are not defined for negated ones, either semantically or pragmatically. This is why negated superlative quantifiers are bad.

We can now explain why (7b), repeated below, does not have the reading where *at most* is under the scope of *may*.

(99) You may have at most two martinis.

The scope of *may* licenses *any*, as demonstrated by (100).

(100) You may pick any card.

This is explained by the fact that the purpose of permissions is to lift restrictions for the addressee: a permission to pick a card with unspecified characteristics is stronger than a permission to pick, say, the Ace of Spades, since it gives the addressee more options.

Chierchia (2004) points out that, indeed, implicatures do not survive under *may*: (101) does not mean that you are not allowed to both drink and smoke.

(101) You may drink or smoke.

Since implicature is crucial to the derivation of the truth conditions of sentences involving superlative quantifiers, the reading of (99) where it is under the scope of *may* is uninterpretable, and this is why it is not attested. If *at most* takes scope over *may*, there is, of course, no problem.

When *at most* has wide scope, we get the intended meaning, in the following way. The utterance means that the maximal n s.t. I GRANT that you may have n martinis is 2, i.e. for all values of $n > 2$, I deny that you may have n martinis. By implicature, I GRANT that you may have two martinis, I GRANT that you may have one martini, 1, and I GRANT that you may have no martinis at all. Now, recall that to GRANT Φ is to explicitly refrain from denying $\neg\Phi$. To explicitly refrain from denying that you may have n martinis is, in effect, to permit you to have n martinis. Hence, we get the correct interpretation: the addressee is prohibited from having more than two martinis, but is permitted to have less.

If superlative quantifiers are bad in downward entailing environments, it follows that that they are positive polarity items. Indeed, they behave similarly to PPIs. For example, it is well known that PPIs are typically not good with positive questions, but require negative questions (Borkin 1971; Pope 1972):

(102) a. $\begin{Bmatrix}\text{*Is}\\ \text{Isn't}\end{Bmatrix}$ it rather cold for this time of year?

b. $\begin{Bmatrix}\text{*Are}\\ \text{Aren't}\end{Bmatrix}$ you pretty tired?

c. $\begin{Bmatrix}\text{*Does}\\ \text{Doesn't}\end{Bmatrix}$ he have TONS of money?[38]

The same behavior is observed when a superlative quantifier is embedded under a question operator:

(103) $\begin{Bmatrix}\text{*Did}\\ \text{Didn't}\end{Bmatrix}$ John have at least three martinis?

Of course, in an echoing or contrast context, a superlative quantifier can be acceptable in a positive question. Suppose that at a meeting of Alcoholics Anonymous, it was decided that anyone who drank no more than three martinis would be honored by a public mention of the person's name. We know John to be a very heavy drinker, so when we hear his name announced, we can certainly ask, incredulously:

(104) Did John have at most three martinis?

5.2.3 …but sometimes good

The facts concerning embeddings of superlative quantifiers in downward entailing environments are not so simple, however. There are cases when superlative quantifiers are actually perfectly good in such environments, as exemplified by the following minimal pair, adapted from Nilsen (2007):

(105) a. Click away at the *Finalize your order* button: You will get a discount if you click it at least twice.

b. ??Don't click incessantly on the *Finalize your order* button: You will generate multiple orders if you click it at least twice.

The difference in acceptability between (105a) and (105b) is striking: yet they appear to make very similar statements, and in both, the superlative quantifier is in a downward entailing environment. The only difference appears to be that in (105a), the consequences of clicking multiple times on the button are "good", whereas in (105b), the consequences are "bad".

What counts as "good" or "bad" consequences is, of course, not easy to define. Yet, it is generally fairly clear intuitively what the speaker judges to be good or bad. We are not claiming that it is a strict rule that superlative quantifiers *never* occur in the antecedents of conditionals with "bad" consequents, but the tendency is quite strong. Indeed, a corpus study (Shapira 2010) reveals that only 5.7 % of conditionals with *at least* in the antecedent have "bad" consequents. Shapira found that, in contrast, 45.05 % of conditionals with *more than* in the antecedent have "bad" consequents.

[38] As Bolinger (1978) points out, these negative questions actually have another reading, where PPIs are *not* licensed; under this reading, the speaker is seeking confirmation for the propositions that it is *not* cold, that the hearer is *not* tired, and that he does *not* have much money, respectively.

Nilsen (2007) attempts to account for this phenomenon by stipulating that *at least* n presupposes that n is the least useful of the alternatives. But apart from being stipulative, this is incorrect: in (1a), there is no reasonable sense in which three rabbits is the least "useful" among the alternatives.

How, then, can we account for these facts?

To answer this question, let us consider the antecedent of the conditional as an asserted full sentence:

(106) You will click at least twice.

According to our theory, (106) means that the speaker asserts that the hearer will not click exactly n times for $n = 0$ and $n = 1$.

As we have seen, if the hearer clicks exactly zero or one times, (106) is false. If the hearer clicks $n \geq 2$ times, (106) is true, but its truth follows pragmatically, by way of conversational implicature, rather than semantically: by not explicitly excluding higher values of n, the speaker implicates that she is only committed to the assertions that $n \neq 0$ and $\neq 1$; since the contents of both these assertions are satisfied for $n \geq 2$, (106) is true.

Thus the falsity of (106) follows semantically, but its truth requires an implicature. So (106) does not have standard truth conditions: to get a proposition from it, we need a scalar implicature. Since scalar implicature cannot be embedded in a downward entailing contexts, superlative quantifiers are predicted to be bad in such environments.

Yet, as we have seen, this prediction is not borne out. While (105b) is indeed bad, (105a) is fine. How can we explain this fact?

If there were an alternative way to interpret (106), one that would provide us with truth conditions without implicature, it would be possible to embed the superlative quantifier in a downward entailing environment. In other words, we need to get the strengthened meaning of (106), namely that you click twice or more, without implicature.

It so happens that *at least* does have such an alternative interpretation. Kay (1992) identifies three senses of *at least*, one of which is particularly relevant here.[39] This is the interpretation Kay calls the *evaluative* sense of *at least*, which can be exemplified by the following sentence:

(107) At least this hotel is centrally located.

Several points should be made regarding this sense.[40]

First, note that (107) presupposes that being centrally located is a desirable property for hotels. If this presupposition is not satisfied, as in (108), the sentence is odd.

(108) # At least this hotel is noisy.

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[39] The other two are *scalar*, which is roughly the sense we have been dealing with so far in this paper, and *rhetorical retreat*, which is exemplified by:

(i) a. Mary is at home—at least John's car is in the driveway.
 b. Mary is at home—at least I think so.
 c. Mary is at home—at least that's what Sue said.
 d. Mary will help me—at least on the first draft, if it doesn't rain, when I've finished the outline,…

[40] Rullmann and Nakanishi (2009) propose a not dissimilar interpretation of *at least* that they call *concessive*.

Kay describes the evaluative use of *at least* when applied to a root clause. When *at least* is embedded in the antecedent of a conditional, the felicity of the evaluative reading requires that the property be a "positive" one; but sometimes we can only tell this from the consequent. For example, there is nothing inherently good or bad about being on Walnut St.; the felicity of the following sentences depends on the "goodness" of the consequent: taking the hotel is "good", going elsewhere is "bad".

(109) a. If this hotel is at least on Walnut St., I will take it.
b. #If this hotel is at least on Walnut St., I will go elsewhere.

Second, it follows from (107) that the hotel is centrally located, and that being centrally located is the minimal requirement for the speaker's goals (presumably, staying at the hotel), though the hotel may possess additional good qualities.

Third, Kay points out that the property of being centrally located is not the maximally positive one, and there are some positive properties that the hotel fails to have. In our view this last point is implicated, rather than entailed by (107), because it can be canceled:

(110) You should consider this hotel. At least it's centrally located, and possibly it's the perfect hotel for you.

Compare this with the impossibility of canceling the entailment that being centrally located is the minimal requirement:

(111) #This hotel is at least centrally located, but it would be ok even if it were far away.

Fourth, the evaluative reading, unlike the reading of *at least* we discussed up to now, ought not to be analyzed as a speech act modifier. One piece of evidence for this is the fact that (107) is perfectly felicitous even when the speaker knows all the properties of this hotel, whereas, as pointed out above, (1a) would be odd if the speaker knew exactly how many rabbits John petted.

Kay discusses briefly the syntax of the evaluative *at least*, and argues that it functions as an unfocused parenthetical: "Initial, final, and preverbal position are favorite places for parenthetical insertions in English, though they are not the only such positions available" (p. 318). We assume that, perhaps depending on intonation and context, the evaluative reading of *at least* is, in principle, always available.

We suggest that embedding a sentence such as (106) in the scope of a conditional is another way to force it to receive an evaluative interpretation, since the illocutionary reading is not available. The resulting meaning is as follows:

1. Presupposition: Clicking (exactly) twice is good
2. Implicature: You will click less than some maximally good limit
3. Entailment: Two is the minimal number of clicks you will perform.

When (106) is embedded in a downward entailing environment, as in (105a), the presupposition is satisfied, since getting a discount is a good thing. As discussed above, the implicature disappears, because of the Strength Condition. We are left with the entailment, which gives us the desired truth conditions: if twice is the minimal number of clicks you will perform, you will get a discount.

In contrast, we cannot embed (106) in (105b), because, in this case, the presupposition is not satisfied: clicking twice is not good, because it will generate multiple orders.

Kay (1992) does not discuss the possibility of an evaluative reading for the other superlative quantifier, *at most*, but it seems to have an evaluative interpretation too:

(112) This is a bad hotel; at most, it's centrally located.

Just as with *at least*, being centrally located is still presupposed to be a good thing. The difference is in the entailment: being centrally located is less than the minimal requirement, so that the sentence entails that nothing better than being centrally located can be said about the hotel.

Now consider (113).

(113) You will click at most twice

If (113) receives the evaluative reading, the result is this:

1. Presupposition: Clicking (exactly) twice is good
2. Implicature: you will click more than some maximally bad limit
3. Entailment: the maximal number of clicks you will perform is two.

When (113) is embedded as in (114a), the presupposition is satisfied, the implicature disappears, and the entailment provides us with the correct truth conditions: if two is the maximal number of clicks you perform, you will get a discount. This is impossible in (114b), because the presupposition is not satisfied.

(114) a. If you click at most twice, you will get a discount.
b. #If you click at most twice, the transaction will be canceled.

The behavior of both superlative quantifiers in the antecedent of a conditional is thereby explained.

There is further evidence that the superlative quantifiers in these examples really get the evaluative reading and not the illocutionary interpretation such quantifiers usually receive. Recall that one of the characteristics of superlative quantifiers, as opposed to comparative ones, is that they are not ambiguous under a deontic modal. So, (115) can only mean that the hearer is not allowed to have more than one drink, not that it's ok if the hearer has no more than one.

(115) You may have at most one drink

However, this reading *is* available with superlative quantifiers in the antecedents of conditionals with "good" consequents. For example, suppose Mary is invited to a party. She is a very moderate drinker, so she decides that if she is allowed to have no more than one drink, she will come. Mary can then say:

(116) If I may have at most one drink, I will come to the party.

Note that, in this case, the superlative quantifier is in the scope of *may*, which it is not supposed to be able to do under the illocutionary interpretation. This is an indication that the superlative quantifier receives a different reading. Since the consequent is "good", this different reading is plausibly the evaluative interpretation.

In contrast, note what happens if the consequent is "bad." Suppose John is also invited to the party. Unlike Mary, John believes that a party is not good unless there is plenty of alcohol, and demands that guests not be allowed to have fewer than two drinks. Yet, he cannot say:

(117) #If guests may have at most one drink, I will not come to the party.

The reason is that the consequent is "bad", so the superlative quantifier cannot be in the antecedent of the quantifier.

Additional evidence that the reading in question is different from the "normal" reading of superlative quantifiers (which we analyze as illocutionary operators) comes from the following. As we have seen above, usually *at most three* does not entail *at most two* (Geurts et al. 2010; Cummins and Katsos 2010). For example, subjects judge the inference from (118a) to (118b) to be invalid.

(118) a. Berta has had at most two drinks
b. Berta has had at most three drinks

Interestingly, Cummins and Katsos have found that the inference from (119a) to (119b) *is* judged to be valid.

(119) a. If Berta has had at most three drinks, she is fit to drive. Berta has had at most two drinks.
b. Berta is fit to drive.

They point out that this inference can only be valid if, in this context, (118a) does entail (118b), and conclude that, in general, such inferences are valid if the superlative quantifier is embedded in the antecedent of a conditional.

But note that the consequent of the conditional in (119a) is "good": it is a good thing for Berta to be fit to drive. The sentence would be much worse if the consequent were "bad", as in (120), a fact that Cummins and Katsos would be hard-pressed to explain.

(120) ??If Berta has had at most three drinks, she won't enjoy the party.

Under the theory argued for here, when *at most* is interpreted as an illocutionary operator, (118a) pragmatically clashes with *never* entails (118b). The superlative quantifier in the antecedent of the conditional in (119a) is interpreted differently, as evaluative. This provides a natural explanation of the facts.

The reason why (120) is bad is simple: the presupposition of the evaluative readings, namely that the consequent be "bad", is not satisfied (and the illocutionary reading of the superlative quantifier is ruled out, because, as discussed above, the implicature does not survive a downward entailing context and the sentence lacks truth conditions).

In contrast, (119a) is fine, because although the superlative quantifier cannot be interpreted as an illocutionary operator (because it is embedded in a downward entailing environment), it can receive the evaluative reading (because the consequent is "good"). Under the evaluative reading (but *only* under the evaluative reading) (118a) says that the maximal number of drinks that Berta has had is two, and (118b) says that the maximal number of drinks that Berta has had is three. Clearly, under this reading, (118a) entails (118b). If, as we argue, the superlative quantifier in (119a) receives the evaluative interpretation, the inference to (119b) follows immediately.

Hence, the fact that the inference in (119) goes through *only* if the consequent of the conditional is “good” provides further evidence that, in these environments, the superlative quantifier receives the evaluative interpretation.

Interestingly, NPIs behave in a way that is exactly the opposite of that of superlative quantifiers—they are fine if the consequent is “bad”, but ruled out if the consequent is “good” (Lakoff 1969)[41]:

(121) a. If you eat $\left\{\begin{array}{l}\text{some}\\ \text{*any}\end{array}\right\}$ spinach I will give you \$10

b. If you eat $\left\{\begin{array}{l}\text{*some}\\ \text{any}\end{array}\right\}$ candy I will whip you.

Regine Eckardt (pc) shows that the effect is even more pronounced with strong NPIs:

(122) If you budge an inch, I will $\left\{\begin{array}{l}\text{kill}\\ \text{*thank}\end{array}\right\}$ you.

We have already seen that superlative quantifiers are PPIs; it is tempting to speculate that our account can be extended to other PPIs, i.e. that they have an evaluative interpretation too. Possibly, NPIs also have something like an evaluative interpretation, except that they presuppose that an undesirable, rather than a desirable property. However, we will not explore these intriguing possibilities further here.

As noted above, the conditional form helps to make it clear whether the relevant property is “good” or “bad”, but it is not necessary for the evaluative interpretation. Note, for example, the difference between (123a) and (123b).

(123) a. No one would accept at most three weeks additional holiday in exchange for having to work an extra half day every week (Anita Mittwoch, pc)

b. * No one would accept at most \$100 a month pay reduction in exchange for a company car.

In both cases, the superlative quantifier is in the scope of the downward entailing operator *no one*, hence the illocutionary interpretation is impossible. Sentence (123a) is fine, because having additional holiday is a good thing[42]; while (123b) is bad, because pay reduction is a bad thing.

Indeed, this is how the evaluative reading behaves in general: (124a) is bad but (124b) is good, because being clean is good but being noisy is bad.

(124) a. No one would accept a hotel that is at most clean

b. * No one would accept a hotel that is at most noisy

Nilsen (2007) notes that superlative quantifiers in the restrictor of a universal behave the same way as in the antecedent of a conditional—they are acceptable if the consequent is “good”, but are ruled out if it is “bad”:

(125) a. Everybody who donates $\left\{\begin{array}{l}\text{more than}\\ \text{at least}\end{array}\right\}$ 10 BGN will get a thank you postcard.

[41] We are thankful to Barbara Partee for drawing our attention to this paper.

[42] The sentence says that it is not *sufficiently* good to compensate for the extra work, but having a holiday is still a good thing in itself.

b. Everybody who uses $\left\{\begin{array}{l}\text{more than}\\ \text{\#at least}\end{array}\right\}$ three exclamation marks is a fool.

The explanation we propose is the same as we have proposed for conditionals. The restrictor of a universal is a downward entailing context, in which conversational implicatures are canceled: the quantification domain of (126) includes individuals who drink *and* smoke.

(126) Everybody who drinks or smokes will become ill.

Therefore, only the evaluative sense of superlative quantifiers is allowed, and it requires that the nuclear scope will be perceived to be positive. Since getting a thank you postcard is a good thing, (125a) is fine; but since being a fool is bad, (125b) is odd.

If the consequent of the conditional is a deontic modal, there is a clear difference in acceptability, depending on the type of modal. Superlative quantifiers are fine if the consequent is a permission, but bad if the consequent is an obligation:

(127) a. If you're $\left\{\begin{array}{l}\text{more than}\\ \text{\#at least}\end{array}\right\}$ 30 minutes late you must report immediately to the Office of the Registrar, Room 2122, South Building (Nilsen 2007).

b. If you arrive at least 30 minutes early, you may come to my office for a coffee.

(128) a. If you have $\left\{\begin{array}{l}\text{less than}\\ \text{\#at most}\end{array}\right\}$ \$50 in your pocket, you ought to go to the bank to get more.

b. If you make at most \$500 a month, you may apply for a stipend.

Again, the explanation appears to be related to the good/bad distinction. In general, obligations are "bad," but permissions are "good." Thus, the presupposition of the evaluative sense of superlative quantifiers is satisfied with obligations, but violated with permissions.

The same phenomenon obtains regarding superlative quantifiers in the restrictor of quantifiers: obligations are considered "bad," whereas permissions are "good":

(129) a. Campaign finance laws make it a campaign's responsibility to disclose the occupation and employer of everybody who contributes $\left\{\begin{array}{l}\text{more than}\\ \text{\#at least}\end{array}\right\}$ \$200 (Nilsen 2007)

b. Everybody who contributed at least \$200 may vote in the Primaries.

c. Every man who had $\left\{\begin{array}{l}\text{more than}\\ \text{\#at least}\end{array}\right\}$ two martinis must refrain from driving.

Epistemic modals present an interesting problem. Geurts and Nouwen note that superlative quantifiers may be embedded in the antecedent of a conditional, if the consequent contains an epistemic modal, either of necessity or possibility:

(130) If Betty had at least three martinis, she must/may have been drunk.

They admit, however, that their theory cannot handle this example.

Similar facts obtain for superlative quantifiers in the restrictor of a quantifier:

(131) Everybody who had at least three martinis, must/may have been drunk.

As we have seen, the antecedent of a conditional and the restrictor of a universal are downward entailing context, in which scalar implicature does not survive; since an implicature is necessary to derive truth conditions, the antecedent does not have derived truth conditions, hence the sentences ought to be ruled out; yet they are acceptable.

One may wonder whether such sentences are acceptable because the superlative quantifier receives an evaluative interpretation, but this is implausible. Epistemic modals are neutral with respect to "good" or "bad": in most contexts, there is nothing inherently "good" or "bad" about believing something to a high or low degree. Hence, the presupposition of the evaluative interpretation is not satisfied.

A possible solution to this behavior of superlative quantifiers may be provided by the type of conditionals that Sweetser (1996) calls *meta-metaphorical conditionals*. They are exemplified by the following:

(132) a. If the Île de la cité is the heart of Paris, the Seine is the aorta.
b. If life is a candle-flame, then people are the moths burned on the flame.

These conditionals relate two metaphors that are literally false (the Île de la cité is not a heart, the Seine is not an aorta, etc.). However, people may still refuse to assert their falsity, but accept them as metaphors, and be willing to assert them. The intended meaning appears to be that if the speaker is willing to assert the antecedent, she would also be willing to assert the consequent.

We propose that conditionals containing a superlative quantifier in the antecedent and an epistemic modal in the consequent are interpreted in a similar way: the conditional does not relate two propositions, but rather two speech acts.[43] It is the main claim of this paper that superlative quantifiers are interpreted as (meta) speech acts; hence the antecedent of (130), containing a superlative quantifier, is a speech act: the speaker denies that Betty had two martinis, denies that Betty had one martini, and denies that Betty had zero martinis. Following the theory of epistemic modals as illocutionary operators (Cohen 2010a; cf. Sect. 5.1), the consequent, too, is a speech act: it is an assertion that Betty was drunk, with a high (for *must*) or low (for *may*) degree of strength. What the conditional says is that if the speaker is willing to make the first speech act, then she will be willing to make the second speech act too.

In a similar way, if the speaker utters (131), she is saying that for every individual x s.t. she is willing to deny that x had n martinis if $n < 3$, she is willing to assert with a high/low degree of belief that x was drunk.

Other cases where conditionals appear to relate two speech acts are exemplified by the following attested examples. Significantly, both describe rules of a card game:

(133) a. Ace-High [is a] five-card hand with an ace but no pair; if nobody has at least a pair, it's the winning hand.[44]
b. If nobody reveals a hand (that is, nobody has at least 3 of a kind) everyone still in may make another exchange.[45]

[43] We make no claim here about the meaning of conditionals in general, but only about this specific construction.

[44] http://faq.holdemmanager.com/glossary.php/letter/A.

[45] http://wardbaxter.com/poker/drawgames.htm.

In both examples, the superlative quantifier is in a downward entailing environment, and the consequent appears to be "good"; hence, one might think that this is the evaluative interpretation. But note that the sentences remain equally acceptable if we make the consequent "bad":

(134) a. If nobody has at least a pair, the game is canceled.
b. If nobody has at least 3 of a kind, everyone loses.

We propose that in the context of stating the rules of the game, the consequent is interpreted not as a proposition, but as a declarative speech act, declaring the winner, the cancellation of the game, etc. Hence, the conditional relates two speech acts: the antecedent of (133a) says that there is no participant x for which the speaker denies that x has only two pair, one pair, and so on (for all weaker hands); the consequent says that the speaker declares Ace-High the winning hand. And the entire sentence says that if the former (meta) speech act obtains, so does the latter speech act. The other card game rules are interpreted similarly.

5.2.4 Back to negation

If the superlative quantifier in the antecedent of a conditional is negated, the judgments are reversed: if the consequent is "good" the sentence is bad, whereas if the consequent is "bad", the sentence is good:

(135) a. If you don't click at least twice, the system won't respond to your request.
b. #If you don't click at least twice, you will get a discount.

The reason is simple. Under the evaluative interpretation, clicking at least twice requires a "good" consequence; hence *not* clicking at least twice requires a bad consequence.

Indeed, we observe the same behavior with unambiguously evaluative uses of *at least*:

(136) a. If this hotel isn't at least on Walnut St., I will go elsewhere.
b. #If this hotel isn't at least on Walnut St., I will take it.

We have seen that since permissions are "good" and obligations are "bad", superlative quantifiers are fine with the former but odd with the latter. When the superlative quantifiers are negated, these judgments are reversed: they are bad with permissions, but good with obligations.

(137) a. If Betty didn't have at least 3 martinis, she should be barred from our club.
b. #If Betty didn't have at least 3 martinis, she can drive.
c. Everybody who didn't drink at least 3 martinis should be barred from our club.
d. #Everybody who didn't drink at least 3 martinis can drive.
e. If you don't have at least \$50, you should go to the bank to get more.
f. #If you don't make at least \$500 a month, you may apply for a stipend.

The judgments are reversed with epistemic modals too, but for a different reason. We have seen that epistemic modals are fine with superlative quantifiers, because they naturally express a connection between two speech acts. However, if the superlative quantifier in the antecedent is negated, the result is bad:

(138) #If Betty didn't have at least three martinis, she must/may have been sober.

The reason is that (138), unlike (130) and (131), does not convey any natural connection between two speech acts.

Recall the discussion of (89b), repeated below:

(139) ?Betty didn't have at least three martinis.

As we have seen above, a negated superlative quantifier is a rather unclear statement. The antecedent of the conditional, (139), only says that the speaker GRANTs the statement that Betty had n martinis for some $n < 3$; but there may be other values of $n < 3$ for which she does *not* GRANT this, while there may be values of $n \geq 3$ for which she *does*. Hence, there is no clear connection between this statement and the speaker's level of confidence that Betty was sober.

As we have seen, (130) and (131) cannot be helped by the evaluative reading, since there is nothing inherently "good" or "bad" about epistemic modals. In fact, one may take this point even further. Why can't (139), on its own, receive the evaluative interpretation, and, consequently, be acceptable?

It turns out that, in general, the evaluative interpretation is bad in the scope of negation, regardless of whether the predicated property is "good" or "bad":

(140) a. ??This hotel isn't at least centrally located.
b. ??This hotel isn't at least far away.

We are not sure what the reason for this behavior is.

5.3 Propositional attitudes

Propositional attitudes can, of course, allow for recursive embedding:

(141) Annabelle believes that Matthew suspects that Darcy wants to kill Guy.

Can speech acts also be embedded under propositional attitudes?

They ought to be, if speech acts are sufficiently similar to propositional attitudes, i.e. if speaking is sufficiently similar to thinking (Krifka to appear-a). According to some, this is indeed the case: the only difference is that the speaker uses a natural language, whereas the thinker uses a *language of thought* (Fodor 1975).

We will not settle the language of thought issue here, nor do we need to: superlative quantifiers are not regular speech acts, but rather meta-speech acts. Meta-speech acts express willingness or unwillingness to make certain speech acts, hence they are quite similar to propositional attitudes (like desire). Since propositional attitudes can be embedded under other propositional attitudes, if superlative quantifiers are some sort of propositional attitude, they are also predicted to be embeddable under them.

This prediction is, indeed, borne out:

(142) Mary thinks that John petted at least three rabbits.

Note that (142) is, in fact, ambiguous, between two readings that can be paraphrased as (143a) and (143b).

(143) a. Mary thinks: "John petted n rabbits," for some number n, and the speaker says that n is at least three.
b. Mary thinks: "John petted at least three rabbits."

The two readings are distinct: only (143b) implicates that Mary is not sure how many rabbits John petted. How can this ambiguity be accounted for?

Note that (144), with a comparative quantifier, is similarly ambiguous.

(144) Mary thinks that John petted more than two rabbits.

This ambiguity can straightforwardly be accounted for as involving the relative scopes of the attitude verb and the comparative quantifier.[46] This suggests that the ambiguity of (142) is also a scope ambiguity.[47]

Specifically, in our system, (143a) means that the minimal n s.t. the speaker GRANTs that Mary thinks that John petted exactly n rabbits is 3. Note that under this reading, the propositional attitude is inside the scope of the illocutionary operator. But (143b), which is, in fact, much more prominent, means that the minimal n s.t. *Mary* GRANTs that John petted exactly n rabbits is 3. Now, the illocutionary operator is inside the scope of the propositional attitude operator.

According to Schlenker (2002), propositional attitudes quantify not over possible worlds, but over contexts. Contexts determine a possible world, but also a speaker and addressee. Non-shiftable indexicals, such as the first person pronoun in English, can only refer to the speaker in the context of utterance. But shiftable indexicals, such as the first person pronoun in Amharic, may refer to the speaker of the embedded context. This is why (145) can only mean that John says that the speaker is a hero, while its counterpart in Amharic can mean that John says that John is a hero.

(145) John says that I am a hero.

Although first person pronouns do not shift in English, speaker-oriented expressions do seem to shift. Moreover, they appear to shift obligatorily. For example, (146a) means that the speaker considers the fact that it stopped raining lucky; but when embedded in (146b), the sentence can only mean that *Mary* considers it lucky.

(146) a. Luckily it stopped raining.
b. Mary thought that luckily it had stopped raining.[48]

Recall that a meta-speech act, like other speech acts, has two indices, in addition to the propositional content: the speaker and addressee. Since if speech-acts are clearly speaker oriented, if anything is, it is attractive to propose that the variable s, indicating the speaker, is an obligatorily shiftable indexical. Treating the speaker variable of a

[46] Note that *think* is a bridge verb, allowing for wh-movement from its complement, and hence for wide-scope construal of scope-bearing elements.

[47] Superlative quantifiers, although illocutionary operators, are still syntactically quantifiers, hence subject to a scoping mechanism. We remain agnostic on the nature of this mechanism—QR, *quantifying in*, or whatever.

[48] See Coniglio (2011) for such shifts of speaker-oriented expressions in German. A possible exception to this generalization may be predicates of taste, which have been claimed to shift only optionally; but it is quite controversial whether predicates of taste are indeed speaker-oriented—see, e.g., Cohen (2010b), and other papers presented at that workshop.

speech act as a shiftable indexical, and ignoring the addressee, we can formulate[49] the respective logical forms of readings (143a) and (143b) as follows (where c_0 is the context of utterance):

(147) a. $c_0 : \forall n(\text{GRANT}_{\text{Sp}(c_0)}(\mathbf{think}_{c_1}(\mathbf{m}, (|\mathbf{rabbits} \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| = n))) \rightarrow n \geq 3)$

b. $c_0 : \mathbf{think}_{c_1}(\mathbf{m}, \forall n(\text{GRANT}_{\text{Sp}(c_1)}(|\mathbf{rabbits} \cap \lambda x.\mathbf{pet}(\mathbf{j}, x)| = n) \rightarrow n \geq 3))$

Since c_1 is the context of Mary's thought, both "speaker" and "addressee" of c_1 are Mary herself, hence (147b) means that Mary thinks to herself that John petted at least three rabbits. This is the intended interpretation.

5.4 Requirements

Sentence (148) is ambiguous between (149a) and (149b).

(148) John needs at least three martinis.

(149)a. The minimal n s.t. the speaker GRANTs than John needs exactly n martins is $n = 3$.

b. The minimal number n of martinis that satisfies John's needs is $n = 3$.

According to (149a), John needs some number of martinis, and the speaker is not sure how many, but is sure it is no less than 3. Reading (149b), which is probably more prominent, says that the minimal needs of John are satisfied with 3 martinis (though more might be better). Büring (2007) refers to the readings exemplified by (149a) and (149b) as the *speaker insecurity* and *authoritative* readings, respectively.

Geurts and Nouwen account for the speaker insecurity reading in a straightforward, compositional way. In order to get the more prominent authoritative reading, they propose a mechanism of modal concord: the epistemic modal of *at least* becomes deontic as a consequence of the explicit requirement indicated by *need*.

Nouwen (2010) has a different solution. Like Geurts and Nouwen (2007), he believes that superlative quantifiers are modal operators; but, for him, the epistemic modal is not part of their lexical meaning, but is introduced as a silent epistemic modal operator. When the superlative quantifier is in the scope of a verb like *need*, this silent operator is not introduced, and no epistemicity comes into the picture. However, the price he has to pay is high: "Underlying this analysis is the assumption that there exist silent modal operators. I can offer no independent evidence for this assumption" (p. 17).

In contrast, we account for the two readings of (148) as a straightforward case of scope ambiguity, without the need to posit any additional device. When GRANT has wide scope, the value of the speaker variable comes from the context of utterance, and we get the reading where the speaker is not certain; when GRANT in the scope of **need**, the value of its speaker variable is the *needer*, and we get the reading where the speaker *is* certain. The corresponding formulations are as follows:

[49] Here, and in the following section, we will only use intuitive formulations, leaving complete formalizations in terms of changes of commitment spaces for another occasion.

(150) a. $c_0 : \forall n(\text{GRANT}_{\text{Sp}(c_0)}(\textbf{need}_{c_1}(\textbf{j}, |M \cap \lambda x.\textbf{have}(\textbf{j}, x)| = n)) \rightarrow n \geq 3)$ (speaker insecurity).

b. $c_0 : \textbf{need}_{c_1}(\textbf{j}, \forall n(\text{GRANT}_{\text{Sp}(c_1)}(|M \cap \lambda x.\textbf{have}(\textbf{j}, x)| = n) \rightarrow n \geq 3))$ (authoritative).[50]

Note that in order to obtain the authoritative readings, we are treating the verb *need* as a sort of meta-speech act. Specifically, the *needer* must be an entity that is capable of GRANTing (typically a human). Indeed, with an inanimate *needer*, this reading is not available:

(151) The project needs at least three years to finish.

Sentence (151) can only get the speaker insecurity reading, namely that the speaker denies that the project will be completed in less than three years (but is not sure how long it will actually take). Crucially, it does not receive the authoritative reading, where three years will definitely be enough to provide whatever is necessary for the project to finish (though more time may perhaps be better).

The human argument of *need* does not have to be explicit. The following example, suggested to us by an anonymous reviewer, does seem to have an authoritative readings:

(152) (The safety guide clearly states that) the walls of a nuclear plant are required to be made out of at least three layers of concrete.

Sentence (152) can mean that three layers of concrete are definitely enough (though more may be better). But note that without the material in parenthesis being available (at least contextually), the sentence would not be good. The reason is that it is not really the walls that need three layers of concrete: it is the author or follower of the guidebook, who needs three layers of concrete in order to approve the plant, declare it safe, or whatever.

The other superlative quantifier, *at most*, is treated similarly. Consider the following example:

(153) John is required to have at most three martinis.

Sentences (153) is ambiguous: under the speaker insecurity reading, it says that there is some number of martinis that John must drink (in order to qualify as a member of the gang, or whatever), but the speaker is not sure what this number is (except that it is not greater than three). Under the authoritative reading, three is the upper limit on the number of martinis John is allowed to drink (though possibly fewer martinis would be even better). The two readings can, as before, be expressed as a scope ambiguity:

[50] Recall that (7b), repeated below, only allows the reading where the superlative quantifier scopes over the modal.

(i) You may have at most two martinis.

The sentence therefore has the *form* of speaker insecurity, but not the interpretation. The reason is that whereas explicitly refraining from denying something is not tantamount to asserting it, explicitly refraining from prohibiting something *is* tantamount to permitting it. With a non-deontic modal, as in (ii) (suggested to us by an anonymous reviewer) we readily get a speaker insecurity reading.

(ii) He can do at most 20 pushups.

(154) a. $c_0 : \forall n(\mathrm{GRANT}_{\mathrm{Sp}(c_0)}(\mathbf{need}_{c_1}(\mathbf{j}, |M \cap \lambda x.\mathbf{have}(\mathbf{j}, x)| = n)) \rightarrow n \leq 3)$ (speaker insecurity).

b. $c_0 : \mathbf{need}_{c_1}(\mathbf{j}, \forall n(\mathrm{GRANT}_{\mathrm{Sp}(c_1)}(|M \cap \lambda x.\mathbf{have}(\mathbf{j}, x)| = n) \rightarrow n \leq 3))$ (authoritative).

Of course, the formulations in (150) and (154) above are not complete, so long as we do not provide an analysis of the relation **need**. In fact, we argue that, together with such an appropriate definition, our approach provides the way to solving a difficult puzzle involving the meaning of expressions of requirements. Consider the following sentence:

(155) John needs to have a drink in order to go to sleep.

On the face of it, this sentence expresses a necessary and sufficient condition for John's going to sleep: if he has a drink he will fall asleep, and if he doesn't—he won't. But von Fintel and Iatridou's (2005) point out that the condition, although necessary, is not sufficient: in order to go to sleep John also needs not to be working, to have a place to sleep on, to breathe, etc.

There are, however, two problems with the view that sentences like (148) express only necessary conditions. One problem is that every entailment of a necessary condition is also a necessary condition: for example, having a drink entails being alive. Now, although, upon reflection, most people will agree that John needs to be alive in order to sleep, (156) certainly does not sound very natural (cf. von Stechow et al. 2005).

(156) John needs to be alive in order to go to sleep.

Nouwen (2009) points out another problem, which is the following. If requirements are necessary conditions, (157a) would entail (157b), which would entail (157c). From these, (158) would follow, but, again, this sounds wrong.

(157) a. John needs three martinis to fall asleep.

b. John needs two martinis to fall asleep.

c. John needs one martini to fall asleep.

(158) The minimal number of martinis that John needs to fall asleep is 1.

If a sentence is true yet sounds funny, this is often indicative of an ambiguity: under one reading it is true, but under another, it is odd. Perhaps this is the case with (156); this suggests an alternative way to interpret von Fintel and Iatridou's observation.

We maintain that requirements are, in fact, necessary and sufficient conditions. But we suggest that statements of requirements contain an implicit *at least*. Verbs like *need*[51] subcategorize for a nominal expression (DP) that expresses a minimal requirement that can be made explicit with *at least*.

Some evidence for this comes from examples such as (159a), which can only mean that the number n such that you need at least n good deeds to go to heaven is $n \leq 3$. In contrast, (159b) has a perverse reading saying that if you did more than three good deeds you won't go to heaven.

[51] We leave open the question of which, exactly, these verbs are.

(159) a. You need at most three good deeds to go to Heaven.
b. You need to have done at most three good deeds to go to Heaven.

Additional evidence comes from Musolino (2004). He found that usually children give number words an "exactly" interpretation. For example, in a situation in which a troll figure places four hoops on a pole, children were asked the following question:

(160) Goofy said that the troll had to put two hoops on the pole in order to win the coin. Does the troll win the coin?

The response was overwhelmingly negative, indicating that children treat Goofy's statement as requiring *exactly* two hoops; therefore, by putting four hoops on the pole, the troll failed to satisfy the requirement.

Interestingly, Musolino obtained different results in his second experiment. In this experiment, children were presented with a situation where, for example, Goofy had four cookies, and were asked the following question:

(161) Let's see if Goofy can help the troll. The troll needs two cookies. Does Goofy have two cookies?

Now, the majority of children responded in the affirmative. This is easily explained if we assume that children derive an "exactly" interpretation for bare number words, but that the verb *need* (and probably other verbs of requirement) contains an implicit *at least* as part of its lexical entry.[52]

Assuming an implicit *at least* in the representation of requirements, the result is ambiguous, depending on whether this superlative quantifier takes scope above or below the modal. Thus, (157a) actually means (162), which is ambiguous between (163a) and (163b), with the latter being clearly the dominant reading.

(162) John needs at least three martinis (to fall sleep).

(163) a. The set of necessary and sufficient conditions for John's falling asleep includes at least the condition of having three martinis (i.e. the conjunction of the necessary and sufficient conditions entails having three martinis).
b. The necessary and sufficient condition for John's falling asleep is that he has at least three martinis

This ambiguity falls out of the formulations in (150), if the order '≥' is generalized to entailment, as discussed in Sect. 3.6. We formulate (163a) and (163b), respectively, as (164a) and (164b) (ignoring issues of modality and the context variables):

(164) a. $\forall P(\text{GRANT}(\mathbf{sleep(j)} \leftrightarrow P) \rightarrow P \geq {}^\wedge|M \cap \lambda x.\mathbf{have}(\mathbf{j}, x)| \geq 3)$
b. $\mathbf{sleep}(j) \leftrightarrow \forall P(\text{GRANT}(P) \rightarrow P \geq {}^\wedge|M \cap \lambda x.\mathbf{have}(\mathbf{j}, x)| \geq 3)$

[52] Musolino's own explanation is that children obtained an "exactly" interpretation in (160) because "in the context of a game, children would be pickier about keeping an accurate record of what gets scored as opposed to what is missed" (pp. 21–22). But this is implausible, since Papafragou and Musolino (2003) have already found "exactly" interpretations in situations that do not involve games. For example, (i) was judged false in a situation in which three horses jumped over the fence.

(i) Two of the horses jumped over the fence.

The formulation in (164a) means that if it is GRANTed that P is a necessary and sufficient condition for John's sleeping, then P is greater than or equal to (i.e., entails) the condition that John have three martinis or more. In other words, the speaker knows that the set of necessary and sufficient conditions for John's falling asleep includes at least having three martinis.

The formulation in (164b) means that it is a necessary and sufficient condition for John's falling asleep that any GRANTed statement (contextually restricted to statements about the number of martinis John has) is that John had a number of martinis that is greater than or equal to 3.

Note that (164a) entails (156): if a necessary and sufficient condition for John's falling asleep entails having three martinis or more, and having three martinis or more entails breathing, then a necessary and sufficient condition for John's falling asleep entails breathing. However, (164b) does not entail (156): if having at least three martinis is a necessary and sufficient condition for John's falling asleep, it does not follow that breathing is also a necessary and sufficient condition for John's falling asleep. We believe this is why most people would agree that (156) follows, but feel uncomfortable with this conclusion.

Similarly, (157b) only follows from (164a), but not (164b). It follows from (164a), for if the necessary and sufficient condition for John's falling asleep entails having three martinis or more, it entails having two martinis or more. But (157b) does not follow under reading (164b), for if having at least three martinis is necessary and sufficient for falling asleep, it does not follow that having at least two martinis is necessary and sufficient for John's falling asleep.

Therefore, (158) only follows under the following reading: the minimal number n s.t. the set of necessary and sufficient conditions for John's falling asleep entails having n martinis is 1. But since this reading is extremely awkward, most people would refuse to accept this inference.

An anonymous reviewer drew our attention to the important fact that the same type of ambiguity between speaker insecurity and authoritative readings obtains not just with modal quantifiers such as requirements, but with nominal quantifiers as well. Consider the following:

(165) Every kid petted at least three rabbits.

This sentence is ambiguous in the same way. The speaker insecurity reading is rather implausible, for according to it there is some number of rabbits n such that each kid petted n rabbits, but the speaker is not sure what n is, except that it is no less than three. The much more plausible reading is that every kid petted no fewer than three rabbits. Again, the two readings can be expressed as a scope ambiguity:

(166) a. $\forall n(\text{GRANT}(\forall x(\mathbf{kid}(x) \rightarrow |R \cap \lambda y.\mathbf{pet}(x, y)| = n)) \rightarrow n \geq 3)$ (speaker insecurity).

b. $\forall x(\mathbf{kid}(x) \rightarrow \forall n(\text{GRANT}(|R \cap \lambda y.\mathbf{pet}(x, y)| = n) \rightarrow n \geq 3))$ (authoritative).

6 Conclusions and implications

In this paper we have argued that superlative quantifiers are quantifiers over meta-speech acts, and developed a framework for modeling speech acts and meta-speech acts. In this framework, GRANTing a proposition is the denegation of asserting its negation. This framework also provides a natural analysis of conjunction and, to some extent, disjunction of speech acts.

We have proposed that *at least 3* ϕ means that the minimal number n s.t. the speaker GRANTs $\phi(n)$ is 3. And *at most 3* ϕ means that the maximal number n s.t. the speaker GRANTs $\phi(n)$ is 3.

Furthermore, we have argued that the falsity of a superlative quantifier is determined semantically, but its truth is determined pragmatically, via scalar implicature. Hence, superlative quantifiers can be embedded in environments where scalar implicature survives.

We have shown how this theory explains the facts concerning the distribution and interpretation of superlative quantifiers better than competing approaches, while maintaining correct, objective truth conditions.

If this account is on the right track, it has significant implications that go beyond an account of a particular linguistic phenomenon. We have argued that interlocutors often express meta-speech acts: they are not moves in the conversation, but indicate which moves are possible. Moreover, we have demonstrated that speech acts (including meta-speech acts) can be modeled as changes of commitment spaces. Thus they become semantic objects, and hence part of semantic recursion, although they do not have truth conditions. Therefore they can be embedded, provided their embedding is interpretable.

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